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## Yield behavior of sand under generalized stress conditions

### L'effet du sable sur les conditions de stress général

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#### ABSTRACT

The yield behavior of Leighton Buzzard sand was studied for different magnitudes of the three principal stresses and different orientations of the major principal stress using a hollow cylinder apparatus. Stress paths were followed that allowed well defined portions of different yield surfaces to be determined under generalized stress conditions. The Matsuoka-Nakai and Lade yield criteria were both found to adequately predict the onset of yielding under generalized stress conditions. The experimental data indicated that the onset of yielding as predicted by both yield criteria was independent of the orientation of the major principal stress and magnitude of the intermediate principal stress.

#### RÉSUMÉ

L'influence sur le comportement du sable de Leighton Buzzard a été étudié à différentes échelles sur les trios principaux types de contrainte et le retour périodique de contrainte principal. On a suivi des chemins de contrainte qui ont permis aux parties bien définies de différentes surfaces de rendement d'être déterminées dans des conditions généralisées de contrainte. Les critères d'influences de Matsuoka-Nakai et Lade ont toutes deux conduites à prédire clairement le début de l'influence sur les conditions générales de contrainte. Le début de l'influence comme prévu par l'ensemble des critères se trouve être indépendant du renouvellement et la cause principal de contrainte.

#### 1 INTRODUCTION

The principal stresses in the ground generally rotate as the foundation load is applied for most geotechnical engineering problems. Figure 1 presents the stress states at five different points (labeled A to E) along a potential failure surface in a soil foundation. Since most sedimentary deposits are inherently anisotropic, the soil deformational response depends on both the magnitudes and the directions of the principal stresses.

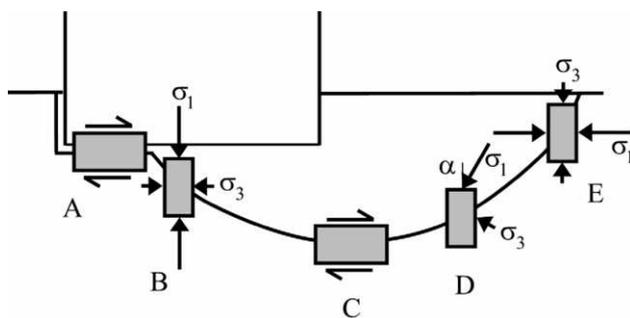


Figure 1. Stress states along potential failure surface in soil foundation.

The soil elements at points A and C are in a state of simple shear. At points B and E, the stress state is such that the major and minor principal stresses,  $\sigma_1$  and  $\sigma_3$ , respectively, act in the vertical and horizontal directions. At point D, the major principal stress has reoriented at an angle  $\alpha$ , to the vertical direction. For this example, the intermediate principal stress  $\sigma_2$ , acts into the page for all cases. The magnitude of the intermediate principal stress can be related

to that of the major and minor principal stresses using the intermediate principal stress parameter  $b$ , with  $0 \leq b \leq 1$ .

The stress states at points B and E can be simulated using the triaxial apparatus assuming  $\sigma_2 = \sigma_3$  and  $\sigma_1 = \sigma_2$  at the respective points. Test devices such as the hollow cylinder apparatus are required to apply more generalized stress states, such that  $0^\circ \leq \alpha \leq 90^\circ$  and  $\sigma_1 \geq \sigma_2 \geq \sigma_3$ .

In this study, the stress states at points B, D and E in Fig. 1 were replicated in sand specimens using a hollow cylinder apparatus. The yield behavior of the sand was studied for different orientations of the major principal stress and different magnitudes of the  $b$  parameter. The experimental yield data was compared with values predicted by the Matsuoka-Nakai and Lade yield criteria to assess the accuracy of these criteria in predicting the onset of yielding in sand under generalized stress conditions.

#### 2 YIELD CRITERIA

The Matsuoka-Nakai yield criteria (Matsuoka and Nakai, 1985), which is a theoretical development of the concept of compoundedly mobilized planes and spatially mobilized planes, can be expressed in terms of the effective stress invariants, equation (1). Satake (1982) suggested that the Matsuoka-Nakai yield criteria is the statistically modified form of the Mohr-Coulomb yield criteria.

$$\frac{J_1 J_2}{J_3} = \text{Matsuoka-Nakai constant} \quad (1)$$

where  $J_1$ ,  $J_2$  and  $J_3$  = the first, second and third effective stress invariants.

The Lade yield criteria (Lade and Duncan, 1975), which was developed from analysis of cubical triaxial test data, is best expressed in terms of the first and third effective stress

invariants, equation (2). This yield criteria was developed assuming that the soil behaves isotropically which implies that the yield response is independent of the reorientation of the principal stress axes.

$$\frac{J_1^3}{J_3} = \text{Lade constant} \quad (2)$$

Pradel et al. (1990) compared the Matsuoka-Nakai, Lade and Mohr-Coulomb yield criteria in terms of normalized shear and normalized stress difference (Fig. 2). In general, all three yield criteria appear similar, with the Mohr-Coulomb criteria predicting the onset of yielding at the lowest stress level and the Lade criteria predicting the onset of yielding at the highest stress level. The Matsuoka-Nakai yield criteria appear to lie between the Mohr-Coulomb and Lade criteria. For  $\alpha < 45^\circ$  the Lade yield prediction is closer to the Mohr-Coulomb prediction whereas for  $\alpha > 45^\circ$  the Matsuoka-Nakai yield prediction is closer to the Mohr-Coulomb prediction.

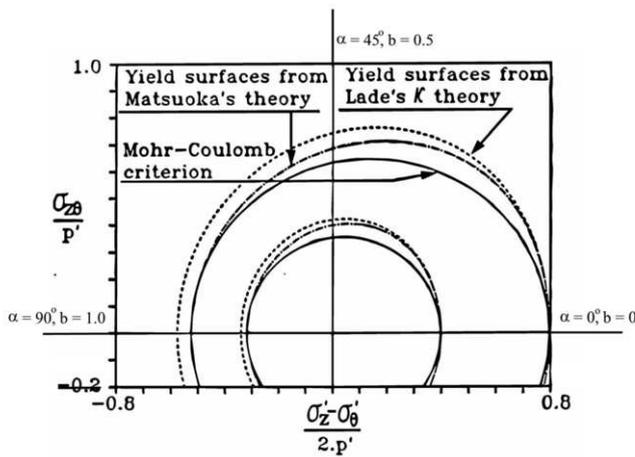


Figure 2. Comparison of Matsuoka-Nakai, Lade and Mohr-Coulomb yield criteria (after Pradel et al., 1990).

### 3 THE HOLLOW CYLINDER APPARATUS

The University College Dublin hollow cylinder apparatus (O'Kelly and Naughton, 2003) was used to apply axial and torsional loads ( $M$  and  $T$ ), and independently controlled confining pressures ( $p_o$  and  $p_i$ ) to the hollow cylindrical test specimens, 100mm outer diameter by 71mm inner diameter by 200mm long (Fig. 3a). The apparatus was closed-loop controlled allowing precise regulation of the stress components  $\sigma_z$ ,  $\sigma_r$ ,  $\sigma_\theta$ ,  $\tau_{z\theta}$  induced in an element of the sample wall (Fig. 3b) to  $\pm 0.5$ kPa. The radial confining stress ( $\sigma_r$ ) equals the intermediate principal stress. The application of the torque causes rotation  $\alpha$ , of the major and minor principal stresses (Fig. 3c). The strain response ( $\epsilon_z$ ,  $\epsilon_r$ ,  $\epsilon_\theta$ ,  $\gamma_{z\theta}$ ) of the test specimen was recorded using internal instrumentation which recorded the deformation response to a resolution of better than  $5 \times 10^{-5}$  strain. The apparatus automatically compensated for membrane penetration effects using the method developed by Sivathayalan and Vaid (1998) and for membrane restraint effects using corrections developed by Tatsuoka et al. (1986).

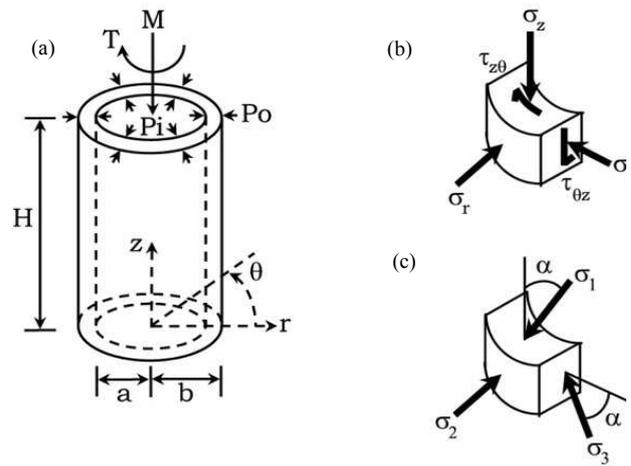


Figure 3. Stress state in hollow cylindrical test specimen.

### 4 PROPERTIES OF TEST SAND AND SAMPLE PREPARATION

Well graded, fine-to-medium Leighton Buzzard sand was used in this study (Table 1). Hollow cylindrical specimens of the test sand were prepared using the a wet-pluviation technique. Pluviation has the added benefit in that it replicates the sedimentation process and hence the anisotropic fabric of many natural sand deposits.

The sand specimens were formed in a single layer and densified by tapping around the sides of the sample preparation moulds with the aim of producing identical samples. The actual relative density of the test specimens ranged from 72% to 78%. Naughton and O'Kelly (2003) showed that this level of variation did not significantly affect the constitutive response of the test specimens for isotropic loading conditions.

Table 1: Physical properties of test sand.

Coefficient of uniformity	Coefficient of curvature	Specific gravity	$D_{50}$ (mm)	Max void ratio	Min void ratio
1.32	0.96	2.64	0.52	0.77	0.50

### 5 EXPERIMENTAL TEST PROGRAM

The stress paths were developed using the methodology of Tatsuoka and Ishihara (1974) to identify portions of yield surfaces for the test sand in generalized stress space. After establishing an initial point on a yield surface, the stress path test would search for other points on that yield surface for the same test specimen. In this study segments of two different yield surfaces, corresponding to different stress levels and fabrics, were determined for a single test specimen. In total, stress path tests that involved different rotations of the major-to-minor principal stress axis were conducted on eight identically prepared test specimens. The stress paths consisted of four stages, described below, with a typical stress path (plotted in terms of the effective major-to-minor principal stress ratio  $R'$ , versus the  $b$  parameter) shown in Fig. 4.

- (i) The test specimen was isotropically consolidated to a mean effective stress of 100kPa to ensure that it was in a normally consolidated condition, point A, Fig. 4.
- (ii) The specimen was then anisotropically consolidated which included rotation of the major principal stress to  $\alpha = 0^\circ, 30^\circ, 60^\circ$ , or  $90^\circ$ , and with the  $b$  parameter value

initially set equal to zero or 0.5. The mean effective stress and the orientation of the major principal stress were then held constant for the remainder of the stress path test.

- (iii) An initial point on a yield surface (point B) was established by increasing the effective stress ratio  $R'$ , until plastic straining of the specimen commenced. The initial point on the yield surface was established at  $R' = 2.1$  for each specimen in this test series.
- (iv) Once the initial yield point had been established, the  $R'$  value was reduced (point C) and the specimen was reconsolidated in order to increase the magnitude of the  $b$  parameter by 0.25 (point D). The effective stress ratio  $R'$ , was increased again to search for another point on the yield surface with the magnitude of the  $b$  parameter held constant (point E). A second yield surface, corresponding to a different stress state, was also determined using this specimen. Two points were identified on a particular yield surface in this manner.

After establishing the initial point, on the second yield surface (point F), the value of  $R'$  was reduced (point G) and the specimen reconsolidated to a higher value of the  $b$  parameter (point H). The magnitude of  $R'$  was increased again in order to locate another point on the second yield surface (point K). The stress path started from  $b = 0$  or  $b = 0.5$ , and the value of the  $b$  parameter was incremented in steps of 0.25. Segments of two yield surfaces were identified from a single specimen using this technique.

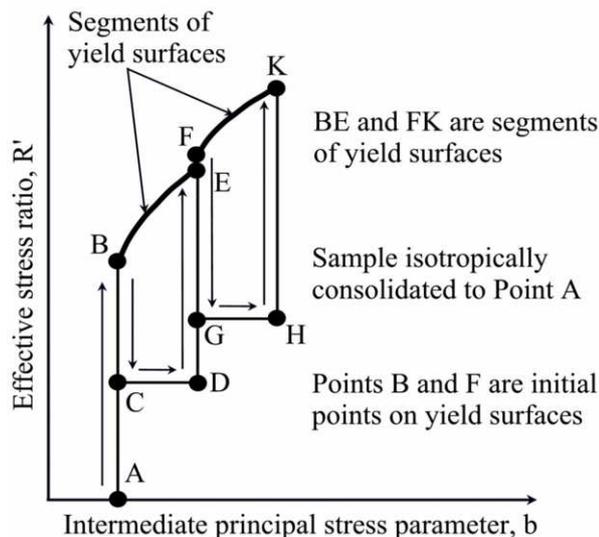


Figure 4. Stress paths used to identify parts of yield surface.

The location of a point on a yield surface was defined by a marked change in the stiffness response which was established using a graphical analysis of the experimental stress-strain curves. The idealized linear-elastic and plastic portions of the stress-strain curves were extended with the point of intersection defining one point on the yield surface. Joining other yield points (determined for higher values of the  $b$  parameter using the same technique) back to their initial

yield point identified different portions of yield surfaces in generalized stress space.

Care was taken when reconsolidating the test specimen to limit the development of plastic deformations between loading stages. Significant plastic deformations would increase the size of the transition zone between idealized linear-elastic and plastic behavior thus making it more difficult to accurately define the value of the yield point.

The value of the yield point for each stress probe was determined from a minimum of four stress-strain curves. Considerable scatter was observed in the yield point determined using the graphical technique, with the coefficient of variation for  $R'$  at the yield point varying from 0.2 % to 10.9 %.

## 6 EXPERIMENTAL TEST RESULTS

Figures 5 and 6 present the experimental yield surfaces plotted in terms of the Matsuoka-Nakai and Lade yield constants versus the  $b$  parameter for different orientations of the major principal stress. Both sets of experimental are normalized by the initial value of each yield criteria on the yield surface to facilitate a direct comparison of the experimental and theoretical yield surfaces defined using both yield criteria.

## 7 DISCUSSION

Different portions of yield surfaces in generalized stress space were identified for Leighton Buzzard sand by following stress paths which rotated the major principal stress by different amounts during the initial consolidation stage of the tests. The experimental yield data indicated that both the Matsuoka-Nakai and Lade yield criteria can be used to adequately predict the onset of yielding for generalized stress conditions.

When expressed in terms of the Matsuoka-Nakai and Lade yield criteria, the shapes of the experimental yield surfaces appear to be largely independent of the rotation of the major principal stress at different magnitudes of the  $b$  parameter. The experimental yield criteria follow the same general pattern relative to the theoretical yield surfaces, slightly underestimating the onset of yielding for  $0 \leq b \leq 0.5$ , and slightly overestimating the onset of yielding for  $0.5 < b \leq 1$ .

## 8 CONCLUSIONS

The experimental techniques allowed well-defined portions of yield surfaces for Leighton Buzzard sand to be identified in generalized stress space. The values of the yield points were successfully determined from a number of stress-strain curves for each stress probe.

The Matsuoka-Nakai and Lade yield criteria were both found to adequately predict the onset of yielding for Leighton Buzzard sand under generalized stress conditions which were achieved using a hollow cylinder apparatus.

The experimental yield surfaces indicated that yield predictions by both yield criteria are predominately independent of the rotation of the major principal stress during the initial consolidation stage.

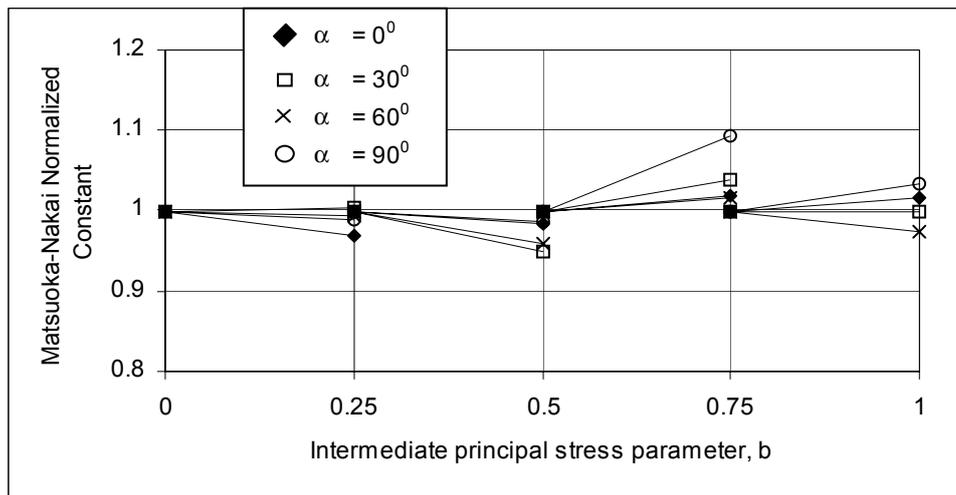


Figure 5. Experimental and theoretical yield surfaces expressed in terms of the Matsuoka-Nakai yield criteria.

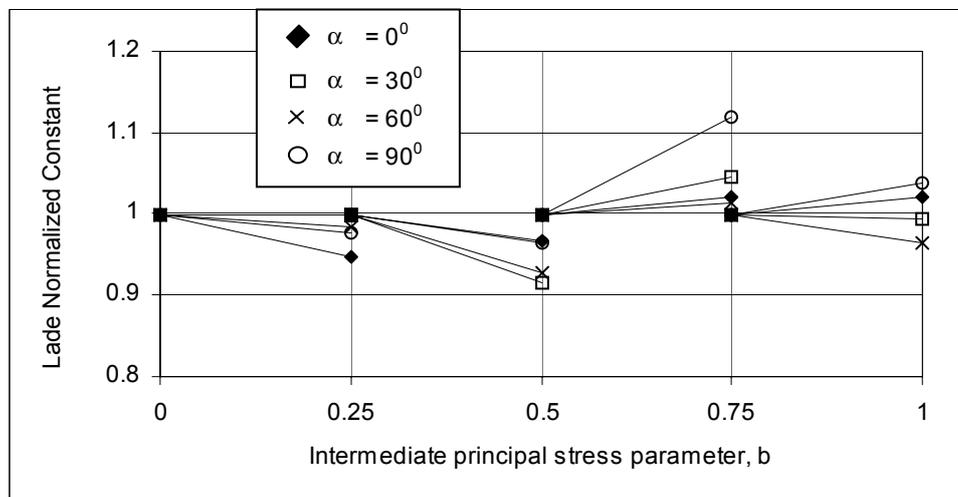


Figure 6. Experimental and theoretical yield surfaces expressed in terms of the Lade yield criteria.

#### ACKNOWLEDGEMENTS

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