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State limits and attractors of psammoids and peloids

D'états aux limites et attracteurs de péloides et psammoides

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ABSTRACT
Simplified hypoplastic relations are briefly outlined for simple shearing. Therein state limits are asymptotic solutions independently of the initial state, i.e. attractors, for certain strain paths. Elasto-hypoplastic relations are adequate for sand-like rate-independent soils (psammoids). Hypoelastic behaviour holds for cycles with small amplitudes. Visco-hypoplastic relations hold for clay-like rate-dependent soils (peloids). With easily determined parameters, initial and boundary conditions realistic predictions are obtained for a wide spectrum. Extensions are indicated.

RÉSUMÉ
Relations hypoplastiques simplifiées s'expliquent brièvement pour cisaillement simple. États aux limites sont solutions asymptotiques indépendamment d'état initial, i.e. attracteurs, pour certains chemins de déformations. Relations elasto-hypoplastiques sont adéquates pour des sols non-visqueux sableux (psammoides). Relations visco-hypoplastiques valent pour des sols argileux (péloides). Avec des paramètres aisément déterminés, et des conditions initiales et aux limites,prédictions réalistes sont obtenus pour une vaste gamme. Extensions sont indiquées.

1 ELASTO-HYPOPLASTICITY FOR PSAMMOIDS

Soils with rather hard grains are idealized as *psammoids* (Greek for sand-like). Consider simple shearing with γ, ε, τ, σ′ and rates $\dot{\gamma} = d\gamma/dt$ etc. (Fig. 1).

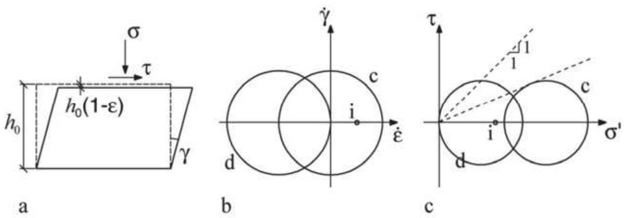

Fig. 1 Simple shearing (a), Mohr diagrams of strain rates (b) and stresses (c) for state limits

State limits are defined by relations of $\dot{\varepsilon}/\dot{\gamma}$ with τ/σ′ and *e* with log σ′ (Fig.2). They range from pure compression (i, γ = 0, τ = 0) via contractant shearing ($\dot{\varepsilon}|\dot{\gamma}|$ > 0, |τ| < σ′ tan φ_{sc}), pure shearing (c, $\dot{\varepsilon}$ = 0, |τ| = σ′ tan φ_{sc}), dilatant shearing ($\dot{\varepsilon}|\dot{\gamma}|$ < 0, 1 > |τ|/σ′ > tan φ_{sc}) to cracking (d, $\dot{\varepsilon}|\dot{\gamma}|$ = -1, |τ|/σ′ = 1). The critical friction angle φ_{sc} is constant. The limit void ratios depend on σ′ via (Bauer 1996)

$$e = e_0 \exp\left[-\left(\frac{3\sigma'}{h_s}\right)^n\right] \quad , \qquad (1)$$

wherein e_0 ranges from e_{i0} via e_{c0} (≈ e_{max} conventionally) to e_{d0} ≈ e_{min}, h_s from ca 0.2 to 20 GPa and *n* from ca 0.25 to 0.45. $e_{i0}/e_{c0}/e_{d0}$ is nearly constant.

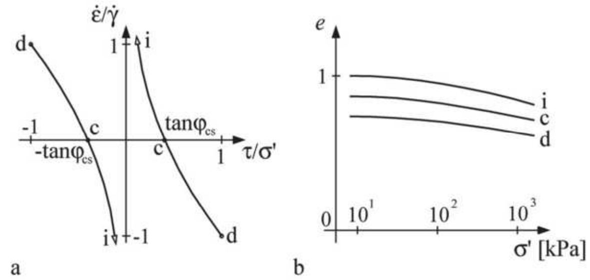

Fig. 2 Strain rate ratio vs. stress ratio (a) and void ratio vs. pressure (b) for state limits of psammoids

The simplified *hypoplastic* relations (Kolymbas 1991 and 2000, Gudehus 1996) are

$$\dot{\tau} = G\left(\dot{\gamma} - \dot{\gamma}_p\right) \ , \quad \dot{\sigma}' = K\left(\dot{\varepsilon} - \dot{\varepsilon}_p\right) \qquad (2)$$

with

$$K = m h_s \left(\sigma'/h_s\right)^{1-n} \ , \quad G = K\nu(1-\nu) \qquad (3)$$

and

$$\dot{\gamma}_p = \left(1 - I_d\right)^\alpha g_\gamma D \ , \quad \dot{\varepsilon}_p = \left(1 - I_d\right)^\alpha g_\varepsilon D \quad . \qquad (4)$$

Therein *m* depends on *e* as outlined below, $D = \sqrt{\dot{\gamma}^2 + \varepsilon^2}$ is the magnitude of strain rate, g_γ and g_ε depend on τ/σ′ as shown in Fig. 3, and the density index is

$$I_d = \frac{e_c - e}{e_c - e_d} \qquad (5)$$

with limit valves e_c and e_d by (1). ν ranges from ca 0.25 to 0.35, α from ca 0.15 to 0.30. g_ε/g_γ equals $\dot{\varepsilon}/\dot{\gamma}$ for state limits. $\dot{e} = -\dot{\varepsilon}(1+e)$ is taken for isochoric grains as usual.

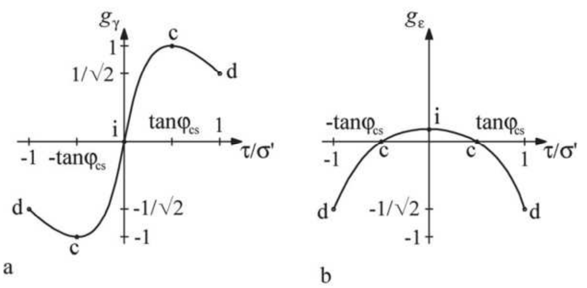

Fig. 3 Factors for plastic strain rates

Starting from arbitrary initial states τ, σ′, e within the limits monotonous strain paths lead to the state limits for the given $\dot{\varepsilon}/\dot{\gamma}$ (Fig. 4). Pure compression (A) leads to τ→ 0 and e_i, thus m in (3a) is determined. Pure shearing (B) leads to |τ| = σ′ tan φ_{sc} and e_c. Contractant shearing (C) leads to |τ| /σ′ = const < tan φ_{sc} and $e_c < e < e_i$. Dilatant shearing (D) with σ′ = const leads to a peak with 1 > |τ| /σ′ > tan φ_{sc} and $e_d < e < e_c$. Anomalous dilatant shearing (E) leads to cracking with |τ| = σ′. Thus the state limits are *attractors* of the constitutive relations.

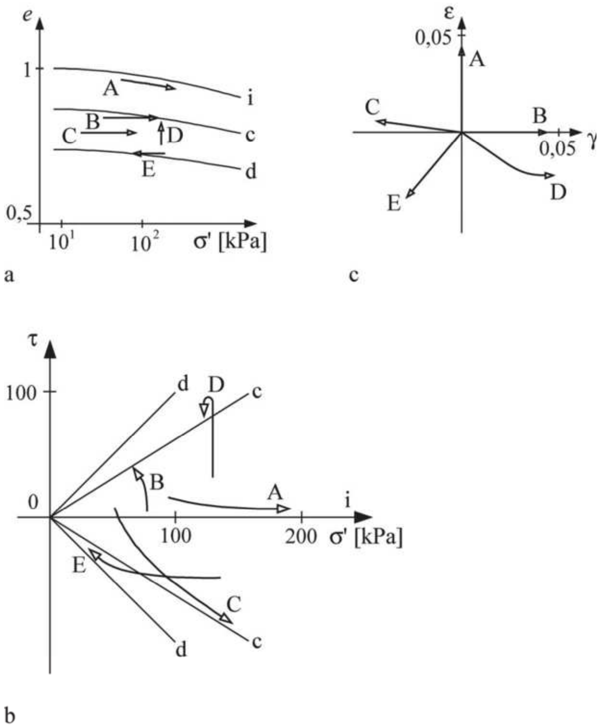

Fig. 4 Evolution of void ratios (a), stresses (b) and strains (c) towards state limits

The differential stiffnesses $\dot{\tau}/\dot{\gamma}$ and $\dot{\sigma}'/\dot{\varepsilon}$ are determined by τ, σ′, e and the sign of $\tau\dot{\gamma}$ and $\sigma'\dot{\varepsilon}$. They tend to minimal values for $\dot{\varepsilon} > 0$, $\dot{\gamma} = 0$ and $\dot{\varepsilon}/\dot{\gamma} > 0$, and to zero for $\dot{\gamma}$ = const for $\dot{\varepsilon} = 0$ and $\dot{\sigma}' = 0$. The latter case implies peak and dilatancy. After a reversal, i.e. $\tau\dot{\gamma} < 0$ and/or $\sigma'\dot{\varepsilon} < 0$, $\dot{\tau}/\dot{\gamma}$ and $\dot{\sigma}'/\dot{\gamma}$ are higher than before except for $I_d = 1$, and the contractancy ratio $\dot{\varepsilon}/\dot{\gamma}$ exceeds the previous dilatancy ratio $-\dot{\varepsilon}/\dot{\gamma}$ .Cyclic shearing with σ′ = const and | τ | < σ′ tan φ_{sc} thus leads to a densification up to e_d.

The *ratcheting*, i.e. the accumulation of γ for τ-cycles between zero and σ′ tan φ_{sc} with σ′ =const, or the reduction of σ′ by τ- or γ-cycles with e = const, is exaggerated with (2) to (5). This is avoided by the *intergranular strain*, here with components Δ_γ and Δ_ε. For $\rho = \sqrt{\Delta_\gamma^2 + \Delta_\varepsilon^2} = 0$, which represents a minimum spatial fluctuation of intergranular forces, $\dot{\gamma}_p = \dot{\varepsilon}_p = 0$ holds. The response by (2) and (3) is then *hypoelastic*. For ρ = R, ranging from ca 10^{-4} to 10^{-6} for fine- to coarse-grained soils, the fluctuation is maximal and the response is hypoplastic in case of $\tau\dot{\gamma} + \sigma'\dot{\varepsilon} > 0$ and $\dot{\gamma}\,\Delta_\gamma + \dot{\varepsilon}\,\Delta_\varepsilon > 0$. An interpolation is employed for intermediate cases with the two switch functions given above (Niemunis and Herle 1996). In this *elasto-hypoplastic* concept the intergranular strain is a genuine state variable with limits ρ = 0 and ρ = R.

Granular soils are characterized already by φ_{sc}, h_s and e_{c0}. This suffices to define geometrically simplified regions. The further parameters e_{i0}, e_{d0}, n, α and R can be estimated from grain properties and determined by element tests. For evaluation and further applications the constitutive relations are written with cylindrical and tensor components (e.g. v. Wolffersdorff 1997, Gudehus and Herle 1999). Explicit stiffness and strength values are not needed a priori, but derived.

The initial effective stress field is estimated with K_0 or a hypoplastic calculation for a geometrically simplified model including ground water conditions. I_d is determined by penetration sounding with hypoplasticity (Cudmani and Ossinev 2001). The initial intergranular strain field can be estimated from the recent past. The *evolution of state and position* is calculated with boundary conditions for technical and natural actions (Gudehus 2003). This has successfully been done for a wide spectrum of cases, viz.

- filling, excavation and ground improvement,
- placement, casting and penetration of structural parts,
- buildings with shallow and deep foundations,
- retaining structures with filling or excavation,
- cyclic and dynamic loading, e.g. by earthquakes.

State limits in the large can result, also with shear localization. This enables simplified assessments for design with ultimate or serviceability limit states. Observational methods are more powerful with elasto-hypoplastic predictions.

2 VISCO-HYPOPLASTICITY FOR PELOIDS

Saturated soils with fine soft particles are idealized as *peloids* (Greek for clay-like). *State limits* are defined for them again with $\dot{\varepsilon}/\dot{\gamma}$ vs. τ/σ′ and e vs. σ′ relations. Fig. 2a is used as for psammoids, φ_{cs} has a wider range from ca 10° for highly plastic to ca 55° for diatomaeceous clay. (1) is used again, but with higher e_0 and lower h_s ranging from ca 0.5 to 50 MPa. Thus e vs. log σ′ is nearly linear in the usual σ′-range (Fig. 5). h_s depends on $D = \sqrt{\dot{\gamma}^2 + \dot{\varepsilon}^2}$ by

$$h_s = h_{sr}\left(D/D_r\right)^{I_v} \tag{6}$$

with the *viscosity index* I_v ranging from ca 0.02 to 0.05 for lowly to highly plastic clays. The reference rate D_r is conveniently chosen as the $\dot{\varepsilon}$ at the end of primary lab compression.

The *equivalent pressure* can be approximated by

$$\sigma_e = \sigma_{ei} / [1+ (\tau/\sigma' \tan \varphi_{sc})^2] \qquad (7)$$

with σ_{ei} from (1) for e_{i0} and h_{sr}.

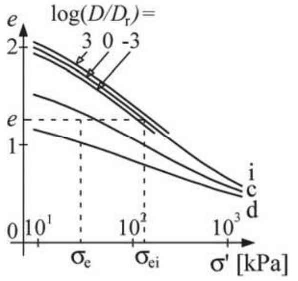

Fig. 5 Isotachs of void ratio vs. effective pressure for a peloid

The simplified *visco-hypoplastic* relations (Niemunis 1996) are again (2) and (3), but now with

$$\dot{\gamma}_p = D_r g_\gamma \mathrm{OCR}^{-1/I_v} \quad , \quad \dot{\varepsilon}_p = D_r g_\varepsilon \mathrm{OCR}^{-1/I_v} \qquad (8)$$

with g_γ and g_ε as in Fig. 3. The overconsolidation ratio is

$$\mathrm{OCR} = \sigma_e/\sigma' \ . \qquad (9)$$

State limits are reached from arbitrary initial states by monotonous deformations with D = const and are thus attractors (Gudehus 2004b). Then OCR = $(D/D_r)^{I_v}$ holds. This is supported by the findings of Bjerrum (1973), Leinenkugel (1976) and many others. For OCR $(D/D_r)^{I_v}$ > ca 1.5 at the onset the initial response is hypoelastic. The empirical relation $\tau_p = c' + \sigma' \tan \varphi'$ is reproduced for drained shearing with OCR > 1 initially and realistic D-values. The excess $|\tau| - \sigma' \tan \varphi_{sc}$ is solely due to OCR- and D-dependent dilation. This is nearly original Cam Clay (Schofield 2002).

Creep with constant σ' and $|\tau| < \sigma' \tan \varphi_{cs}$ is obtained with constant $\dot{\varepsilon}/\dot{\gamma} = \dot{\varepsilon}_p/\dot{\gamma}_p$ by (1) and (4). It tends to

$$e = e_0 - \lambda I_v \ln (1+D_r t) \qquad (10)$$

with $\lambda = e_0 n/2.72$ from (1), which agrees with observations (e.g. Bjerrum 1973, Leinenkugel 1976, Mesri 1987). Stationary creep occurs for $|\tau| = \sigma' \tan \varphi_{cs}$ = const with

$$\dot{\gamma} = D_r \mathrm{OCR}^{-1/I_v} \quad (\dot{\varepsilon} = 0) \ . \qquad (11)$$

For $e = e_c$ this implies a critical state.

For $|\tau| > \sigma' \tan \varphi_{sc}$ (10) holds with $-D_r$ instead of $+D_r$ (Gudehus 2004b), this explains a delayed creep rupture. The latter is also obtained without drainage and constant σ for $|\tau| > \sigma \tan \varphi_{sc}$.

Relaxation is obtained with constant γ and ε. τ/σ' tends to zero, and σ' to

$$\sigma'/\sigma_0 = 1 - I_v \ln (1+D_r t). \qquad (12)$$

This agrees with observations (eg Lacerda and Houston 1969). Leinenkugel (1976) has also shown that I_v in (12) is the same as in (10) and in $\sigma' = \sigma_e \ (D/D_r)^{I_v}$ for critical states.

The intergranular strain can be incorporated for cycles with small amplitudes (Niemunis 2003), thus ratcheting is reduced. The parameters φ_{cs}, e_{c0}, h_{sr} and I_v suffice for characterization. The conventional limit water contents w_L, w_p, w_s are related with state limits (Gudehus 2004b). Further parameters can be determined by element tests with remoulded samples. Representations with cylindrical and tensor components are available (Niemunis 2003, Gudehus 2004a). Realistic stiffness and strength values of undisturbed samples are derived with allowance for D.

The initial effective stress field is estimated with K_0 or by calculation, of course with initial pore pressures. The initial OCR-field is derived from penetration or vane shear sounding data. Changes of position and state are again predicted by means of boundary conditions. State limits and design limit states can be reached with delay due to seepage and skeleton viscosity. This has been validated for a wide spectrum of cases (e.g. Gudehus 2003, Gudehus et al 2004).

3 EXTENSIONS, COMBINATIONS AND LIMITATIONS

For *shear localizations* in psammoids hypoplastic relations with polar quantities are available (Gudehus and Nübel 2004). Further state limits appear for polar stresses and work as attractors. The shear band with d_s is proportional to the mean grain size d_g and decreases with increasing I_d. Shear bands and patterns of them are obtained realistically. Presumably cracking is an anomalous shear localization with $d_s \approx d_g$.

Temporal fluctuations around mean state values of psammoids cannot be treated with elasto-hypoplasticity because of limited computer power and numerical error accumulation (Wichtmann et al 2004). *Vibro-hypoplasticity* (Gudehus 2004c) fills the gap. It resembles visco-hypoplasticity, but I_v and D_r are proportional to a granular temperature, and D_r also to a mean granular frequency f_c. Realistic pseudo-creep and –relaxation is obtained.

The *specific pore water volume* v_w is below the v_{w0} of free water. $\chi = 1 - v_w/v_{w0}$ is related with the state limits of the skeleton. A constitutive relation for $\dot{\chi}$ yields χ state limits as attractors and explains a number of observations. This is relevant for deformations without seepage which are no more isochoric for S_r = 1, in particular for localizations (Gudehus 2004b).

Pore gas can principally be allowed for. Bubbles between solid particles make the pore fluid more compressible. Bigger bubbles weaken the skeleton, whereas gas channels strengthen it due to suction. Both can principally be allowed for by composite approaches. Patterns of fissures and shear bands can also be modelled by composites, and sandwich soils.

The combination of psammoids and peloids at different scales is no principal problem, but it requires judgment (e.g. Karcher et al. 2003). As usual, many layers can be lumped into a few, but potential slip surfaces, seals or drains must not be omitted. The same holds true with faults, inclusions, fills and granular columns.

As any theory the proposed concept has limitations. State limits imply permanent solid particles and ionic strength. This is never exactly given, but often a good approximation. It can be overcome by evolution models for the constitutive parameters. As any kind of effective tensile strength is left aside, soils with macropores cannot be covered. In addition to mean values like σ' and ε they require measures of spatial fluctuation. Intergranular strain and polar stress represent already spatial fluctuations of interparticle forces, so there is an opening. Soils with cementation are excluded as they are not plastic.

4 CONCLUSION

The proposed comprehensive concept is apt to predict changes of position and state for a wide spectrum of cases.

State limits of soils are defined by relations of strain rate ratios with stress ratios and of void ratios with pressure. Psammoids, i.e. sand-like materials, are characterized by a critical friction angle, a critical void ratio and a solid hardness. For peloids, i.e. clay-like soils, A viscosity index is needed in addition. Hypoplastic constitutive relations yield state limits for monotonous strain paths and arbitrary initial states as attractors. The key state parameter is a density index I_d for psammoids and an overconsolidation ratio OCR for peloids. Differential stiffness and strength are derived and agree with test results. For peloids both are rate-dependent, this explains also creep and relaxation.

The ground model consists of geometrically simplified psammoid and peloid zones. Hydraulic conditions enter as usual. The initial effective state field is estimated with K_0 or a hypoplastic calculation. The initial field of I_d and OCR is determined from penetration and vane shear data by hypoplasticity. Ground movement and improvement is allowed for by changes of boundaries and initial state. Natural and technical actions are represented by boundary conditions. Placement and penetration of structural parts are included. Realistic predictions include objective state limits in the large, which can be close to subjective limit states for design and observational methods.

The concept has been extended to allow for many small cycles leading to vibro-hypoelasticity or vibro-viscosity. Polar extensions enable predictions of shear bands. Dilation of bound pore water is important for localized shearing and cracking of peloids. Changing solid particles, macropores and cementation are as yet excluded.

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