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3D finite element analysis of bearing capacity failure in clay

L'analyse tridimensionnelle de la rupture de portance de l'argile par la méthode des eléments finis

Mark D. Evans
United States Military Academy, West Point, NY, USA
D.V. Griffiths
Colorado School of Mines, Golden, CO, USA

ABSTRACT

Three-dimensional finite element analysis is a useful tool for practical problem solving as well as for parametric analysis and failure mechanism visualization. In this paper, Mohr-Coulomb, elasto-viscoplastic, bearing capacity analyses were performed using finite element code running on massively parallel computers. With the recent advent of PC Clusters, parallel computing is readily available to those who may not have access to parallel supercomputers. Square, rectangular, and strip footings are considered to determine bearing capacity factors. Values obtained by these finite element analyses are comparable to those values reported in the literature. Speedup values for increased numbers of processors in the parallel environment are also presented and discussed.

RÉSUMÉ

L'analyse tridimensionnelle de la portance par la méthode des éléments finis est un outil utile pour résoudre des problèmes pratiques tels que l'analyse paramétrique et la visualisation de mécanismes. Dans cet article, les résultats élastiques/plastiques de Mohr-Coulomb appliqués à l'analyse de la portance ont été retrouvés en utilisant un algorithme de méthode des éléments finis avec un nombre important d'ordinateurs montés parallèlement en réseaux. Avec l'arrivée récente des groupements de PC, le calcule en parallèle est facilement accessible pour ceux qui n'ont pas accès aux superordinateurs en parallèles. Des carrés, des rectangles et des bandes sont posés puis pris en compte afin de déterminer les facteurs de portance et de comparer les valeurs déterminantes à celles présentes dans les livres. Les valeurs obtenues par cette méthode des éléments finis sont comparables à celles trouvées dans les livres. Les valeurs d'accélération en fonction de l'augmentation du nombre de processeurs dans l'environnement parallèle sont également présentées et discutées.

1 INTRODUCTION

Three-dimensional finite element analysis is a useful tool for practical problem solving as well as for parametric analysis and failure mechanism visualization. In this paper, Mohr-Coulomb, elasto-viscoplastic, bearing capacity analyses were performed using finite element code running on massively parallel computers. Square, rectangular, and strip footings are considered in these analyses to determine bearing capacity factors and compare the determined values to those in the literature. With the recent advent of PC Clusters, parallel computing is readily available to those who may not have access to parallel supercomputers. Speedup values for increased numbers of processors in the parallel environment are also presented and discussed.

2 BEARING CAPACITY OF SHALLOW FOOTINGS

When a uniform pressure is applied to the soil, the surface settles. Settlement curves are commonly used to relate pressure and settlement. For stiff soils, the bearing capacity is usually well-defined. However, for loose or soft soils, the bearing capacity is not so well defined and is typically described as the point on the curve where the settlement-pressure curve becomes steep and straight (Terzaghi and Peck, 1967)

For frictionless, cohesive soils, where friction angle, $\phi = 0$ and cohesion, $c > 0$, Prandtl (1921) determined that the bearing capacity per unit area is:

$$q_{ult} = N_c c = (2 + \pi)c = 5.14c \qquad (1)$$

In this paper we shall consider only weightless, cohesive soil, so other bearing capacity factors that the reader will be familiar with, namely N_γ and N_q, are equal to zero and one, respectively. Those two parameters will be investigated in greater detail for soils with $\phi > 0$ in future studies.

Prandtl showed that the shape of the failure surface and the results of equation 1 remain valid whether the soil has self weight or is weightless. Thus, for the analyses in this paper, the case of weightless soil was considered. Further, for simplicity, only the case where the loaded area is acting directly on the ground surface is considered.

Skempton (1951) proposed that equation 1 be multiplied by a so-called "shape" factor to account for changes in bearing capacity for footings not of infinite length. He proposed the following shape factor, s:

$$s = (1 + 0.2B/L) \qquad (2)$$

where B is the footing width and L is the footing length, such that:

$$q_{ult} = 5.14c (1 + 0.2B/L) \qquad (3)$$

Thus, for an infinite, strip footing, s = 1 and for a square footing, s = 1.2. Square, strip, and rectangular footings are considered in this paper.

3 FINITE ELEMENT ANALYSES

This paper presents the results of three-dimensional elasto-viscoplastic analyses of bearing capacity problems using a Mohr-Coulomb failure criterion. Plasticity is modeled via the elasto-viscoplastic algorithm (Zienkiewicz and Cormeau, 1974) and the algorithm employs 20-node hexahedral elements with 8 integrating points (2x2x2) per element. This level of "reduced integration" has been shown to work effectively in both the stiffness and stress redistribution phases of the algorithm.

The program used in these studies is essentially the same as Program 12.2 in the text by Smith and Griffiths (2004) with minor modifications to allow different sizes of footing. All the source code from Smith and Griffiths (over 60 main programs for solving a wide range of engineering problems) is written in Fortran 95 and available on-line at the web site: www.mines.edu/fs_home/vgriffit/4th_ed. Many of the programs are written for execution using parallel processing computers such as those described in Section 7 of this paper. Three-dimensional analyses such as those demonstrated in this paper are greatly facilitated by the use of iterative equation solvers that avoid the need to assemble and store large matrices such as the global stiffness matrix. In these studies, the Preconditioned Conjugate Gradient (pcg) method has been used (Smith and Griffiths, 2004) with diagonal preconditioning. The pcg method enables sophisticated 3-d analyses to be performed in a routine manner on either inexpensive desktop computers or massively parallel supercomputers like that used in this study.

4 STRIP FOOTINGS

Infinitely-long strip footings are expected to exhibit a shape factor of 1, thus, according to Equation 1, $q_{ult} = 5.14c$. Applied footing pressures were normalized by cohesion (normalized pressure = q/c) and applied incrementally to the footing area so that the normalized, dimensionless applied pressure was plotted directly versus settlement, as shown in Figure 1. The numerical values of settlement shown in the figure are not particularly meaningful, as they are a function of element and footing dimensions and elastic properties, thus the units for settlement are deliberately not shown. What is important to note in the figure is the small, gradual increases in settlement as pressure is increased followed by a rapid, steep increase in settlement after a particular value of normalized pressure is applied. The point on the pressure-settlement curve where the settlement increases steeply and becomes straight was defined by Terzaghi and Peck (1967) as the bearing capacity. Thus, the normalized pressure (normalized pressure = q/c) at that break-point on the curve is defined as Bearing Capacity Factor, N_c, and in Figure 1, that value is approximately 5.16. Since the N_c value obtained in this study using the FEM, pcg, and Mohr-Coulomb failure criterion is almost exactly the anticipated Prandtl theoretical value of 5.14, the authors judged that this model was sufficiently accurate to warrant further investigation.

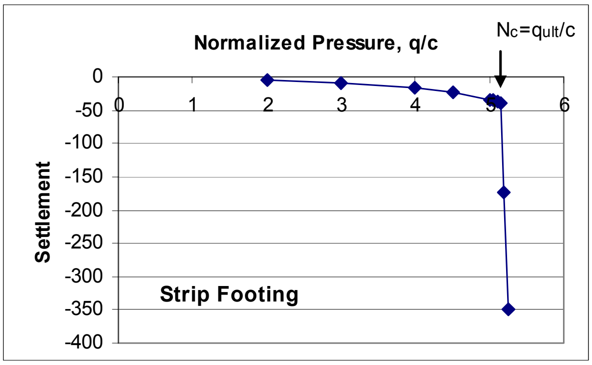

Figure 1. Settlement vs. Pressure for Strip Footing

Shown in Figure 2 is the deformed mesh and in Figures 3 and 4 are 3D and 2D displacement vectors, respectively, for the strip footing at an applied pressure close to the bearing capacity. All three figures used in concert show the zone of applied strip-footing pressure along the left-hand edge, the downward soil movement below the footing and the final deformed shape of the ground surface. Similarly, to the right of the strip footing, outside the footing-load area, upward soil movement vectors may be observed along with a bulging and uplift of the ground surface. These observations indicate a failure mechanism and zones of plastic equilibrium consistent with those described by Prandtl (1921) and Terzaghi and Peck (1967).

Once again it should be emphasized that the units and dimensions are not shown in these figures since they are not particularly meaningful in the context of this paper and our investigations of bearing capacity. We are not attempting to predict settlement for a given set of conditions. Rather, we are attempting to illustrate that the 3D FEM used in this paper is a valid tool for predicting failure, illustrating failure zones, and determining bearing capacity factors.

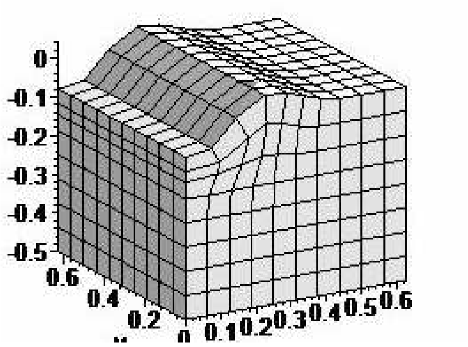

Figure 2. Deformed Mesh for Strip Footing

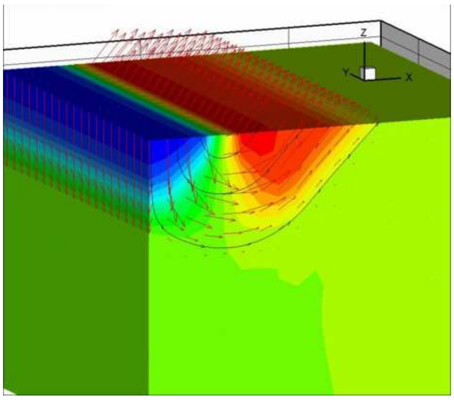

Figure 3. 3D Displacement Vectors for Strip Footing

5 RECTANGULAR FOOTINGS

Rectangular footings with 1:2 and 1:3 aspect ratios were investigated, having B/L ratios of 0.33 to 0.50, with essentially no noticeable difference in results between them. A settlement vs. pressure curve for the 1:3 rectangular footing is shown in Figure 5. It may be seen that the normalized pressure value at which the settlement curve becomes steep and straight is around 5.8, which results in a shape factor of between 1.10 and 1.15. The expected shape factors for these aspect ratios are between 1.07 and 1.10, thus these analyses are producing results very close to the expected range illustrating that the 3D FEM approach described in this paper can accurately predict bearing capacity failure for rectangular footings, accurately accounting for shape factors.

The deformed mesh and 3D displacement vectors for the rectangular footing are shown in Figures 6 and 7, respectively. It may be seen in these figures that even though aspect ratios of only 2:1 and 3:1 were used, the failure mechanism is dominant on the long side, implying that plane strain behavior starts to dominate quite soon as one side is made longer than the other.

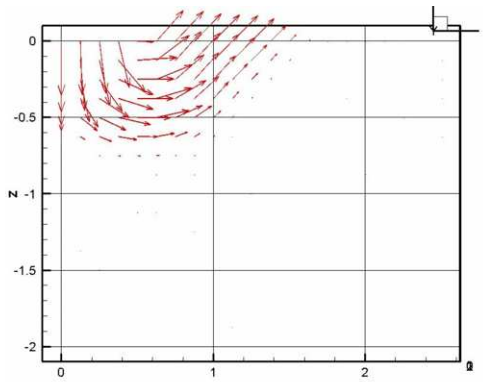

Figure 4. 2D Displacement Vectors for Strip Footing

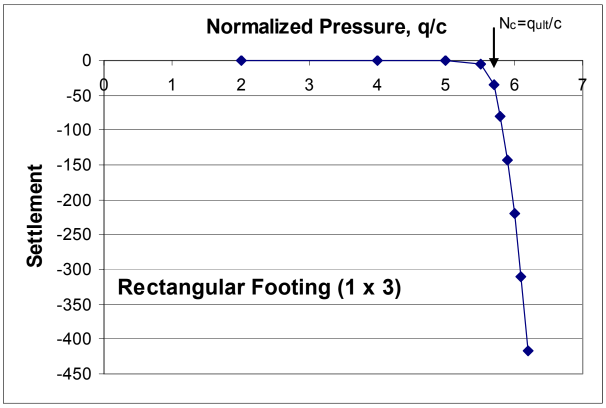

Figure 5. Settlement vs. Pressure for Rectangular Footings

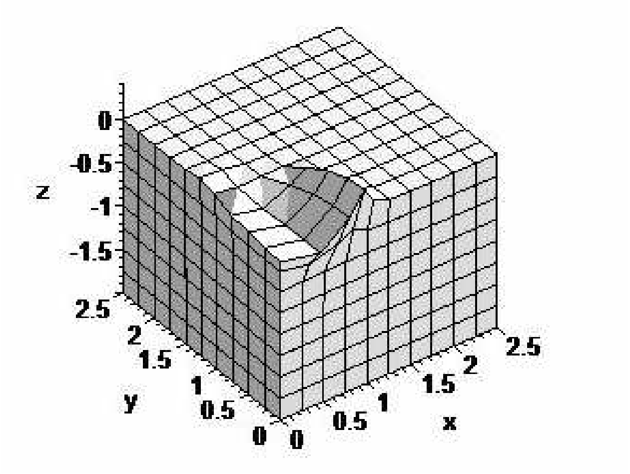

Figure 6. Deformed Mesh for Rectangular Footings

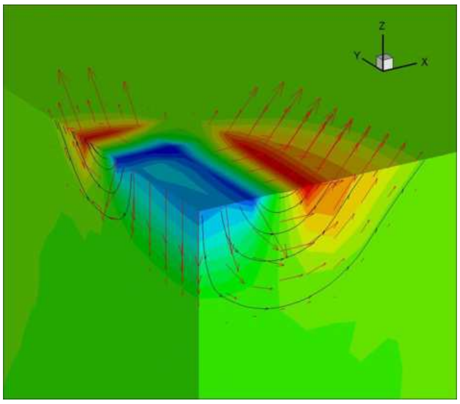

Figure 7. 3D Displacement Vectors for Rectangular Footings

6 SQUARE FOOTINGS

Settlement vs. pressure results for square footings was not appreciably different from those results for the 1:2 and 1:3 rectangular footings (see Figure 5). Thus, the bearing capacity factor obtained for the square footings was also in the range of 5.8. This value results in a shape factor for square footings also in the range of 1.1 to 1.15, just slightly lower than the value of 1.2 that would be anticipated. Squares of both coarser grids (10x10x8) and finer grids (20x20x16) were analyzed with no appreciable difference in results. A deformed mesh for the square footing is shown in Figure 8 and displacement vectors are shown in Figure 9. It may be seen that the failure mechanism develops equally in both the x and y directions, as should be expected. Note that the square footing failure mechanism appears very constrained and localized suggesting that the mesh boundaries do not need to be so far away. The influence of mesh size will be investigated more fully in future work.

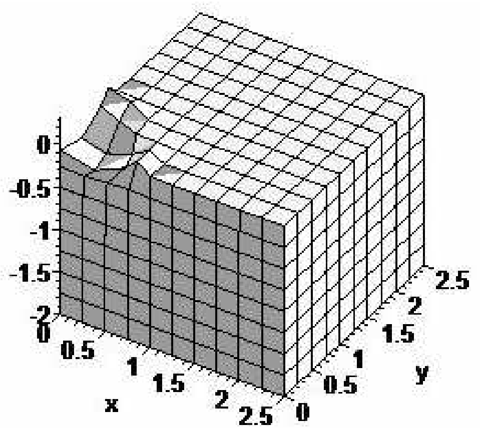

Figure 8. Deformed Mesh for Square Footing

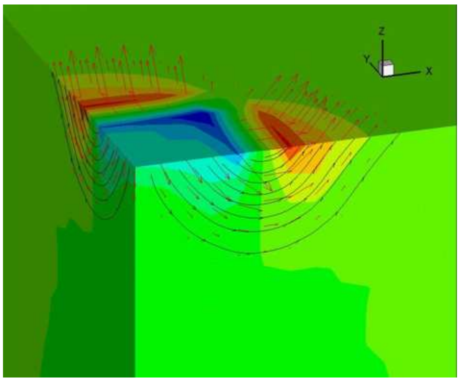

Figure 9. 3D Displacement Vectors for Square Footing

7 PARALLEL PROCESSING COMPUTERS

The analyses described in this paper were performed on a US Department of Defense (DOD) supercomputer operated by the High Performance Computing Modernization Program (HPCMP) at the Army Research Laboratory (ARL) at Aberdeen Proving Ground, MD. The name of the particular machine used is Zornig. Zornig is an SGI Origin 3800, operating on the IRIX 6.5 operating system. On the unclassified side of Zornig, there are 512, R12000 processors operating at a processing speed of 400 MHz. The SGI 3800 is a shared memory multi-processor system, and Zornig has 384 Gbytes of shared memory (RAM) and employs MPI – Message Passing Interface – to pass information between processors. Zornig has a hard disk capacity of 5 Tbytes backed up by RAID storage.

The first author has also constructed a 16-node pc cluster at his institution for parallel computing. The details of this system will be described elsewhere, but by constructing such a reasonably affordable system, parallel computing is accessible to all.

Parallel processing allows the main processor to slice up the work to be performed and scatter those computations to the various compute nodes (processors) allocated to the work. Each compute node is responsible for completing the assigned computations, after which the main processor will gather up all the data and reassemble it for output or for further computations.

Parallel processing allows very large jobs to be run in much shorter time. For example, one of the data sets described in this paper entailed loading a 2 x 2 square footing acting over the corner of a 10 x 10 x 8, 3D grid (with 2 axes of symmetry). This was one of the smaller grids used in this study, consisting of 800 elements with 4037 nodes and 10,064 equations of equilibrium. For these data, 10 pressure increments were applied. When a single processor is used to solve this problem it would take 472 minutes (7.9 hours) to complete. However, by parallelizing the program, it might take only 26 minutes to run on 32 processors. Table 1 shows the number of processors used versus the time to complete the analyses for this footing.

Table 1: Number of Processors vs. Compute Time (min)

Number of Processors	Compute Time (min)
1	472
2	236
4	118
8	62
16	36
32	26
64	26

These data are plotted in Figure 10 where a 45-degree line is shown for comparison. The 45 degree line indicates "perfect speedup" – where there is no time lost sending packets of information between the multiple processors. However, in reality, there is inefficiency, or latency, in parallel computations where compute nodes may not be kept busy 100% of the time as data packets are passed between processors. In spite of this latency, the value of parallel processor computations should be evident in that the overall compute time decreases dramatically for greater numbers of processors. For the program used in these analyses, it may be seen that there is near-perfect speedup up to 16 processors, after which processing time begins to flatten, indicating that there is no value in using more than 32 processors for this particular program and data.

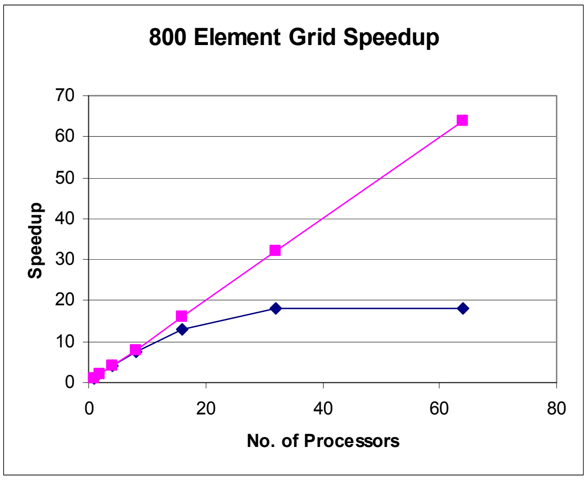

Figure 10. 800 Element Grid Speedup

8 SUMMARY AND CONCLUSIONS

The central point of this paper is to demonstrate that real life 3D analyses in geomechanics may be accurately performed using finite elements, pcg algorithms, and parallel processing. Bearing capacity factors and failure mechanisms were investigated for weightless, cohesive soil. A bearing capacity factor very close to Prandtl's value of 5.14 for strip footings was determined from these analyses for strip footings, indicating the accuracy of the method. The failure mechanism for rectangular footings is observed to be strongly dominated by the long side of the footing, and that point is clearly illustrated in this paper. The failure mechanism for square footings indicates that much smaller mesh sizes may be used in future analyses without adversely affecting results. Shape factors for square and rectangular footings were determined in this study to be very close to values determined by Equation 2. Parallel processing is shown to be a valuable tool for performing 3D finite element computations in a reasonable amount of time. If supercomputers are not available, PC Clusters or Beowulfs can provide parallel processing power at a reasonable cost. This is an on-going project in which a range of soils parameters, including frictional soils, will be investigated.

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