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Applicability of the meshless method to soil-water coupled problems

Applicabilité de la méthode sans maille pour les problèmes mixtes eau-sol

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ABSTRACT
A formulation of the Element-Free Galerkin Method (EFG) is presented for consolidation under a large deformation and its applicability to soil-water coupled problems is examined through numerical analyses by comparing it with the conventional finite element computation. The accuracy of the proposed numerical strategy is examined through an analysis of unconfined compression and simple shear tests under undrained and plane strain conditions by a comparison of stress paths integrated directly from the finite strain Cam-clay model. The results obtained from the numerical tests show that the use of Delaunay triangles as background cells leads to a higher accuracy of the numerical solutions than those under the usual square background cells. It is also revealed that the particular type of weight functions to be adopted in the moving least-square approximation and the density of the nodal points can determine the resultant shape functions of EFG for both the pore water pressure and the displacement field such that they are advantageous in avoiding spatial instability in the numerical solutions of pore water pressure under the undrained conditions appearing in the patch tests.

RÉSUMÉ
Une formulation de la méthode de Galerkin sans élément (EFG) est présentée pour la consolidation sous grandes déformations et son applicabilité aux problèmes mixtes eau-sol est éxaminée par comparaison avec les résultats de l'analyse classique par éléments finis. La précision de la méthode numérique proposée est vérifiée par l'analyse d'un test de compression non confiné et d'un test de cisaillement simple dans des conditions non draînées et de déformation plane et par comparaison des chemins de contrainte obtenus directement par le modèle de déformations finies de Cam-clay. Les résultats numériques indiquent que l'utilisation de triangles de Delaunay comme cellules d'arrière plan permet d'obtenir une précision supérieure par rapport aux cellules d'arrière plan carrées usuelles. Il est aussi montré que le type de fonctions pondérales adoptées dans l'approximation des moindres carrés et la densité de noeuds déterminent les fonctions de formes résultantes de l'EFG pour la pression interstitielle et le champ de déplacement, un choix judicieux permettant d'éviter l'instabilité spatiale de la solution numérique pour la pression interstitielle dans les conditions non draînées du patch test.

1 INTRODUCTION

In solving the particular problems related to predicting the behavior of geomaterials, several difficulties are often encountered in the conventional FEM computation. Such difficulties depend on the mesh distortion and/or the alignment subjected to discontinuous displacement fields within the regime of a large deformation. A numerical remedy using finer meshes at the initial stage of the computation and an iterative remeshing during the subsequent analysis is adopted to obtain an accurate estimation of the relationship between load and dipslacement up to the collapsing stage and the proper profile of deformation, while overcoming the consumption of computational time and effort.

In the recent decade, mesh-free methods have appeared connectivity-free between elements and nodes as alternatives to FEM. Among the mesh-free methods, EFG was proposed by Belytschko *et al.* (Belytschko *et al.*, 1994) based on the diffuse element method originated by Nayroles *et al.* (Nayroles *et al.*, 1992) in the framework of the Galerkin method and has been applied to various boundary-value problems to overcome the above-mentioned numerical difficulties. EFG is proven to be particularly effective for analyzing strain localization, crack propagation, shell problems, and so on.

The first attempt to apply such mesh-free strategies to a soil-water coupled problem was made by Modaressi *et al.* using the EFG method with an elastic model (Modaressi and Aubert, 1998). This was followed by the current authors' application with an elasto-plastic constitutive equation within infinitesimal strain (Murakami *et al.*, 2001) and finite strain for the pirpose of capturing the localization of saturated soil (Murakami *et al.*, 2003; Arimoto *et al.*, 2004). Karim *et al.*, on the other hand, applied the technique to the analysis of a transient response of elastic saturated soil on the seabed under cyclic loading due to wave motion (Karim *et al.*, 2002). Another meshless strategy, on the other hand, by Wang *et al.* (Wang *et al.*, 2001, 2002), based on the point interpolation method (PIM) or the radial point interpolation method (radial PIM), has also been applied to solve Biot's consolidation problem for elastic material under infinitesimal strain in order to overcome the disadvantages of the lack of delta function properties in the shape functions obtained by the MLS approximation in the EFG method. Nogami *et al.* incorporated the double porosity model into the radial PIM to analyze lumpy clay fillings (Nogami *et al.*, 2004).

The main advantage of applying the EFG method to soil-water coupled problems is that it can overcome several numerical difficulties which are confronted when using the conventional FEM computation. Our objective is to point out the advantages of solving a particular problem for a soil-water coupled problem.

The arrangement of the current paper is as follows. The second section briefly summarizes a formulation of the EFG method for a soil-water coupled problem within a large deformation which leads to an updated Lagrangian scheme. In the third section, an unconfined compression test is solved while comparing the integrated solutions under a known deformation gradient tensor with computed ones in order to examine the accuracy of the description of the present algorithm. The use of Delaunay triangles as background cells is also examined to obtain a higher accuracy of the numerical solutions than those under the usual square background cells. In the fourth section, special focus is placed on the shape functions for a soil skeleton and the pore pressure introduced by the MLS approximation of the EFG strategy which allows for stable solutions under undrained conditions despite having the same order. A conclusion follows in the last section.

2 FORMULATION

The EFG method is a mesh-free Galerkin procedure using a moving least-square (MLS; Lancaster and Salkauskas, 1981) interpolant to discretize the weak form of the governing partial differential equations (PDEs), whereas a mesh-free collocation method, such as the smoothed particle hydrodynamics (SPH), deals with the strong form of a PDE. As seen in the textbook of the mesh-free method (Liu, 2002; Li and Liu, 2004), the problem domain, Ω, is represented in the EFG method by a set of nodes scattered in Ω and on the boundary. The variables of both displacement and pore pressure at a point of interest within the problem domain are approximated using the nodal parameters of both variables at the nodes which fall into the circle of the domain influence called 'support', as seen in Fig. 1, and the interpolation function is derived by the MLS approximant. The MLS interpolant $u^h(\mathbf{x})$ of function $u(\mathbf{x})$ is defined in the support, and the resultant interpolation function $\mathbf{N}(\mathbf{x})$ is obtained so as to minimize the weighted residual using a cluster of nodes within the support, whose center is adopted as the integration point of the 'background cells', in which the numerical integration of computing the stiffness matrix is performed, alternative to the finite element meshes shown in Fig.2. The scale fact or, which is defined as the magnification of the support diameter to the second shortest distance between nodes involved in a support, is usually appropriate for computations between 1.2 and 1.5. The shape and the characteristics of the resultant interpolation function, $\mathbf{N}(\mathbf{x})$, is one of the main differences between EFG and FEM.

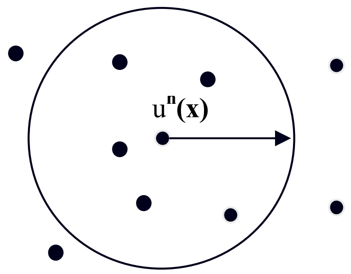

Figure 1. Domain of influence.

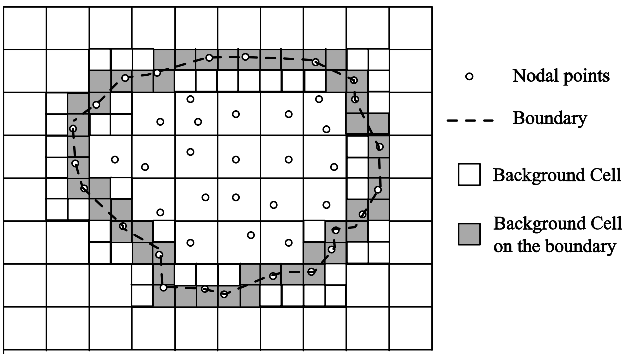

Figure 2. Background cells for the integration of the stiffness.

Another difference is the implementation of the essential boundary conditions for the displacement, because of the lack of a Kronecker delta function property in the MLS based interpolation functions. To avoid this difficulty, the current paper adopts the Penalty method for the imposition of the essential boundary conditions initially suggested by Zhu and Atluri (1998) for 2D linear elastostatic problems.

For the analysis in the regime of finite strain, square background cells, one quarter of background cells, are allocated along the boundary to cover the changing shape of the domain during the deformation, as seen in Fig.2. The formulation is furnished by the discretization of the weak form of a set of PDEs, i.e., the equation of equilibrium forces and the coupling equations via the effective stress, the pore pressure, and the constitutive relation of the soil skeleton, using the MLS interpolation functions in a similar manner to that of FEM (Yatomi *et al.*, 1989; Asaoka *et al.*, 1994). A detailed formulation of the EFG for soil-water coupled problems within a large deformation is given to lead an updated Lagrangian scheme in the literature (Arimoto *et al.*, 2004; Murakami *et al.*, 2005).

3 NUMERICAL EXAMINATION

To examine the numerical accuracy of the discretization scheme based on the EFG method, the behavior of a soil specimen is analyzed under undrained conditions. An unconfined compression test without the friction beneath the pedestal is herein dealt with, because its theoretical solution can be obtained through the direct integration of the constitutive model (Noda, 1994), e.g., the finite strain Cam-clay model, under the prescribed stretching and examined by a comparison with the computed one by EFG.

3.1 *Unconfined Compression Test*

Fig.3 depicts the problem description of the unconfined compression test for one quarter of the specimen because of its symmetricity and initial configuration of 111 nodal point arrangements. At the initial stage of computation, each corner of the square background cells coincides with the nodal point and the finer background cells with one quarter the size allocated along the boundary to cover the changing shape of the specimen.

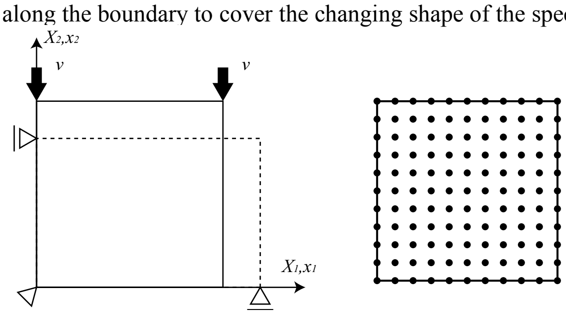

Figure 3. Unconfined compression test.

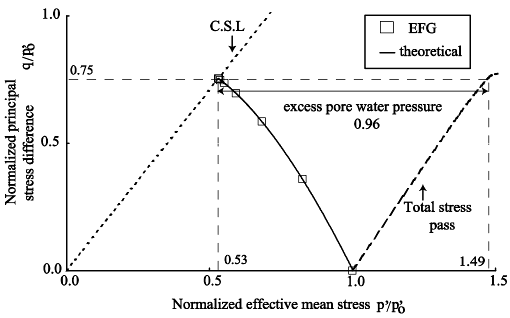

Figure 4. Comparison of the numerical solution with the theoretical one.

Fig.4 reveals that the numerical solutions are in good agreement with those obtained through the integration of the constitutive equation under the uniform deformation.

3.2 *Delaunay Triangulation for Background Integration*

The Delaunay triangulation of a point set is a collection of edges satisfying an 'empty circle' property, namely, for each edge we can find a circle containing the edge's endpoints but not any other points. Its dual diagram is known as the Voronoi diagram, as seen in Fig.5. The Delaunay triangulation is often adopted for mesh generation in the FEM computation.

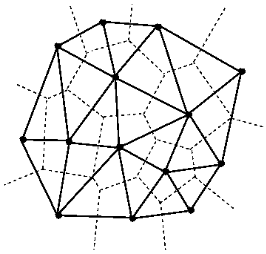

Figure 5. Delaunay triangles and their dual Voronoi diagrams.

The Delaunay triangles based on the nodal points changing their coordinates during deformation, are adopted herein as the background cells to minimize the numerical errors and the computation effort, because the triangles effectively cover the domain at each stage of the computation and integration points are reduced compared with those for square background cells. Fig.6 describes the improved accuracy in the case of the Delaunay triangles interpolating 111 nodal points as background cells for the analysis of the same problem which appeared in the previous subsection. The triangulation of background cells is carried out at both the initial stage and subsequent stages of the computation. An index related to the numerical accuracy is adopted as 'ERR' in the following equation:

$$ERR = \frac{\sum_{I=1}^{NP} \sqrt{\left(\frac{D_{22}^{Exact} - D_{22}^{I}}{D_{22}^{Exact}} \right)^2}}{NP} \quad (1)$$

where NP is the number of nodal points, D^{Exact} is a prescribed stretching, and D^{I} is a computed stretching by EFG at each nodal point.

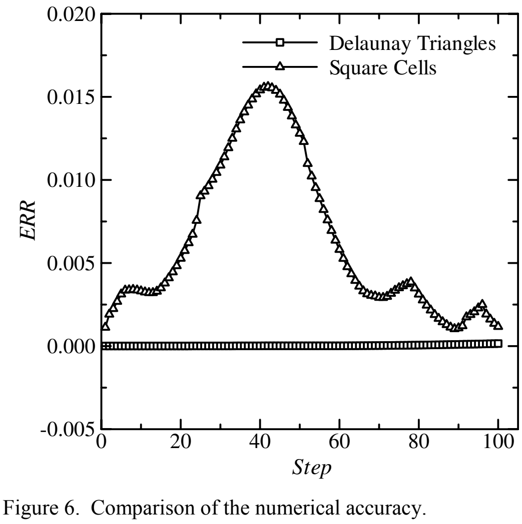

Figure 6. Comparison of the numerical accuracy.

4 CHARACTERISTICS OF THE INTERPOLATION FUNCTION OF THE EFG METHOD FOR A SOIL-WATER COUPLED ANALYSIS UNDER UNDRAINED CONDITIONS

For soil-water coupled problems, numerical instabilities of the FEM computation are often encountered for instantaneous loading under undrained conditions unless certain requirements for the shape functions are met. It is well known that the satisfaction of the so-called patch test, created by Zienkiewicz *et al.* (Zienkiewicz *et al.*, 1986), is a necessary condition and the inf-sup test, created by Chapelle *et al.* (Chapelle and Bathe, 1993), numerically constructs a sufficient condition for the stability of the FEM computation. As the remedy for such numerical instability, a higher interpolation order in displacements than in pore pressures is generally adopted. On the other hand, a stable formulation based on the Simo-Rifai enhanced strain element has been proposed as an alternative stabilization methodology even for the equal order interpolation in both displacements and pore pressures (Mira *et al.*, 2003).

The interpolation functions of the EFG method have an equal order for both displacements and pore pressures, because each nodal point simultaneously has the freedom of both variables. Fig.7 discriminates the shape of the interpolation for the exponential weight function under a combination of different parameters and nodes included within a support. It can be revealed that their shape and characteristics depend on the adopted parameters of the exponential weight function and also on the density of the nodal points.

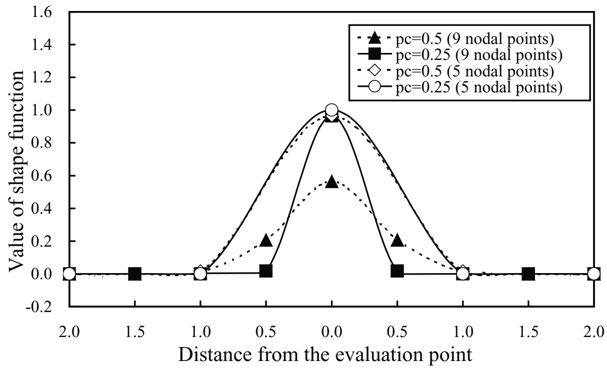

Figure 7. Different shapes of the interpolation functions.

To examine the numerical stability of the EFG computation, based on such an interpolation, the saturated soil column test created by Zienkiewicz *et al.* (Zienkiewicz *et al.*, 1986), shown in Fig.8, is performed using the material parameters listed in Table 1 under different permeabilities, parameters, and densities of the nodal points within a support.

Table 1. Material parameters.

Compression index,	0.11
Swelling index,	0.04
Critical state parameter,	1.42
Poisson's ratio,	0.333
Initial void ratio,	0.83
Initial volume ratio,	1.83
Initial consolidation stress (Pa),	3.0

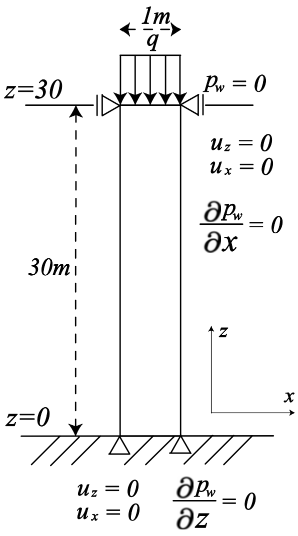

Figure 8. Saturated soil column test.

Numerical analyses are carried out to examine the effect of the density of nodal points and a combination of the weight function, to construct the interpolation function based on the EFG method, on the stability of the solutions. The numerical examination reveals that a certain combination of weight function and density of the nodes under an appropriate scale factor leads to circumvent compatibility (inf-sup) conditions by relaxing the underlying constraint. Table 2 summarizes the results of examinations for the effects of several factors on the numerical behavior of the pore pressures.

Table 2. Summary of the effects of the density of the nodal points and a combination of weight function on the numerical stability.

Combination of weight function		Density of nodal points		
u	pw	poor	⟷	enough
Q	Q	×	⟷	×
E-A	E-A	△	⟷	△
E-A	E-B	△	⟷	○

Q : Quartic spline weight function — × :Bad result
E -A : Exponential weight function (c=0.5) — △ :Better result
E -B : Exponential weight function (c=0.25) — ○ :Good result

5 CONCLUSION

A numerical strategy for a soil-water coupled problem, based on the EFG method and within the framework of finite strain, has been presented to analyze the transient behavior of saturated soil under plane strain conditions. The formulation has been based on the updated Lagrangian scheme with particular concern given to the allocation of finer background cells along the boundary, where the penalty method is employed to implement the essential boundary conditions. The role of the derived interpolation functions, based on the MLS approximation, is validated to allay the unstable behavior of the pore pressure values through the numerical computation for a saturated soil column test which determines the spatial instability of the pore pressure solution under undrained conditions in the case where no particular attention is paid to the selection of shape functions for the displacements or the pore pressure values. The accuracy of the proposed procedure is then examined under the numerical computation of unconfined compression tests where analytical solutions, which are to be compared with the EFG method solutions, are developed by the integration of the constitutive model.

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