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Constitutive equations for Murro clay

Équations constitutives pour l'argile de Murro

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ABSTRACT
This paper concerns formulating the potential function and stress-dilatancy equation for soft clay. The equations have been verified using test results of Murro clay. Empirical stress-strain equations and plastic potential function formed by integrating the yield function have been fitted to results of triaxial stress path controlled tests and shear tests.

RÉSUMÉ
La présente étude répond à la détermination de la fonction potentielle et de l'équation contrainte-dilatation pour argile molle. Les équations ont été vérifiées en utilisant les résultats d'essai de l'argile de Murro. Les équations empiriques de contrainte-déformation et les fonctions potentielles de plastic, qui sont formées en intégrant la fonction d'écoulement, ont été ajustées aux résultats d'essai de cisaillement et aux résultats d'essai triaxial du chemin de contraint contrôlée.

1 INTRODUCTION

The potential function is an important effect in calculating the plastic deformations. For this study, a methodology proposed by Drescher and Mroz (1977) was adopted in modelling the plastic potential.

In this paper, the constitutive equations for Murro clay are considered and verified with experimental data. Murro clay is a soft silty clay (undrained shear strength 26 kPa and clay sized fraction 30%; Karstunen and Koskinen 2004a) from a homogenous clay deposit in Seinäjoki, Finland. The structure of Murro clay is highly anisotropic and bonded. The main clay mineral of Murro clay is illite (Messerklinger et al. 2003), and the clay is rich in sulphur that makes it black in colour. An extensive series of laboratory tests, including oedometer tests, drained and undrained tests as well as stress path controlled triaxial tests on both natural and reconstituted samples of Murro clay, has been carried out at Helsinki University of Technology (Karstunen and Koskinen 2004a). An instrumented test embankment built in Murro in 1993 by the Finnish Road Administration (Koskinen et al. 2002c) provides settlement, horizontal displacement and pore pressure data of more than 10 years so far. After construction, the 2 m thick embankment has settled approximately 80 cm. Due to extensive monitoring of the embankment and field and laboratory testing, Murro clay is a very suitable material for testing constitutive models.

2 YIELD LOCUS AND PLASTIC POTENTIAL

Several models have been developed for calculating plastic deformations in clay and sand layers that are structurally anisotropic and exhibit non-associated flow. In models developed by Dafalias (1986, 1993), Yasufuku et al. (1991) and Boukpeti and Drescher (2000) the yield function (Eq. 2) is obtained by integrating Equation 1

$$\frac{dq}{dp} = \frac{(\eta - \alpha_f)\eta - (L - \alpha_f)L}{\eta - \alpha_f} \tag{1}$$

$$f = (\eta - \alpha_f)^2 + 2L(L - \alpha_f)\ln\left(\frac{p'}{p'_m}\right) \tag{2}$$

where η is the stress ratio q/p', α_f represents the inclination of the yield surface, p'_m is the size of the yield surface, i.e. the value of p' at $\eta=\alpha_f$, and L is a strength parameter defining the slope of the yield peak line passing through the top of the yield surface in p'-q plane (Birgisson and Drescher 1996). The slope of the line passing through the peak of the potential surface is marked with M, and when using an associated flow rule, L≡M. However, if the adopted flow rule is non-associated, M>L.

The general form for the stress-dilatancy relationship can be expressed by Equation 3 (Yasufuku et al. 1991). By integrating Equation 3, the general form (Eq. 4) of the potential function is obtained.

$$\frac{d\varepsilon_v}{d\varepsilon_s} = \frac{[M - (2-c)\alpha_g]M - [\eta - (2-c)\alpha_g]\eta}{c(\eta - \alpha_g)} \tag{3}$$

$$g = \ln p' + \frac{c}{2(c-1)} \ln\left\{(\eta - 2\alpha_g)\eta + \frac{1}{c-1}[M - (2-c)\alpha_g]M\right\} = constant, c \neq 1 \tag{4}$$

where c is a constant and α_g is the inclination of the potential surface. Based on triaxial tests Yasufuku et al. (1991) showed that experimental yield data fits well to the yield curve of Equation 1. However, the same does not apply for the potential function (Eq. 4) e.g. due to scatter in the test data.

Assuming associated yielding behaviour (i.e. L=M), Equation 5 that is derived from Equation 4 by simplifying and setting c=2 has been shown to fit well to results of tests on several natural and reconstituted clays (Dafalias et al. 2003, Koskinen et al. 2002a, 2002b, Karstunen and Koskinen 2004a, 2004b, Näätänen and Lojander 1994, 2000).

$$f = (q - \alpha_f p')^2 - (M^2 - \alpha_f^2)(p'_m - p')p' = 0 \tag{5}$$

where p'_m is the size of the yield curve. It has been assumed Equation 5 that $M_{comp}=M_{ext}=L$.

Figure 1a shows the yield points for natural Murro clay from the depths of 4.0-4.6 m and 7.0-7.5 m normalised with p'_m=25.7 kPa and p'_m=34.5 kPa, respectively, together with the normalised yield curve of Equation 5. The yield points were determined from linear stress-strain plots as explained by Koskinen et al. (2003). The yield curve (Eq. 5) fits well to the experimental data. Figure 1b shows the plastic potential for Murro clay from the depth of 7.0-7.5 m together with the corresponding yield curve. Experimental observations of a drained test CADC 2986 is also shown in Figure 1b. It can be seen based on yield curve and plastic potential that with the values of stress ratio occurring in a case of e.g. an embankment, associated flow rule would be a reasonable estimate of the clay behaviour. The difference between the curves is more obvious below the α_f-line. The potential surface and the drained test shown in Figure 1b will be discussed further in the following sections.

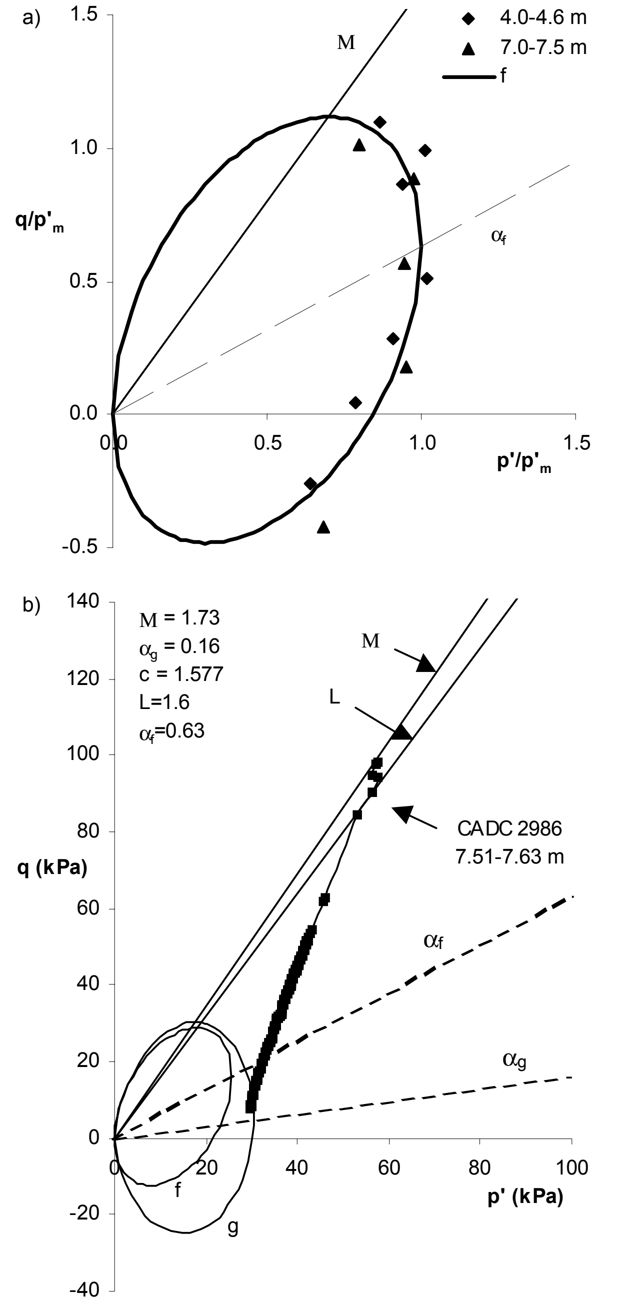

Figure 1. a) Yield curve for natural Murro clay. b) Yield curve, plastic potential and the stress path of the drained test CADC 2986 on natural Murro clay.

3 DEFORMATION EQUATIONS

Models describing mechanical behaviour of soil involve several empirical "material equations" that are an essential part of the models. The equations are also used for determination of parameters from laboratory tests. Empirical correlations that approximate measured relations between two or three variables (p', q, volumetric strain ε_v, shear strain ε_s and η) can also be used in the limited range of stresses and strains used for determination of soil constants and verification of the results calculated with a model. These deformation equations are suitable mostly for the purpose above and they cannot belong to the structure of the model, but the parameters used in the empirical correlations are identical to the model parameters (M, L, λ, κ and so on). Figure 2b shows the relationship of specific volume and plastic shear strains for Murro clay. Based on calculations, the relationship was found to follow Equation 6.

$$v = v_0 - \Delta v(1 - e^{-\lambda_1 \varepsilon_s}) \tag{6}$$

where $\Delta v = v_0 - v_c$, λ_1 is an empirical soil constant, v_0 is the specific volume in the beginning of the shearing stage and v_c is the specific volume in the critical state. Equation 6 can be used for determination of v_c if Δv is estimated accurately enough. It can be seen in the ε_s-η plot of Figure 2a that the critical state is not reached in the test. However, the approximate value of v_c determined based on Equation 6 is usually accurate enough. The soil constant λ_1 in Equation 6 can generally be used as such for calculating shear strains.

Figure 2a shows the relationship between the plastic shear strains and the stress ratio η. The test results have been approximated with Equation 7.

$$\eta = \frac{3\left[q_0 + q_f(1 - e^{-\lambda_1 \varepsilon_s}) \right]}{q_0 + 3p'_0 + q_f(1 - e^{-\lambda_1 \varepsilon_s})} \tag{7}$$

where $q_0 = q(\varepsilon_s = 0)$, $p'_0 = p'(\varepsilon_s = 0)$, and $q_f = q(\varepsilon_s \to \infty)$. Parameter λ_1 is the same as in Equation 6. The test data fit well to the curve calculated with Equation 7, which is valid when the stress path used in the triaxial test is $\dot{q} / \dot{p} = 3$.

When the relationships corresponding to Equations 6 and 7, i.e. $v = v(\varepsilon_s)$ and $\eta = \eta(\varepsilon_s)$, are calculated incrementally with a generally known and approved plastic model suitable for the given soil type, it can be nearly always noticed that an empirical equations calibrated with the test data, e.g. Eqs. 6 and 7, fit to the test data better than the corresponding relationships in a model developed based on theory of plasticity. In such cases where parameters of a model based on theory of plasticity are known reliably and yield function and potential function have been determined based on test results, it has been found that an empirical deformation equation is a better approximate of test data than a model based on theory of plasticity. The incremental deformation equation of such a model may seem sophisticated, but it can be severely more inaccurate than a simple empirical correlation, e.g. Eqs. 6 and 7. However, it is significantly important that equations that are based on the theory of plasticity can be used for all stress paths and densities occurring in geotechnical design that is mostly not the case with empirical correlations. The applicability of empirical equations is limited to such geotechnical conditions where the given values used for generating the equations have been observed. In models that include two yield surfaces and two potential surfaces, i.e. in models that allow separate volumetric and deviatoric hardening, the most important constitutive equations can be determined easily on the basis of the stress- strain relationships (Korhonen 2000).

Equations 6 and 7 will be used for determination of the hardening modulus H (Drescher and Mroz 1977) for Murro clay in a

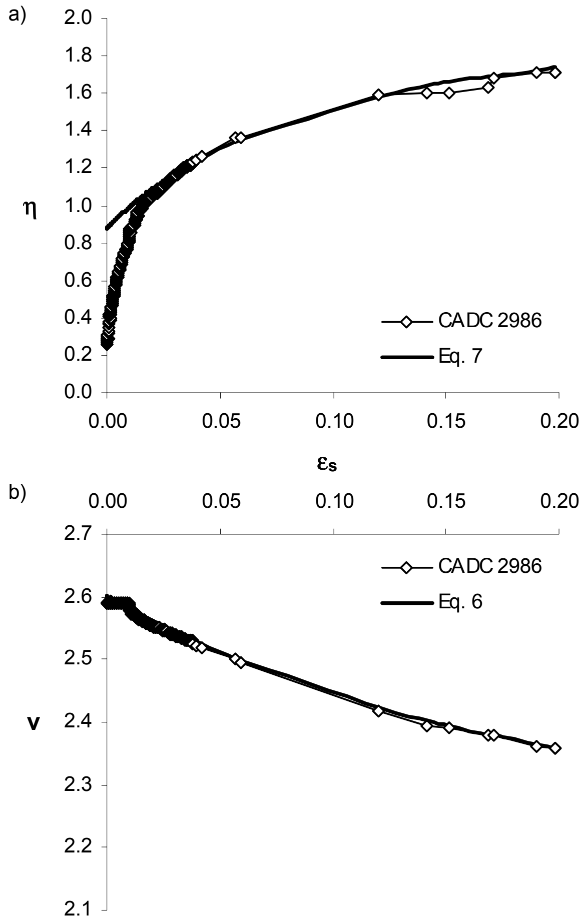

Figure 2. Results of test CADC 2986 on Murro clay from the depth of 7.51-7.63 m and calculations with the deformation equations.

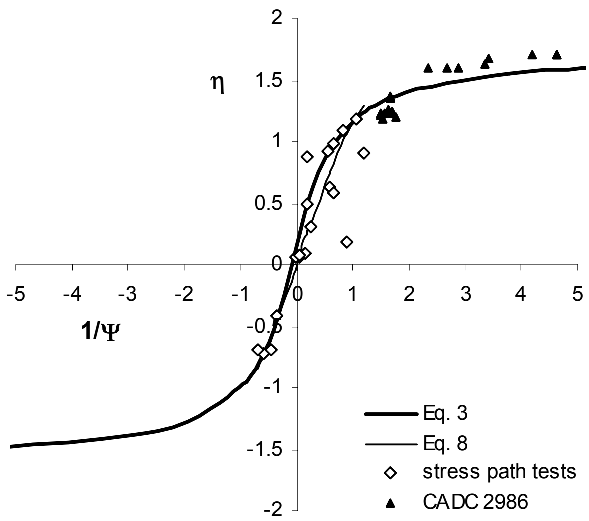

Figure 3. Relationship between dilatancy and stress ratio.

future study that will be a continuation of this paper. The equations are valid for the purpose in the particular stress path and density used in the test.

4 STRESS-DILATANCY EQUATION

In some well-known stress-dilatancy equations (Rowe 1972, Schofield and Wroth 1968, Dafalias 1986) there are only one or two soil constants, typically M and α. In order to find out the applicability of Equation 4 and the effect of constant c, the relationship between the inverse value of dilatancy angle $1/\Psi$ and the stress ratio η was plotted in Figure 3 based on results of tests on Murro clay. A curve based on Equation 3 was also plotted in Figure 3 using values of M=1.73, α=0.16 and c=1.577, i.e. using the same values as used for the plastic potential in Figure 1. It can be seen in Figure 3 that the match between the test data and Equation 3 is rather good. Use of the constant c increases significantly the applicability and accuracy of Equation 4. It seems that the value of c varies between 1.0 and 2.0 (Drescher and Mroz 1977).

A hyperbolic equation (Eq. 8) was also used for calculating the relationship between the inverse value of dilatancy angle $1/\Psi$ and the stress ratio η in order to study the applicability of this function type.

$$\eta = M\left[tanh\left(k\frac{1}{\Psi}\right)\right] \quad (8)$$

A curve calculated with Equation 8 is shown in Figure 3 using value 0.8 for constant k. The calculations with Equation 8 have been carried out within the range of $1/\Psi$-values observed in the stress path controlled tests on natural Murro clay. The first trial functions used showed that the hyperbolic type of function (Eq. 8) can be used for the stress-dilatancy function.

5 CONCLUSION

The aim of this paper was to settle the functional form of the yield curve and the plastic potential based on results of triaxial tests on Murro clay. The calculations show that the yield curve of Equation 5 is in a very good agreement with yield points determined from tests on natural anisotropic Murro clay. In addition, it was shown that the potential function concerning non-associated yielding (Eq. 4) can be formed using the stress-dilatancy equation (Eq. 3) that incorporates three parameters M, α and c. Parameter α representing the amount of anisotropy can be assumed a constant. Use of parameter c increases the accuracy of the calculations. Furthermore, it has been noted that the yield curve and the plastic potential are of the same functional form, and based on test results on natural Murro clay, an associated flow rule is a reasonable assumption in a limited range of stress ratios occurring i.e. in a case of an embankment. Therefore, in calculations it is justifiable to assume the yielding of the clay being associated.

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