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Excess porewater pressures during secondary compression

La génération de pression interstitielle durant la consolidation secondaire

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ABSTRACT

A gradient in an excess porewater pressure conveys water out of soil during secondary compression. This excess porewater pressure is produced by the tendency of soil to continue compression as a result of the disequilibrium produced during primary compression. Because the measurement of excess porewater pressure is frequently specified for establishing the progress of primary consolidation, it is useful to know the order of magnitude of excess porewater pressure associated with secondary compression. Mathematical analyses and experimental results suggest magnitudes of excess porewater pressure at the beginning of secondary compression corresponding to u'_m / σ'_v = 1 to 3 % for C_α / C_c = 0.03 to 0.07, respectively, which decrease rapidly with the progress of secondary compression.

RÉSUMÉ

L′expulsion de l′eau durant la consolidation secondaire est associée à un gradient induit par une surpression interstitielle. Cette surpression interstitielle résulte de la tendance du sol à continuer à se comprimer suite au déséquilibre produit durant la consolidation primaire. Parce que la mesure de la pression interstitielle est utilisée pour le suivi de la consolidation primaire, il est utile de connaitre l′ordre de grandeur de la pression interstitielle associée à la consolidation secondaire. Des analyses théoriques et des résultats expérimentaux indiquent que la surpression interstitielle au début de la consolidation secondaire serait de l′ordre de u_m/σ'_v = 1 à 3% pour des C_α / C_c de 0.03 à 0.07, respectivement, décroissant ensuite rapidement avec l′évolution de la consolidation secondaire.

1 INTRODUCTION

One-dimensional consolidation of saturated soils consists of primary compression which takes place during the increase in effective vertical stress and secondary compression that follows at constant effective vertical stress. Therefore, the least ambiguous definition of end-of-primary (EOP) consolidation has been the full dissipation of excess porewater pressures resulting from construction-related changes in total vertical stress or ground water pressures (e.g. Jones et al., 1986; Jorgenson, 1987; Endicott, 2001). However, an excess porewater pressure is also associated with secondary compresion. This excess porewater pressure, the gradient of which conveys porewater out of soil during secondary compression, is produced by the tendency of soil to continue compression as a result of structural disequilibrium produced during primary compression. Therefore, in general, excess porewater pressure at the EOP consolidation (the begining of secondary compression) need not be zero. For large-scale reclamation projects involving preloading, surcharging, or construction in stages, an unambigious specification of EOP consolidation is an important economic consideration (e.g. Mesri et al., 1994; Endicott, 2001). In case porewater pressure measurement has been specified for control of construction schedule, it would be useful to know the order of magnitude of excess porewater pressures associated with secondary compression.

In this paper, procedures are developed for computing excess porewater pressure during secondary compression, including at EOP consolidation. Simple equations are presented for computing excess porewater pressure for ground conditions with and without vertical drains.

2 ANALYSIS OF EXCESS POREWATER PRESSURE

The rate of secondary compression (i.e. for t equal to or greater than t_p), in terms of void ratio, e, vertical strain, ε_v, or settlement, s, is respectively :

$$\frac{\partial e}{\partial t} = \frac{-0.434\,C_\alpha}{t} \tag{1a}$$

$$\frac{\partial \varepsilon_v}{\partial t} = \frac{0.434\,C_\alpha}{(1+e)t} \tag{1b}$$

$$\frac{\partial s}{\partial t} = \frac{0.434\,C_\alpha\,L}{(1+e)t} \tag{1c}$$

where t = time, t_p = duration of primary consolidation, L = thickness of compressible layer, and $C_\alpha = \Delta e / \Delta \log t$ is the secondary compression index.

2.1 *Compressible layer without vertical drains*

For one-dimensional consolidation with single drainage, the velocity of water at the drainage boundary is equal to the rate of compression (Fig. 1) :

$$(\upsilon_v)_{z=0} = \frac{\partial L}{\partial t} = \frac{\partial s}{\partial t} \tag{2}$$

According to the Darcy flow equation, the velocity of water at the drainage boundary is :

$$(\upsilon_v)_{z=0} = \frac{k_v}{\gamma_w}\left(\frac{\partial u'}{\partial z}\right)_{z=0} \tag{3}$$

where k_v = coefficient of permeability in vertical direction, γ_w = unit weight of water, u' = excess porewater pressure, u'_m = maximum excess porewater pressure, and z = vertical distance from drainage boundary. Note that $\frac{1}{\gamma_w}\left(\frac{\partial u'}{\partial z}\right)_{z=0}$ is the hydraulic gradient at the drainage boundary.

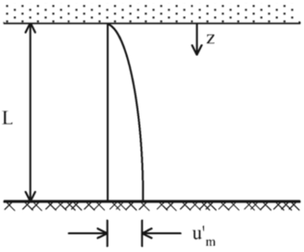

Fig. 1 Excess porewater pressure distribution for vertical compression and vertical water flow

Assuming a parabolic distribution of excess porewater pressure between the drainage boundary and impermeable boundary (Mesri and Feng, 1992) :

$$\left(\frac{\partial u'}{\partial z}\right)_{z=0} = \frac{2}{L}u'_m \tag{4}$$

substituting Eq. 4 into Eq. 3 :

$$(\upsilon_v)_{z=0} = \frac{k_v}{\gamma_w}\frac{2}{L}u'_m \tag{5}$$

substituting Eqs. 1c and 5 into Eq. 2 and rearranging :

$$u'_m = \frac{0.434\gamma_w C_c L^2}{2(1+e)k_v t}\frac{C_\alpha}{C_c} \tag{6}$$

Duration of primary consolidation, t_p, is frequently defined as t_{95} corresponding to 95% average degree of consolidation. Using the solution of Terzaghi theory of consolidation for a linear distribution of initial excess porewater pressure with depth in a single homogeneous layer (Terzaghi et al., 1996), $t_{95} = 1.13L^2 / c_v$.

For $C_k / C_c = 1$, (range 1/2 to 2), where $C_k = \Delta e / \Delta \log k_v$ from Mesri and Rokhsar (1974) :

$$c_v = \frac{k_v(1+e)\sigma'_v}{0.434\gamma_w C_c} \tag{7}$$

and

$$t_{95} = \frac{0.490\gamma_w L^2 C_c}{(1+e)k_v \sigma'_v} \tag{8}$$

substituting Eq. 8 into Eq. 6 and introducing t_p :

$$u'_m = \frac{t_p}{t}\frac{\sigma'_v}{2.3}\frac{C_\alpha}{C_c} \tag{9}$$

Therefore, the maximum excess porewater pressure at the EOP consolidation, or at $t = t_p$ often estimated by the time corresponding to an average degree of primary consolidation at 95%, is a function of the final consolidation pressure, σ'_v, and C_α / C_c .

Values of u'_m / σ'_v as a function of C_α / C_c are listed in Table 1. Therefore, excess porewater pressure at t_p resulting from tendency for secondary compresion is about 1 to 3% of σ'_v. Note that at $t = 10\ t_p$, i.e. after one log cycle of secondary compression, this excess porewater pressure would decrease by a factor of 10. Note also that Eq. 6 may give an impression that u'_m strongly depends on L. However, as L increases, so does t. In this respect Eq. 9 is more meaningful.

Table 1. Magnitude of u'_m / σ'_v at $t_p = t_{95}$ as a function of C_α / C_c

C_α / C_c	u'_m / σ'_v, (%)
0.03	1.3
0.04	1.7
0.05	2.2
0.06	2.6
0.07	3.0

2.2 *Compressible layer with vertical drains*

The rate of vertical compression, assuming equal vertical strains and no vertical flow within the layer, is equal to the rate of radial flow into the vertical drain (Fig. 2) :

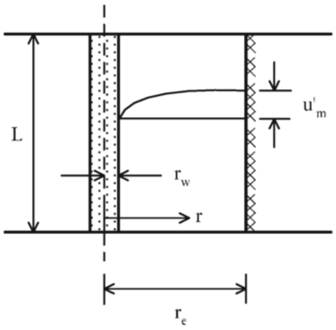

Fig. 2 Excess porewater pressure distribution for vertical compression and radial flow into vertical drain

$$\frac{k_h}{\gamma_w}\frac{du'}{dr}2\pi r L = \pi\left(r_e - r^2\right)\frac{dL}{dt} \tag{10}$$

where k_h = permeability in horizontal direction, r = radial distance from centerline of vertical drain, and r_e = 0.525DS for vertical drains installed in a triangular pattern at a spacing of DS. Equation 10 is rearranged to separate variables r and t :

$$\frac{\gamma_w}{k_h}\frac{1}{2L}\frac{dL}{dt} = \frac{r}{r_e{}^2 - r^2}\frac{du'}{dr} = c \tag{11}$$

where c is a constant independent of r and t.
The solution of Eq. 11 involving u' leads to :

$$u' = c\left(r_e^2 \ln \frac{r}{r_w} - \frac{r^2 - r_w^2}{2} \right) \tag{12}$$

The maximum excess porewater pressure is at $r = r_e$:

$$u'_m = c\left(r_e^2 \ln \frac{r_e}{r_w} - \frac{r_e^2 - r_w^2}{2} \right) \tag{13}$$

Substituting for c from Eq. 11 :

$$\frac{dL}{dt} = \frac{2 k_h L u'_m}{\gamma_w r_e^2 \left[\ln \frac{r_e}{r_w} - \frac{1 - (r_w / r_e)^2}{2} \right]} \tag{14}$$

Let $n = r_e / r_w$ and $1 / n^2 \approx 0$ (n for prefabricated drain installation is generally greater than 10) :

$$\frac{dL}{dt} = \frac{2 L k_h u'_m}{\gamma_w r_e^2 \left[\ln(n) - \frac{1}{2} \right]} \tag{15}$$

Substituting Eqs.1c and 15 into Eq. 2 :

$$u'_m = \frac{0.434\, \gamma_w C_c r_e^2 \left[\ln(n) - \frac{1}{2} \right]}{2\,(1+e)\, k_h t} \frac{C_\alpha}{C_c} \tag{16}$$

The time factor for radial flow is, $T_r = c_h t / r_e^2$, where c_h is coefficient of consolidation for vertical compression and radial flow:

$$c_h = \frac{k_h (1+e) \sigma'_v}{0.434\, \gamma_w C_c} \tag{17}$$

and

$$t = \frac{0.434 \gamma_w T_r r_e^2 C_c}{k_h (1+e) \sigma'_v} \tag{18}$$

The relationship between average degree of consolidation, U for equal strain consolidation with vertical drains is (Terzaghi et al., 1996):

$$U = 1 - \exp\left(-\frac{2}{F(n)} T_r \right) \tag{19}$$

where $F(n) = \ln(n) - 3/4$. Using $n = 10$, $F(n) = 1.553$ and $\ln(n) - 1/2 = 1.803$.
For $U = 95\%$, from Eq. 19 $T_r = 2.34$, and from Eq. 18 :

$$t_{95} = \frac{1.016 \gamma_w r_e^2 C_c}{(1+e) k_h \sigma'_v} \tag{20}$$

Substituting Eq. 20 into Eq. 16 and introducing t_p :

$$u'_m = \frac{t_p}{t} \frac{\sigma'_v}{2.6} \frac{C_\alpha}{C_c} \tag{21}$$

A value of $n = 30$, $F(n) = 2.651$, and $\ln(n) - 1/2 = 2.901$, together with $U = 95\%$, $T_r = 3.971$ also lead to Eq. 21. Therefore, for all practical purposes u'_m is independent of vertical drain spacing. In the present derivation, drain resistance and smear effect have been ignored (Mesri and Lo, 1991). Note that Eqs. 9 and 21 are only applicable for t equal or greater than t_p.

3 EXPERIMENTAL RESULTS

During the past three decades an extensive series of one-dimensional consolidation tests with porewater pressure measurement have been conducted at the University of Illinois in Urbana-Champaign (e.g. Mesri and Choi, 1980; Mesri and Cepeda-Diaz, 1987; Mesri and Castro 1987; Mesri and Feng, 1991, 1992; Mesri and Hayat, 1993; Mesri et al., 1994, 1997). In these tests, on undisturbed specimens of soft clays, fibrous peats, and clay shales, drainage was allowed from the top and porewater pressure was measured at the bottom of specimens. Examples are shown in Figs. 3 – 6, and data from 86 pressure increments of 8 soft clays, 56 increments of 7 clay shales, and 13 increments of 2 fibrous peats are plotted in Fig. 7. The value of excess porewater pressure measured at t_p = the Casagrande t_{100} is compared with excess porewater pressure computed using Eq. 9 i.e. at t_p = the Terzaghi t_{95}.

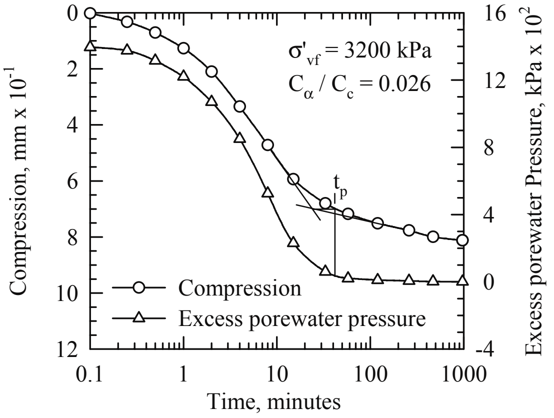

Fig. 3 Boston Blue clay

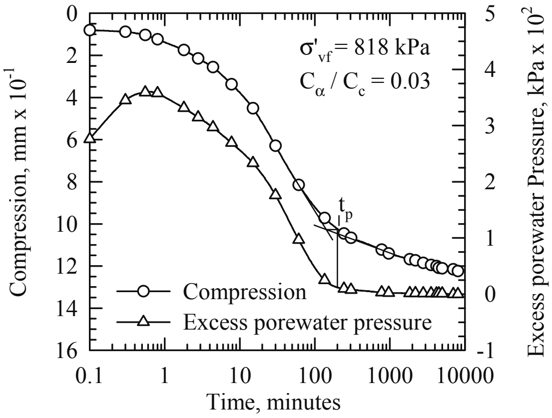

Fig. 4 Batiscan clay

Even though these tests were carried out in special odeometers specifically designed and constructed for accurate measurement of porewater pressure for consolidation pressures up to 15 MPa, and back pressures in the range of 280 to 3,450 kPa were utilized, the primary objective was not precise measurement of excess porewater pressure during secondary compression. This may explain part of the scatter in the data in Fig. 7. However, there is generally good correlation between the measured and computed values of u'_m.

The analyses and data suggest that for soft clay and silt deposits and for fibrous peats, which are rarely subjected to σ'_v values greater than 500 kPa the maximum excess porewater pressure at EOP is expected to be less than 10 kPa. However, for clays and shales subjected to σ'_v values of 8 to 10 MPa, EOP u'_m may be as high as 100 kPa.

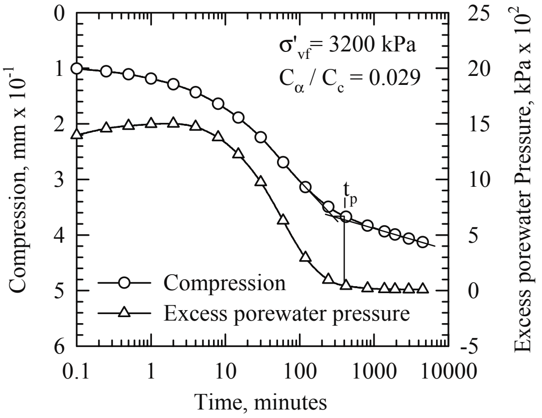

Fig. 5 Bearpaw shale

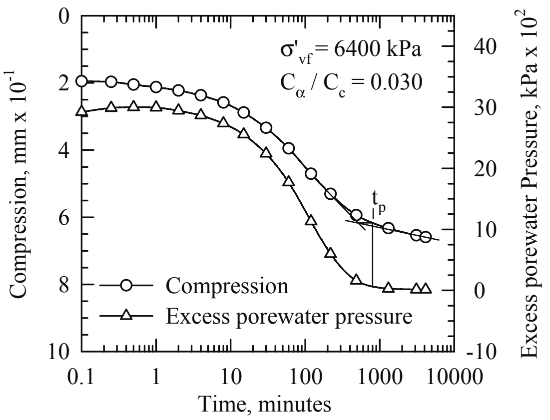

Fig. 6 Pierre shale

4 CONCLUSION

Analysis of rate of secondary compression and oedometer measurements confirm excess porewater pressures during secondary compression. The magnitude of the excess porewater pressure is directly related to C_α / C_c and the consolidation pressure, σ'_v, and is inversely related to t / t_p . The maximum value of excess porewater pressure, u_m', occurs at the beginning of secondary compression stage, i.e. $t / t_p = 1$, and corresponds to u'_m / σ'_v values in the range of 1 to 3%. For soft clay deposits, with u'_m / σ'_v near 2%, which are rarely subjected to σ'_v values in excess of 500 kPa, EOP u'_m is often near 1 kPa and is not expected to exceed 10 kPa. However, for clays and shales subjected to high consolidation pressures, EOP u'_m could be as high as 100 kPa. Information on the order of magnitude of excess porewater pressures associated with secondary compression is useful for interpreting field observation of porewater pressure intended for establishing the progress of primary consolidation.

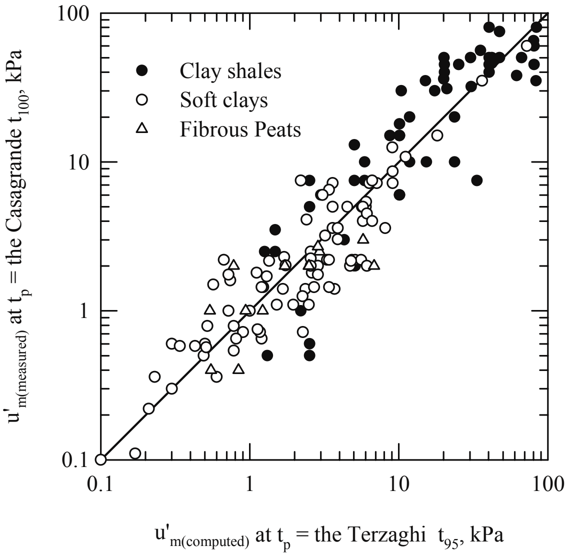

Fig. 7 Computed and measured excess porewater pressure at EOP consolidation

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