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# Dynamic response of a single pile embedded in semi-infinite saturated poroelastic medium using hybrid elements

## Réponse dynamique d'un pieu seul encastré en milieu poro-élastique saturé semi-infini en utilisant des éléments hybrides

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### ABSTRACT

This paper presents a systematic procedure for the dynamic behavior of a single pile embedded in saturated semi-infinite poroelastic medium. The study employs a new developed element, which its main role is to satisfy the radiation condition and a more sophisticated finite element formulation to compute the dynamic response of the pile. The method is verified for a single pile and compared to known solution in order to assess the reliability of the proposed model. A parametric study explores the influence of main factors and some conclusions are made.

### RÉSUMÉ

Cet article présente une procédure systématique de calcul du comportement dynamique d'un pieu seul enfoui dans un milieu poro-élastique saturé semi-infini. L'étude fait appel à l'utilisation de nouveaux éléments récemment développés, dont le rôle est de satisfaire aux conditions de radiation, ainsi qu'à une formulation par éléments finis plus sophistiquée servant à calculer la réponse dynamique du pieu. Afin de s'assurer de la validité du modèle proposé, la méthode est vérifiée pour un pieu seul et elle est comparée à des solutions existantes. Une analyse paramétrique, s'intéressant à l'influence des principaux facteurs, permet de tirer diverses conclusions.

### 1 INTRODUCTION

The dynamic response of a pile embedded in an elastic half-space has been the subject of numerous studies. In general, two types of studies can be distinguished to describe the dynamic behavior of a single pile embedded in semi-infinite elastic medium. In one method, the kinematics of pile and surrounding soil is simplified ignoring the effect of some components of wave assuming simple geometry, Konagai and Nogami (1985), and Gazetas (1991).

The other one is based on rigorous method that is somewhat complicated, Mamoon et al. (1990), Kaynia and Kausel (1991). This method is not feasible for engineering practice because of its complexity for computation, although it can apply to any complicated phenomenon of soil-pile interaction problems.

This paper is concerned primarily with Hankuno (1973) solution for the dynamic response of soil-pile system representing pile as a lumped mass rod and soil as a semi-infinite elastic one phase solid. The research reported herein is an extension of the work presented by Konagai and Matsumoto (1984). They modified the Hankuno approach by replacing the

dynamic Mindlin (1964) solution by the dynamic Kelvin solution combined with mirror image technique satisfying free stress boundary condition at surface, approximately.

Recently, within the framework of Konagai and Hakuno approach, Noorzad and Konagai (1994) solved the buried source Lamb problem in saturated poroelastic medium satisfying the free-stress boundary condition at the surface in exact form. The results of these analyses are presented here with a methodology that allows for a dynamic analysis, which involves radiation element and finite element formulation.

### 2 MATHEMATICAL MODELING AND FORMULATION

In this model, a pile foundation is expressed as a row of rigid circular disks, the so-called radiation element, which represents the propagation of wave from pile foundation to the unbounded soil medium. The rigid circular disks are connected by linear rod elements in the case of vertical excitation and Bernoulli-Euler beam elements in the case of horizontal and flexural excitation, as shown in Figure 1.

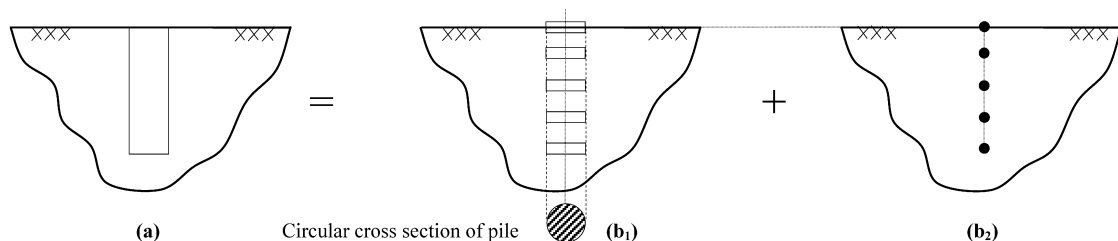


Figure 1. Simulation model for pile foundation; (a) real model, (b) proposed model (b<sub>1</sub>) radiation element, (b<sub>2</sub>) finite element

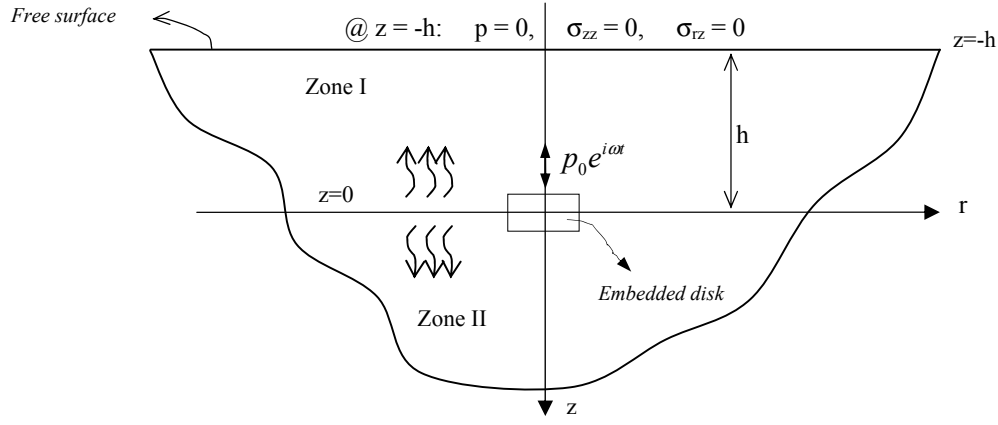


Figure 2. Embedded rigid massless circular plate under vertical excitation in saturated poroelastic medium

The dynamic stiffness of pile embedded in soil can be determined analytically by calculating the dynamic stiffness of embedded rigid disks, Noorzad and Konagai (1994), assembling by beam elements.

The main advantage of this formulation is that the radiation boundary and free surface conditions are exactly satisfied.

### 2.1 Vertical dynamic stiffness of rigid massless circular plate

Considering the radiation element as a rigid massless circular plate embedded in poroelastic medium (Fig. 2), the dynamic stiffness of radiation element has been determined according to the following equations together with the appropriate boundary conditions. For more details, the reader is referred to Noorzad and Konagai (1994).

$$\mu^* \nabla^2 u_j + (\lambda^* + \mu^* + Q_f) u_{i,j} + Q_f w_{i,j} = \rho_f \ddot{u}_j + \rho_f \ddot{w}_j \quad (1-a)$$

$$Q_f (u_{i,j} + w_{i,j}) = \rho_f \ddot{u}_j + \frac{\rho_f}{n} \dot{w}_j + b \dot{w}_j \quad (1-b)$$

where  $u_i$  = displacement component of solid phase;  $w_i$  = relative displacement component of liquid phase relative to solid phase;  $\lambda^* = \lambda(1+2i\beta)$ ;  $\mu^* = \mu(1+2i\beta)$ ;  $\lambda, \mu$  = Lamé coefficients of soil in drained condition;  $\beta$  = hysteresis damping coefficient;

$i = \sqrt{-1}$ ;  $Q_f$  = compressibility of fluid =  $\frac{p_a k_f}{(p_a + (1-s)k_f)n}$ ;

$k_f$  = Bulk modulus of fluid phase;  $n$  = porosity of soil;

$s$  = degree of saturation,  $0.95 \leq s \leq 1$ ;  $p_a$  = air pressure;

$\rho_f$  = density of fluid phase;  $\rho = (1-n)\rho_s + sn\rho_f$ ;  $\rho_s$  = density

of solid phase;  $b = \frac{g\rho_f n}{k}$ ;  $g$  = ground acceleration;

$k$  = coefficient of soil permeability; and  $\omega$  = frequency of excitation.

Two sets of boundary conditions are associated with equations (1-a) and (1-b).

$$\begin{aligned} @ z = -h \text{ (free stress and drained conditions)} \\ (\sigma_{zz})_I = 0 \quad (\sigma_{rz})_I = 0 \quad (p)_I = 0 \end{aligned} \quad (2)$$

where  $p$  is the fluid phase pressure. For convenience, the half space domain is divided into two parts as zone I and II, which is illustrated in Figure 2.

@  $z = 0$

$$\begin{aligned} (u_z)_I = (u_z)_{II}, \quad (u_r)_I = (u_r)_{II}, \quad (w_z)_I = (w_z)_{II}, \quad (w_r)_I = (w_r)_{II}, \\ (\sigma_{zz})_I - (\sigma_{zz})_{II} = f(r), \quad (p)_I = (p)_{II}, \quad (\sigma_{rz})_I = (\sigma_{rz})_{II} \end{aligned} \quad (3)$$

$$\text{and } f(r) = \frac{p_0}{2\pi r_0} \left(1 - \frac{r^2}{r_0^2}\right)^{\frac{1}{2}} \quad (4)$$

where  $p_0$  is the amplitude of load on embedded disk which is transformed from dynamic load at pile head at the mentioned point of pile, and  $r_0$  is the radius of the disk.

### 2.2 Vertical dynamic stiffness of pile element

By solving the dynamic equation of rod in frequency domain, the vertical dynamic stiffness of pile element is obtained.

$$[K_p] = \frac{EA\xi}{\sin(\xi L)} \begin{bmatrix} \cos(\xi L) & -1 \\ -1 & \cos(\xi L) \end{bmatrix} \quad (5)$$

where  $\xi = \left(\frac{m\omega^2}{EA}\right)^{\frac{1}{2}}$ ,  $m$  = unit mass per length of pile,  $E$  =

elastic modulus of pile,  $A$  = area of the cross section of pile. The finite element solution for displacement can be expressed by

$$\begin{aligned} [F_p] &= [K_p] \{u\} \\ [F_s] &= [K_s] \{u\} \end{aligned} \quad (6)$$

where  $[F_p]$  and  $[F_s]$  represent the force transmitted by pile element and soil, respectively. Assuming the full bonding between soil and pile, one may expect the same displacement for both elements.

In accordance to Figure 1, the following equilibrium equation may be written as

$$[F] = [F_p] + [F_s] \quad (7)$$

where  $[F]$  is external force.

Substitution of equation (6) into (7) results in

$$[F] = [K] \{u\} \quad \text{where } [K] = [K_p] + [K_s] \quad (8)$$

$K_p$  and  $K_s$  are dynamic stiffness of pile element and radiation element.

### 3 VERIFICATION

The present study focuses on analytical solution to find out the radiation element stiffness and numerical method to obtain the pile stiffness. Such a combination allows the construction of more realistic simulations, while inducing significant savings.

The proposed approach is verified by comparing predictions with the solutions of rigorous method developed by Kaynia and Kausel (1991) for the case of a single embedded pile. As it is known, the Kaynia and Kausel model is based on boundary element method. Figure 3 shows the non-dimensional vertical impedance versus the dimensionless frequency  $\alpha_0$  with parameters  $\beta=5\%$ ,  $\nu=0.4$ ,  $L/d=15$  and  $E_p/E_s=1000$  for real (stiffness,  $K_{11}$ ) and imaginary (combination of material and radiation damping,  $C_{11}$ ) parts of the impedance function. It is noted that  $\beta$  is damping ratio of soil,  $\nu$ , Poisson ratio,  $L$ , pile length,  $d$ , pile diameter,  $E_p$  and  $E_s$  denote the modulus of elasticity of pile and soil, respectively.

Good agreement is observed between the results of present analysis and the existing solution. The difference at higher frequency is pertinent to the simplifications made.

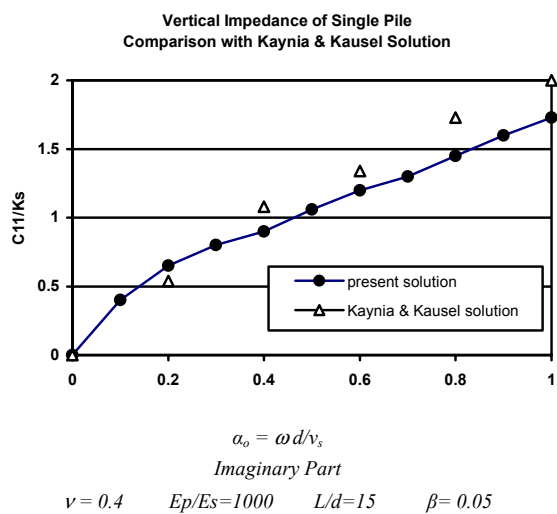
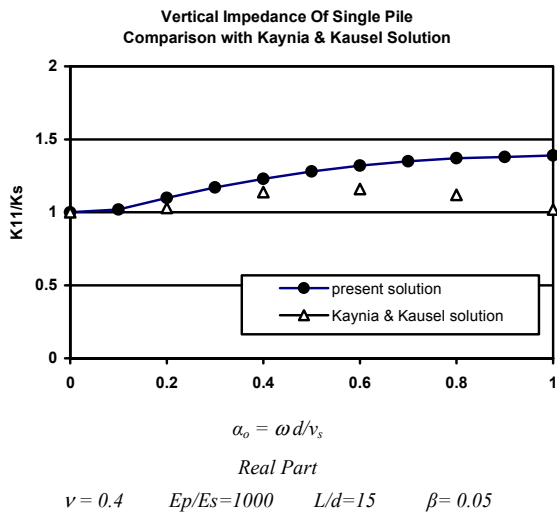


Figure 3. Comparison of the Kaynia's study with this study to compute the pile response

### 4 PARAMETRIC STUDY

To examine the influence of the type of soil, two different values of  $E_p/E_s=100$  (hard soil) and  $E_p/E_s=1000$  (soft soil) for one and two phase materials (dried and saturated conditions) are considered. As shown in Figure 4, the difference between two different cases is more pronounced for high frequency greater than 0.2.

In order to demonstrate the capacity of the model, a parametric study is performed considering the representative values of  $\beta=5\%$ ,  $\nu=0.4$ ,  $L/d=15$ ,  $K_f=2080\text{MPa}$  and different values of porosity 0.3 and 0.375 that correspond to the coefficient of permeability of  $0.001\text{m/s}$  and  $0.01\text{m/s}$ , respectively. The effect of porosity and coefficient of permeability are depicted in Figures 5 and 6 for soft and hard soils.

By keeping the other parameters as constant values, the results indicate that the denser the soil, the higher the vertical impedance.

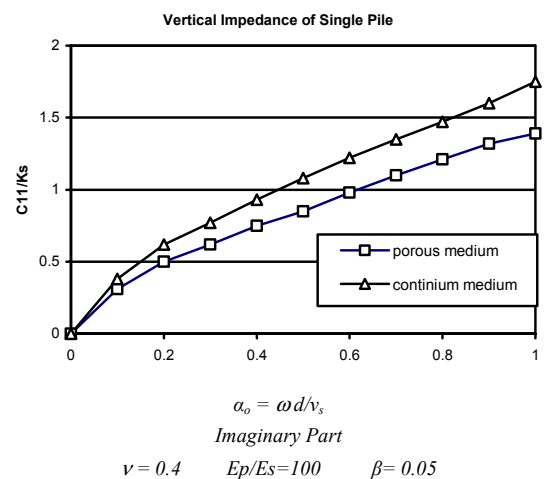
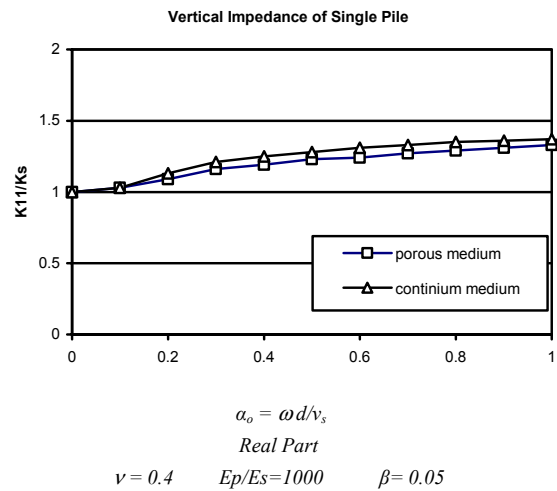


Figure 4. Comparison of vertical impedance of single pile for one and two phase materials

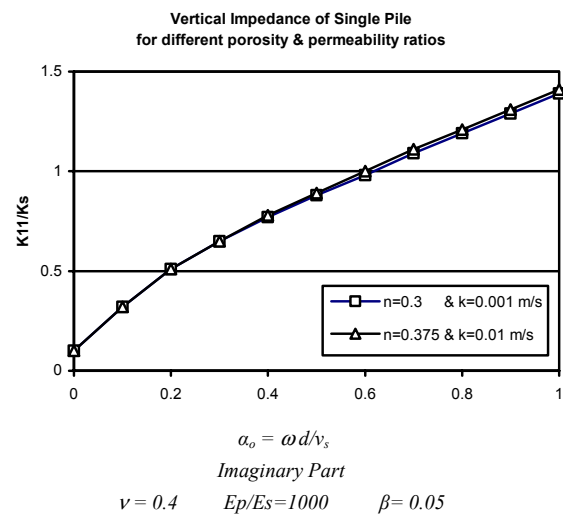
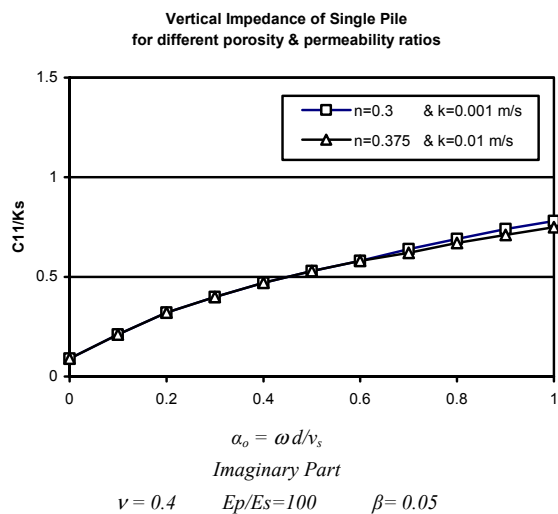
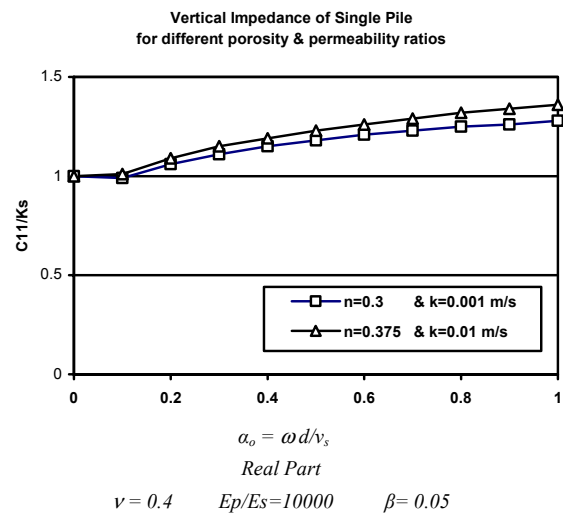
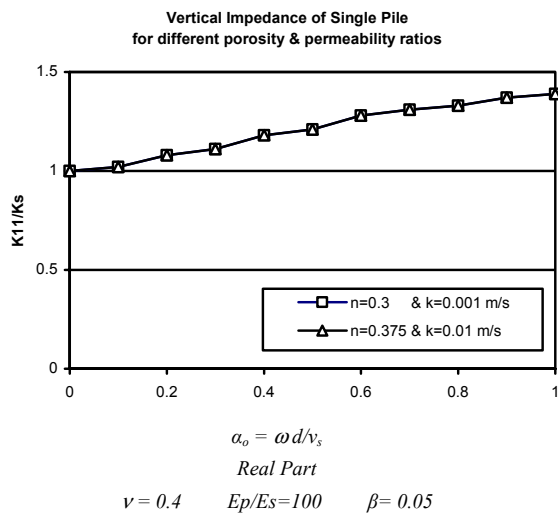


Figure 5. Vertical impedance of single pile for different porosity and permeability coefficients for hard soil

Figure 6. Vertical impedance of single pile for different porosity and permeability coefficient for soft soil

## 5 CONCLUDING REMARKS

Despite the assumptions made to develop this approach, the results are quite acceptable and the conclusions from the present study are as follow:

- The paper outlines the methodology to analyze the dynamic response of a single pile. It is also suitable for combining with the finite element method to derive more generally applicable and more computationally efficient solution.
- It is found that for saturated soil, modeling should be based on saturated poroelastic medium formulation. The results obtained are not reliable considering only one phase material.
- In the case of soft soil, the impedance function of pile does not change for different values of permeability coefficient. However, for the hard soil, the effect of permeability coefficient on impedance function is more pronounced.
- It is worthy to mention that the predictions of the model are in agreement with earlier results, while its simplicity offers a versatile alternative to rigorous solutions.

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