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# The Principle of Natural Proportionality applied to the behaviour of piles

## Le Principe de la Proportionnalité Naturelle appliqué au comportement de pieux

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ABSTRACT
General equations provided by the Principle of Natural Proportionality are applied to describe the behaviour of piles. They include uplift capacity, response of laterally loaded and axial load tests on piles.

RESUMÉ
Trois équations générales derivées du Principe de la Proportionnalité Naturelle ont eté apliquées pour faire la description du comportement de pieux. Une équation concernant la capacité portante de pieux sous charges de tension et les autres deux pour décrire leur comportement sous chargement lateral et axial, respectivement.

## 1 INTRODUCTION

General equations provided by the principle of natural proportionality (Juárez-Badillo 1985 a) have been proven to mathematically describe the mechanical behaviour of geomaterials: solids, liquids and gases. They have been applied to rocks, granular and fine soils and concrete (Juárez-Badillo 1994 b, 1997 a, 1999 a, b, c, 2000). The effect of time has also been considered in the secondary consolidation of fine soils, and also in the creep and relaxation under constant shear stresses or shear strains in the geomaterials (Juárez-Badillo 1985 b, 1988, 1994 a, 2001). The principle of natural proportionality has also been applied to the settlement of embankments (Juárez-Badillo 1991, 1997 b, 2005). This time the principle of natural proportionality is applied to the behaviour of piles.

## 2 THEORETICAL EQUATIONS

Consider a normal pile under an uplift load test. Let $Q$ be the uplift load and $D$ the displacement. It is obvious to think that as $Q$ increases $D$ will increase and that there will exist a peak $Q$ from which $Q$ decreases while $D$ will keep increasing.

The proper variables, that is, the simplest variables to describe the phenomenon, appear to be precisely $Q$ and $D$. Before the peak as $D$ increases from $O$ to $\infty$, $Q$ will increase from $O$ to a final value $Q_F$. The corresponding proper functions, that is, the simplest functions with complete domains, that is, from $O$ to $\infty$, are the same $D$ and $Z = 1/Q - 1/Q_F$. Now when $D$ varies from $O$ to $\infty$, $Z$ varies from $\infty$ to $O$. The principle of natural proportionality states that the relation between $Z$ and $D$ should be

$$\frac{dD}{D} = -v\frac{dz}{Z} \tag{1}$$

where $v$ is the constant of proportionality called the "displacement or deflection exponent".

Integration of (1) gives

$$DZ^v = \text{constant} \tag{2}$$

which may be written

$$D\left(\frac{Q_F}{Q} - 1\right)^v = \text{constant} = D^* \tag{3}$$

where $D^*$ = characteristic $D$ at $Q = ½\ Q_F$ and we have the pre-peak general equation

$$\frac{D}{D^*} = \left[\frac{Q_F}{Q} - 1\right]^{-v} \tag{4}$$

which may also be written as

$$\frac{Q}{Q_F} = \left[1 + \left(\frac{D}{D^*}\right)^{-1/v}\right]^{-1} = Y_s \tag{5}$$

which is precisely the pre-peak "sensitivity" function $Y_s$ for the reason explained in (Juárez-Badillo 1999 c). For our case it is convenient to write (5) as

$$Q = \frac{Q_F}{1 + \left(\dfrac{D}{D^*}\right)^{-1/v}} \tag{6}$$

If the test includes unloadings and reloadings, equation (6) becomes

$$Q = Q_i + \frac{Q_F - Q_i}{1 + \left(\dfrac{D - D_i}{D^* - D_i}\right)^{-1/v}} \tag{7}$$

where $Q_i$ = initial load and $D_i$ = initial displacement. Fig. 1 shows the graph of (5) for different values of $v$.

After the peak and using similar reasoning we may consider that as the load $Q$ varies from $\infty$ to $O$, the displacement $D$ varies from $O$ to $\infty$ and applying the principle of natural proportionality we may write

$$DQ^{v} = \text{constant} = D_1 Q_1^{\ v} \qquad (8)$$

where ($Q_1$, $D_1$) is a known point, (8) may be written as

$$\frac{Q}{Q_1} = \left(\frac{D}{D_1}\right)^{-1/v} = Y_D \qquad (9)$$

which is precisely the "ductility" function $Y_D$ (Juárez-Badillo 1999 c). The constant $v$ now takes the name of ductility coefficient. Fig. 2 shows the graphs of (9) for different values of $v$.

Consider now the case of a normal pile under a lateral load. If $Q$ is the lateral load and $D$ the pile head lateral deflection it is very obvious that all of the above considerations are applicable and therefore all of the above equations are applicable to the response of laterally loaded piles, Figs. 1 and 2.

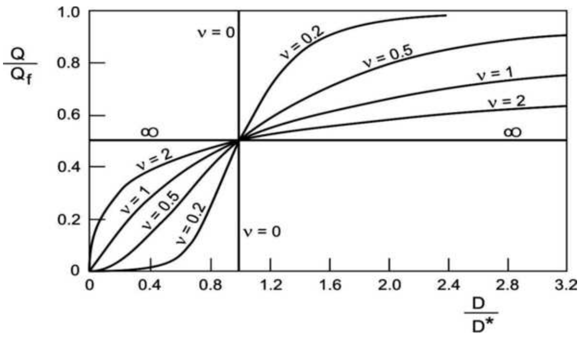


Fig. 1. Graphs of $Y_S = \dfrac{Q}{Q_F} = \dfrac{1}{1+\left(\dfrac{D}{D^*}\right)^{-1/v}}$

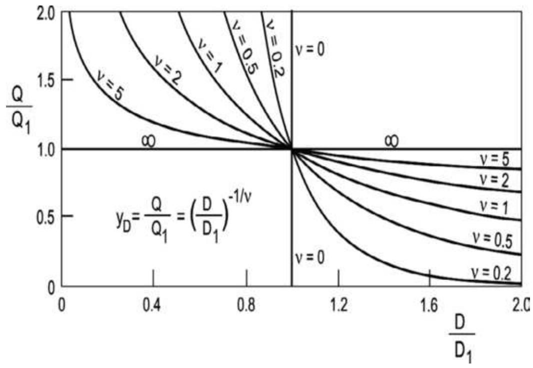


Fig. 2. Graphs of $Y_D = \dfrac{Q}{Q_1} = \left(\dfrac{D}{D_1}\right)^{-1/v}$

Consider now the case of a normal pile under an axial load. Except of very special cases of stratification in the soil we may now consider that there will not exist a peak as the displacement $D$ (or settlement $S$) increases as the load $Q$ is increased. For this case we will find out that $Q_F$ is not a constant and that it will keep increasing as the settlement $S$ (or $D$) increases. For this case we may assume a linear increase of $Q_F$ such that $Q_F$ will be given by

$$Q_F = a + bS \qquad (10)$$

where $a$ and $b$ are constants. (10) should be introduced in (6) and if it is the case in (7). For this important case equations (6) and (7) may be written as:

$$Q = \frac{Q_F}{1+\left(\dfrac{S}{S^*}\right)^{-1/v}} \qquad (11)$$

and

$$Q = Q_i + \frac{Q_F - Q_i}{1+\left(\dfrac{S-S_i}{S^*-S_i}\right)^{-1/v}} \qquad (12)$$

Fig. 3 presents graphs of (11).

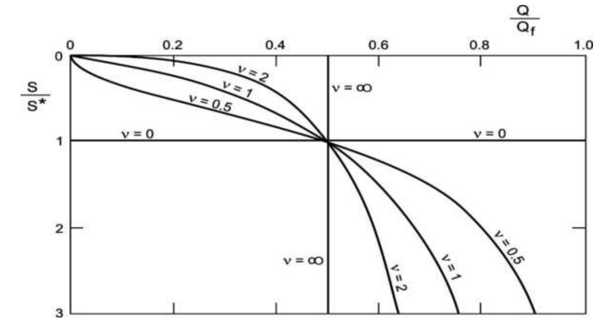


Fig. 3. Graphs of $Y_S = \dfrac{Q}{Q_F} = \dfrac{1}{1+\left(\dfrac{S}{S^*}\right)^{-1/v}}$

## 3 PRACTICAL APPLICATION

The practical application of all of the above equations is very simple.

The author has followed the following steps: 1. Assume $v$ from Fig. 1 or 3. (The author has found out for all the cases that have come to his attention that $v$ = 1). 2. Assume a value of $Q_F$ . 3. Read the value of $D^*$ at $Q_F/2$. 4. Apply the equation to the full experimental curve. 5. Adjust $Q_F$ and $D^*$ until you get a very good equation.

For the post-peak region the application of the post-peak ductility equation is simpler because ($Q_1$, $D_1$) is a fixed good point of the post-peak experimental curve and from Fig. 2 we adjust the value of $v$.

For the case of equation (10) the obtention of $a$ and $b$ for $Q_F$ requires some imagination, sensibility and experience. Just follow a trial and error procedure.

Figures 4 and 5 present two cases of uplift load tests (Konstantinidis et al., 1987). The values of the parameters in equations (6) and (7) appear in these figures. These cases were presented in a Master Thesis at the Graduate School of Engineering of the National University of Mexico (Padrós-Escalante, 2001). Details on the execution of these and all other tests will not be included. They may be consulted in the references cited for each case.

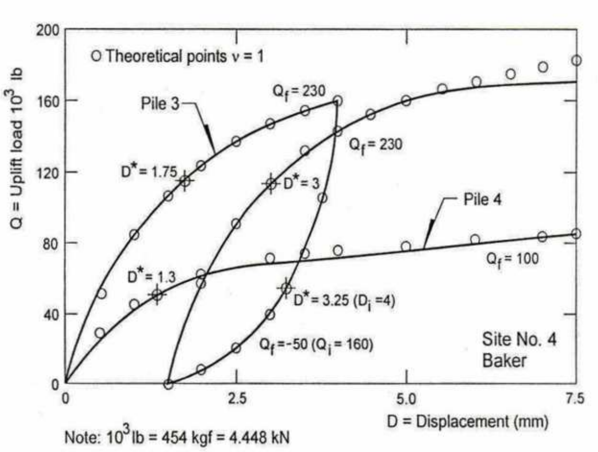


Fig. 4. Uplift load versus displacement for two pile tests in Baker, California, (Konstantinidis et al., 1987).

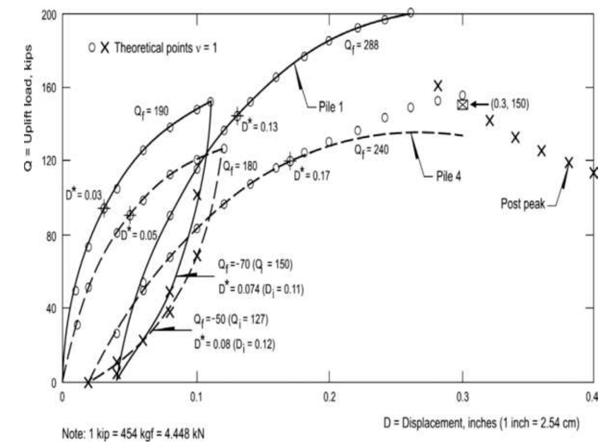


Fig. 5. Uplift load versus displacement for two pile tests in Caliente, Nevada (Konstantinidis et al., 1987).

Figure 6 presents the application of (6) to the load-deformation behaviour for (*a*) a five-pile group and (*b*) a ten-pile group under lateral loads in soft clay (Matlock et al., 1980).

Figures 7 and 8 present the response of laterally loaded large diameter ($D$ = 1.5m) bored pile groups (Ng, Zhang and Nip, 2001). The notation Pm-n$D$ means m = number of piles in the group and n = number of diameters of spacing among the piles. The values of the parameters appear in Table 1. In these figures H and $\rho$ take the place of $Q$ and $D$ respectively.

Figures 9 to 12 present axial load tests on bored piles and pile groups in cemented sands (Ismael, 2001). Application of equations (6) or (11) and (10) was made and the parameter values appear in each of these figures.

Fig. 13 presents the application of (6) to a static load test on a pile (Randolph, 2003). The theoretical equation appears on it together with the computed response by the author of the Reference.

Figures 14 to 17 present the application of (6) to Figs. 14 to 16 and the theoretical equation (13), the inverted pre-peak function $Y_I$, Fig. 18, to Fig. 17. This four figures present furthermore the proposed method of calculation by the author of the Reference and some of them include the lateral deflections obtained using GROUP (Reese and Wang, 1996).

Fig. 18 presents graphs of the pre-peak function $Y_I$ applied to piles, very similar to the pre-peak funcion $Y_I$ for the stress-strain behaviour of geomaterials (Juárez-Badillo, 1999 c) and given by:

$$\frac{Q}{Q^*} = \left[\frac{D_F}{D} - 1\right]^{-1/\nu} = Y_I \qquad (13)$$

where $D_F = D$ for $Q = \infty$ and $Q^* = Q$ at $D = D_F/2$.

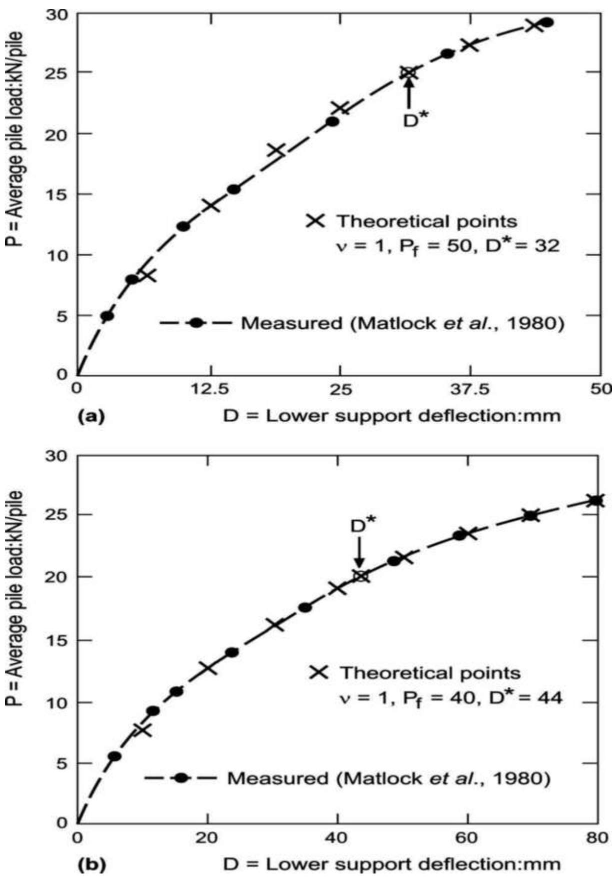


Fig. 6. Load deformation behaviour for (*a*) a five-pile group and (*b*) a ten-pile group.

Table 1. Parameters values and initial points.

| Fig. | Group | Curve | $Hi$ | $\rho_I$ | $\nu$ | $H_f$ | $\rho^*$ |
|---|---|---|---|---|---|---|---|
| 7 | P1 | 1st Loading | 0 | 0 | 1 | 4,800 | 82 |
| | | 3rd unloading | 2,660 | 102 | 1 | -4,400 | 7 |
| | | 4th. unloading | 2,950 | 125 | 1 | -3,250 | 51 |
| | P2-6D | 1st. loading | 0 | 0 | 1 | 4,000 | 25 |
| | P2-3D | 1st. loading | 0 | 0 | 1 | 3,200 | 28 |
| | | 4th. unloading | 2,490 | 97 | 1 | -6,450 | -59 |
| | | 5th.unloading | 2,590 | 127 | 1 | -1,760 | 74 |
| | P3-3D | 1st. loading | 0 | 0 | 1 | 2,200 | 10 |
| 8 | P1 | loading | 0 | 0 | 1 | 4,800 | 0.055 |
| | P2-6D | loading | 0 | 0 | 1 | 8,000 | 0.017 |
| | P2-3D | loading | 0 | 0 | 1 | 6,400 | 0.019 |
| | P3-3D | loading | 0 | 0 | 1 | 6,600 | 0.007 |

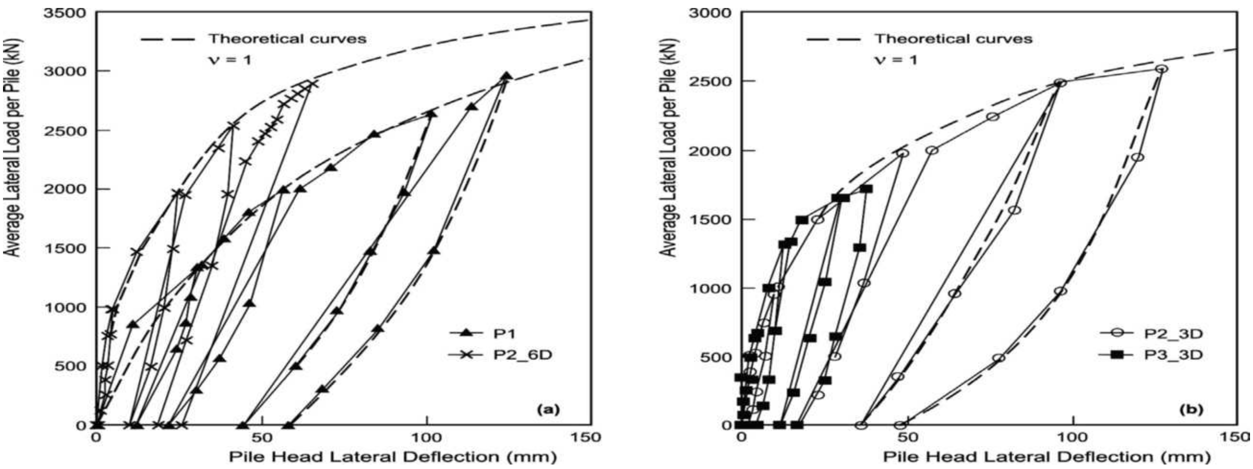


Fig. 7. Load versus defection: (*a*) P1 and P2-6*D* and (*b*) P2-3*D* and P3-3*D*. (Experimental data after Ng et al., 2001).

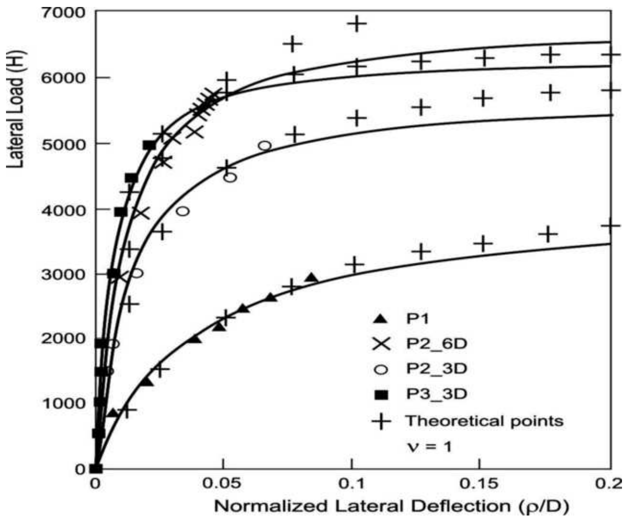


Fig. 8. Load versus normalized lateral deflection. (Experimental data after Ng et al., 2001).

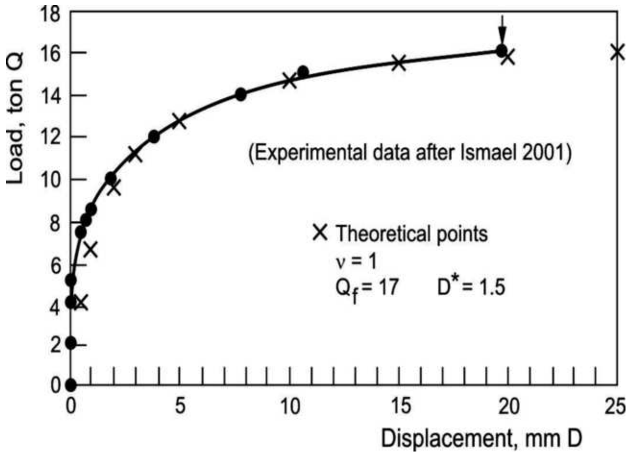


Fig. 9. Load versus displacement for single piles in tension

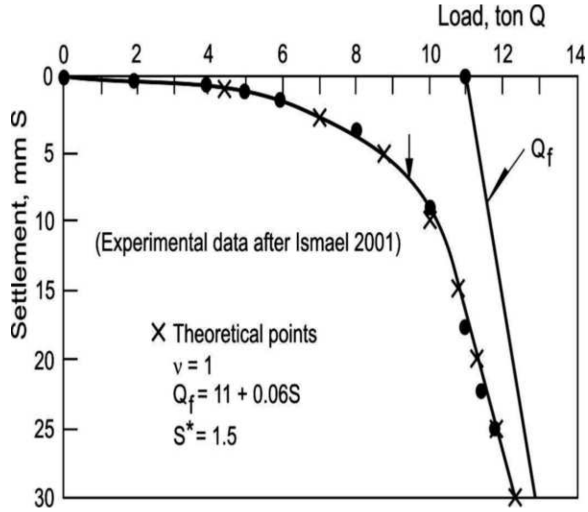


Fig. 10. Load versus settlement for single piles in compression.

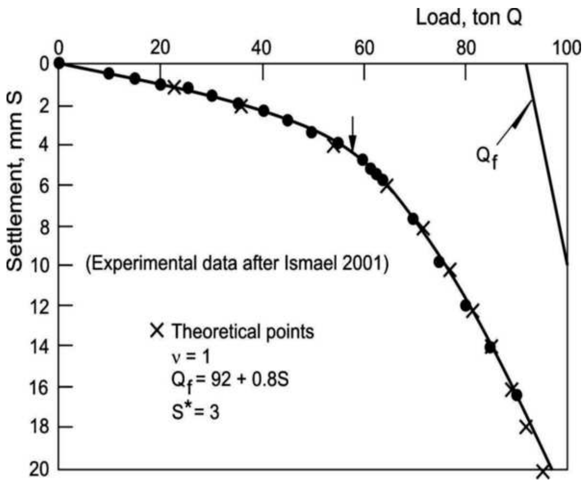


Fig. 11. Load-settlement curve for pile group 3.

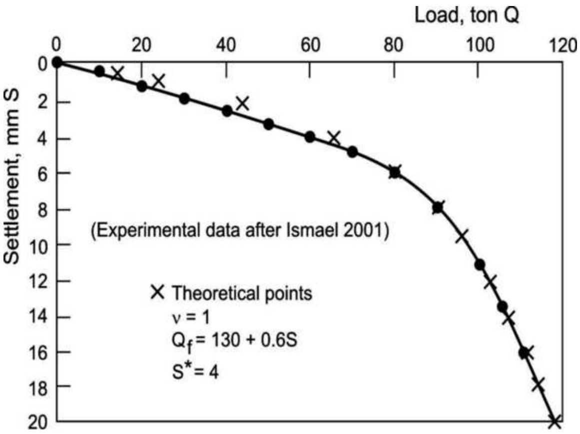


Fig. 12. Load versus settlement for pile group 4.

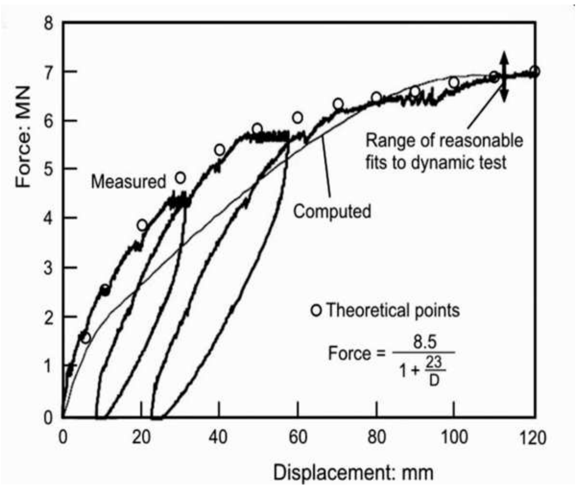


Fig. 13. Comparison of measured, computed and theoretical pile responses (Randolph 2003)

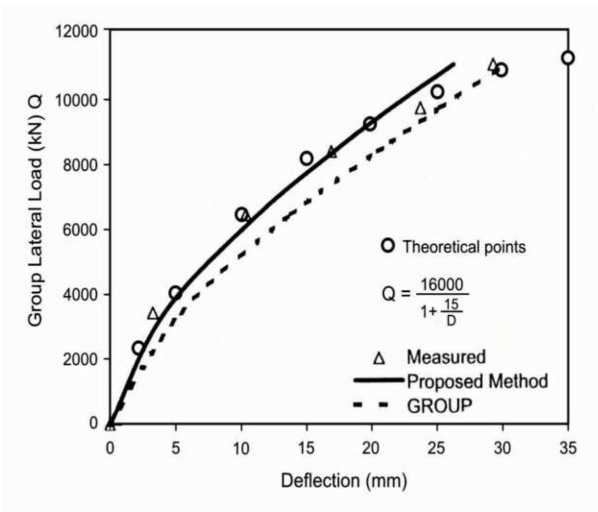


Fig. 14. Comparison of theoretical, predited and measured lateral displacements for Chaiyi, Taiwan load test (Ooi et al, 2004)

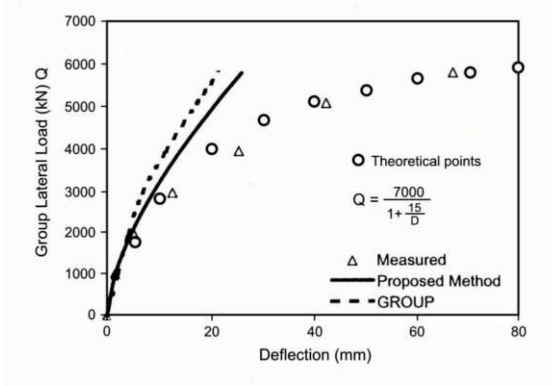


Fig. 15. Comparison of theoretical, predicted and meeasured lateral displacements for Hong Kong load test (Ooi et al, 2004)

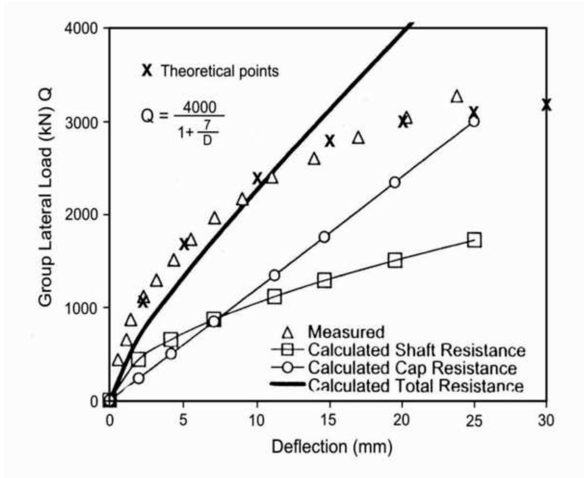


Fig. 16. Comparison of theoretical, predicted and measured lateral displacements of first Las Vegas load test (Ooi et al, 2004).

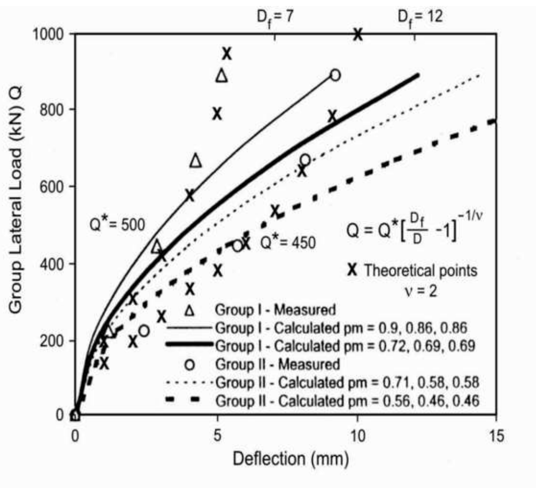


Fig. 17. Comparison of theoretical, predicted and measured lateral displacements for Lewisburg load test (Ooi et al, 2004).

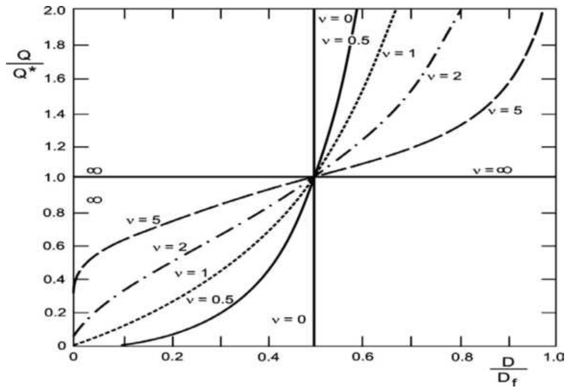


Fig. 18. Graphs of $Y_I = \frac{Q}{Q^*} = \left[\frac{D_F}{D} - 1\right]^{-1/v}$

## 4 CONCLUSIONS

The principle of natural proportionality that provides very simple equations for natural phenomena appears to be very general, not only for laboratory tests on geomaterials but also for describing the behaviour of engineering works in nature as are the settlement of embankments and the behaviour of piles which has been the subject of this paper.

The theoretical equations are so simple that a colleague of the author dare to express: it just appears to be "an exercise in matching experimental behaviour with empirical equations".

I would like to conclude with:

"We will understand the simplicity of the Universe when we accept how strange it is"

John Archibald Wheeler

## REFERENCES

Ismael Nabil F. 2001. Axial load tests on bored piles and pile groups in cemented sands. *Journal of Geotechnical and Geoenvironmental Engineering,* Vol. 127:766-773.

Juárez-Badillo, E. 1985 a. General volumetric constitutive equation for geomaterials. Special Volume on Constitutive Laws of Soils. *XI International Conference on Soil Mechanics and Foundation Engineering*, San Francisco. Japanese Society for Soils Mechanics and Foundation Engineering, Tokyo, pp. 131-135.

Juárez-Badillo, E. 1985 b. General time volume change equation for soils. *XI International Conference on Soil Mechanics and Foundation Engineering,* San Francisco, Vol. 2:519-530

Juárez-Badillo, E. 1988. Postsurcharge secondary compression equation for clays. *Canadian Geotechnical Journal,* 25:594-599.

Juárez-Badillo, E. 1991. Thirty years of secondary consolidation in sensitive marine clay: Discussion. *Canadian Geotechnical Journal,* Vol. 28:466-467.

Juárez-Badillo, E. 1994 a. General stress-strain time equation for soils. *XIII International Conference on Soil Mechanics and Foundation Engineering*, New Delhi, India, Vol. 1:337-348.

Juárez-Badillo, E. 1994 b. Deviatoric stress-strain equation for soils. *XIII International Conference on Soil Mechanics and Foundation Engineering*, New Delhi, India, Vol. 1:349-356.

Juárez-Badillo, E. 1997 a. General equations to describe the mechanical behaviour of granular soils. *XIV International Conference on Soil Mechanics and Foundation Engineering*, Hamburg, Vol. 1:133-138.

Juárez-Badillo, E. 1997 b. Case histories illustrate the importance of secondary-type consolidation settlements in the Fraser River delta: Discussion. *Canadian Geotechnical Journal,* 34:1007-1008.

Juárez-Badillo, E. 1999 a. General equations to describe the mechanical behaviour of concrete. *Fourth International Conference on Constitutive Laws for Engineering Materials.* Rensseleaer Polytechnic Institute, Troy, New York, U.S.A., pp. 403-408.

Juárez-Badillo, E. 1999 b. Improved general stress-strain equations for granular soils. *XI Panamerican Conference on Soil Mechanics and Geotechnical Engineering,* Foz du Iguassu, Brazil, Vol. 1:297-306.

Juárez-Badillo, E. 1999 c. Static liquefaction of very loose sands: Discussion. *Canadian Geotechnical Journal,* 36:967-973.

Juárez-Badillo, E. 2000. General equations to describe the mechanical behaviour of rocks. *Year 2000 Geotechnics,* Asian Institute of Technology, Bangkok, Thailand, pp. 39-52.

Juárez-Badillo, E. 2001. General relaxation equations for soils. *XV International Conference on Soil Mechanics and Geotechnical Engineering*, Istanbul, Vol. 1:141-144.

Juárez-Badillo, E. 2005. Kansai International Airport, Future Settlements. *XVI International Conference on Soil Mechanics and Geotechnical Engineering,* Osaka, Japan. (Accepted abstract).

Konstantinidis, B., Pacal, A.J. and Shively, A.W. 1987. Uplift capacity of drilled piers in desert soils. Foundations for transmission line towers. *Geotechnical Special Publication* No. 8 (edited by Jean-Louis Briaud) ASCE, New York.

Matlock, H., Ingram, W.B., Kelley, A.E. and Bogard, D.C. 1980. Field tests on the lateral load behaviour of pile groups in soft clay. *Proc. 12th Offshore Technology Conference*, Houston, 163-174.

Ng Charles, W.W., Zhang Limin and Nip Dora, C.N. 2001. Response of laterally loaded large diameter bored pile groups. *Journal of Geotechnical and Geoenvironmetal Engineering,* Vol. 127:658-669.

Ooi, P. S. K., Chang, B. K. F. and Wang, S. 2004, Simplified lateral load analysis of fixed-head piles and pile groups. *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 130:1140-1151.

Padrós-Escalante, G. 2001. Determinación de la capacidad de carga por fricción en pilas de concreto de sección constante, construidas en depósitos granulares, a partir de resultados obtenidos en pruebas de extracción. *Master's Thesis,* directed by Boris Simpser Dubovoy, Graduate School of Engineering, National University of Mexico.

Randolph, M. F. 2003. Science empiricism in pile foundation design. *Geotechnique* 53, No. 10, 847-875.

Reese, L. C., and Wang, S. T. 1996. *Group 4.0 for Windows, Analysis of a group of piles subjected to axial and lateral loading*, Ensoft, Inc. Austin, Tex.