Proceedings of the 16th International Conference on Soil Mechanics and Geotechnical Engineering

doi:10.3233/978-1-61499-656-9-2069

End-bearing capacity and tip settlement of piles in sandy soils

Résitance ultime et tassement de pointe des pieux aux milieux sableux

J. Yang, L.G. Tham, P.K.K. Lee & F. Yu
Department of Civil Engineering, The University of Hong Kong, Hong Kong, China
junyang@hku.hk

ABSTRACT
A new approach is presented for analysis of the end bearing capacity and load settlement behavior of piles installed in sandy deposits. The approach is developed based on the cavity expansion theory, the finite strain theory and the principle of energy conservation, and takes account of the reduction in soil strength and the particle crushibility under high stress in a practical yet rational manner. The accuracy and versertility of the approach are assessed using a number of field case histories.

RÉSUMÉ
La communication présente une nouvelle approche de l'analyse de la résitance ultime et du comportement de déplacement du charge des pieux dans le sable. La approche est développée découlant de la théorie de l'expansion de cavité, de la théorie des déformations finites et du principe de conservation d'énergie. Elle rend compte à la rèduction de la résistance du sol et au cassement de grain en raison des hautes contraintes dans une facon practique et résonable. La précision du calcul est vérifiée par plusieurs essais in-situ.

1 INTRODUCTION

The capacity of piles installed in sandy soil has been identified as being the area of greast uncertainty in foundation design (Randolph et al., 1994). While there are various means for estimating the capacity (Meyerhof, 1976; Poulos, 1989; Randolph, 2003), such as the limit equilibrium, the cavity expansion, the strain path and the finite element approaches, current design methods still depend largely on empiricism and pile load tests. The failure to understand the underlying mechanisms is due mainly to the complexity of the interaction between pile and surrounding soil and the complexity of the sandy soil behavior.

Based on existing spherical cavity expansion solutions, a new approach is proposed in this paper for analyzing piles driven/jacked in sandy soil, where the reduction in soil strength under high stress is taken into account in a rational and practical manner. The initial strength and stiffness parameters can be derived from the overburden pressure and SPT-N value at the pile tip and are varied as the cavity expands or as the soil approaches the critical state condition. Furthermore, the mean volumetric strain in the plastic zone around the cavity is determined analytically by adopting the finite strain theory, and therefore, it is no longer necessary to estimate such strain from laboratory test results. By incorporating the energy conservation principle, the approach is further extended to allow prediction of the load settlement behavior of piles.

2 PILE END-BEARING CAPACITY

It is well recognized that pile tip under vertical compressive loading behaves similarly to the expansion of a spherical cavity, which can lead to soil yielding and form a plastic zone surrounded by an elastic zone. Based on the cavity expansion theory, Vesic (1972) derived a general solution for the cavity limiting pressure p_u as follows:

$$p_u = c'\cot\phi'(F-1) + p_0'F + \gamma_w h_w \tag{1}$$

where c' and ϕ' are strength parameters of soil mass; the notations of $\gamma_w h_w$ and p_0' denote respectively the water pressure and the isotropic effective stress prior to cavity expansion; F is the cavity pressure factor, relating to the reduced rigidity index I_{rr} as:

$$F = \frac{3(1+\sin\phi')}{3-\sin\phi'}\left(I_{rr}\right)^{(4\sin\phi')/(3+3\sin\phi')} \tag{2}$$

$$I_{rr} = \frac{\eta I_r (1+\Delta)}{1+\eta I_r \Delta} \tag{3}$$

$$\eta = \frac{3-\sin\phi'}{3\cos\phi'} \tag{4}$$

where Δ is the mean volumetric strain within the plastic zone, I_r is the rigidity index depending on the ratio of shear modulus to strength:

$$I_r = \frac{G}{(c'+p_0'\tan\phi')} \tag{5}$$

The mean volumetric strain Δ plays a key role in estimating the cavity limiting pressure. Laboratory triaxial tests are usually required in determining the value of Δ (Vesic, 1972; Yasufuku & Hyde, 1995), causing inconvenience in practical use. In consideration of the large deformation due to pile tip penetration, the plastic strain field around the pile tip should be described more accurately by using the finite strain theory (Gupta, 1991). In this context, the value of Δ can be calculated analytically as:

$$\Delta = \frac{3}{R_p^3 - R_u^3}\int_{R_u}^{R_p}(\varepsilon_{vp1}+\varepsilon_{vp2})r^2 dr \tag{6}$$

where R_u is the radius of cavity at the limiting state and R_p is the radius of the induced plastic zone; ε_{vp1} is the plastic volumetric strain due to isotropic compression and ε_{vp2} is the volumetric strain due to shear distortion. They can be determined by

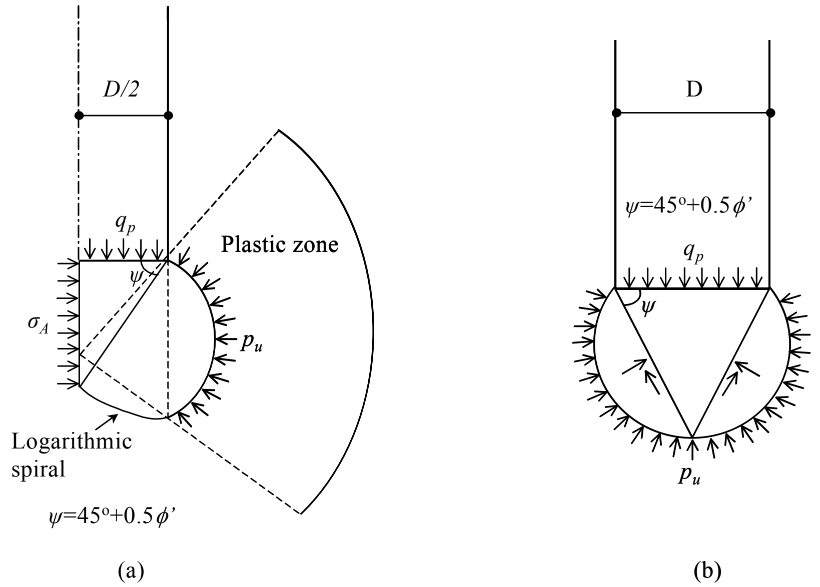

Figure 1. Typical failure patterns assumed for piles in sandy soil: (a) Local shear failure, and (b) punching shear failure

$$\varepsilon_{vp1} = 3(\varepsilon_1 - \varepsilon_1^2) + \varepsilon_1^3 \tag{7}$$

$$\varepsilon_{vp2} = \frac{(3\varepsilon_2^2 + \varepsilon_2^3)}{4} \tag{8}$$

$$\varepsilon_1 = \left(\frac{1-2\upsilon}{E}\right)\left(\frac{(3-\sin\phi')\sigma_{rp} - 4c'\cos\phi'}{3(1+\sin\phi')} - p_0'\right) \tag{9}$$

$$\varepsilon_2 = -\frac{4(1+\upsilon)}{3E}\frac{(\sin\phi'\sigma_{rp} + c'\cos\phi')}{(1+\sin\phi')} \tag{10}$$

where E and υ are the elastic modulus and the Poisson's ratio of the soil, respectively; σ_{rp} is the radial stress in the plastic zone. Note that the derivation of the mean volumetric strain involves the principle of superposition.

Further, the cavity limiting pressure can be linked with the end-bearing capacity of pile q_b as:

$$q_b = \lambda p_u + (\lambda - 1)(c'\cot\varphi' - \gamma_w h_w) \tag{11}$$

in which λ is a factor accounting for the failure pattern of the soil beneath the pile tip and normally is a function of the angle of shearing resistance of the soil.

While a number of failure patterns under piles have been proposed, the local shear or punching shear failure, such as those shown in Fig. 1, are thought to be dominant for piles in sandy soil in many cases. Hence, the following expressions for the factor λ, developed mainly based on the local or punching shear failure mechanisms, are used in the calculation of q_b:

$$\lambda_v = \frac{\exp(0.5\pi - \phi')\tan\phi'}{(1-\sin\phi')} \tag{12}$$

$$\lambda_{sh} = \tan^2\left(\frac{\pi}{4} + \frac{\phi'}{2}\right) \tag{13}$$

$$\lambda_r = 1 + 1.732\tan\phi' \tag{14}$$

$$\lambda_{yh} = \frac{1}{(1-\sin\phi')} \tag{15}$$

Equations (12)-(15) are from Vesic (1972), Sayed and Hamed (1987), Randolph et al. (1994) and Yasufuku and Hyde (1995), respectively.

3 SETTLEMENT AT PILE TIP

In the context of performance-based design, a better understanding of the load-settlement behavior of piles becomes increasingly important. The hyperbolic function has been well accepted for evaluating the pile settlement in the virgin loading. In this study, it is assumed that the pile tip resistance q_t and the tip settlement, expressed in the normalized form, (δ_t/D), obeys the hyperbolic relation as:

$$q_t = \frac{(\delta_t/D)}{n + m(\delta_t/D)} \tag{16}$$

in which D is the pile diameter, n and m are two parameters usually determined on the basis of pile load test data (e.g. Hirayama, 1990). Here, a method that is of theoretical nature is proposed for determining the values of n and m. Assuming that the pile tip settlement at the state of limiting pressure is s_t, the two parameters can thus be taken as

$$m = \frac{1}{q_b} \qquad \frac{1}{n} = \frac{q_b}{(s_t/D)} \tag{17}$$

It is reasonable to relate the tip settlement s_t with the radius of the cavity R_u by

$$s_t = (16R_u^3/3D^2) \tag{18}$$

Further, the radius R_u can be determined by using the principle of energy conservation, which specifies that the work input to expand the cavity is equal to the work consumed by the soil mass. The consumed energy is divided into three components that can be calculated as:

$$E_1 = \iiint (\frac{1}{3}\sigma_{rp} + \frac{2}{3}\sigma_{\theta p} - p_0')\varepsilon_{vp1} dv_p \quad (19)$$

$$E_2 = \iiint (\frac{2}{3}\sigma_{rp} - \frac{2}{3}\sigma_{\theta p})\varepsilon_{vp2} dv_p \quad (20)$$

$$E_3 = \iiint (\frac{2}{3}\sigma_{re} - \frac{2}{3}\sigma_{\theta e})\varepsilon_{ve2} dv_e \quad (21)$$

where σ_{rp} and $\sigma_{\theta p}$ are the stress components in the radial and circumstantial directions in the plastic zone, σ_{re} and $\sigma_{\theta e}$ are the stress components in the radial and circumstantial directions in the elastic zone.

4 STATE-DEPENDENT SOIL PROPERTIES

The behavior of sandy soil is complex and influenced by various factors. The stress level and the void ratio/relative density are identified as being dominant. Both the shear strength and the stiffness of sandy soil depend strongly on the two factors (Bolton, 1986; Yang & Li, 2004), rendering it necessary to take account of them in estimating the capacity and settlement of piles in sandy soil. A practical framework is proposed in this study to allow for the effects of stress and relative density. While it is preliminary, the framework has sufficient flexibility to allow refinements.

Firstly, the in-situ relative density of sandy soil is evaluated based on the Standard Penetration Test (SPT), one of the commonly used field tests in site investigation. To date, a number of empirical correlations have been developed between the SPT-N value and the relative density (e.g. Cubrinovski & Ishihara, 1999). Based on the available database and for the first instance, the following correlation is suggested for practical use:

$$D_r = 0.12\left(\frac{98N^2}{\sigma_v'}\right)^{0.25} \quad (22)$$

where σ_v' is the effective overburden pressure in the units of kPa.

As far as the friction angle of the sandy soil is concerned (Bolton, 1986; Yang & Li, 2004), the following relation is suggested to simultaneously account for the effects of stress and relative density:

$$\tan\phi' = 0.634\exp(0.413D_r)\left(\frac{p'}{p_a}\right)^{-0.08} \quad (23)$$

in which p_a=100 kPa is a reference pressure and p' is the effective mean normal stress. Since the friction angle will approach a steady or critical state value as particles crush under very high stress (Vesic & Clough, 1968; Yang & Li, 2004), Eq. (23) should be supplemented with the condition that ϕ' should approach the critical state friction angle under high stress.

Fig.2 shows the variations of the friction angle with the stress and relative density. While in reality the critical state friction angle may vary with the grading and angularity of soil, the range of its values for many sandy soils is limited. As a first approximation, a typical value (32 deg) is assumed in Fig. 2.

The stiffness of sandy soil has been extensively studied through laboratory experiments and some empirical relations are available for characterizing the influence of stress and relative density. In the first instance, the following relation (Lo Presti, 1987) is used in this study:

$$\left(\frac{G_0}{p_a}\right) = \mu\exp(0.7D_r)\left(\frac{p'}{p_a}\right)^{0.43} \quad (24)$$

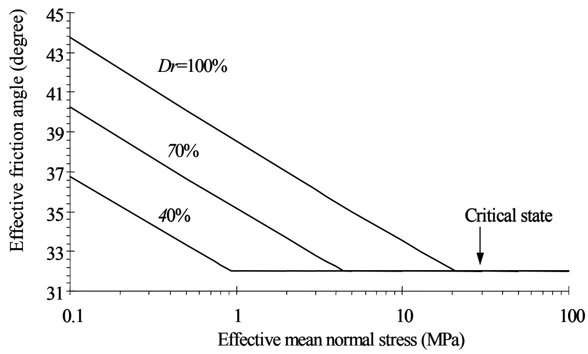

Figure 2. State-dependent friction angle of sandy soil

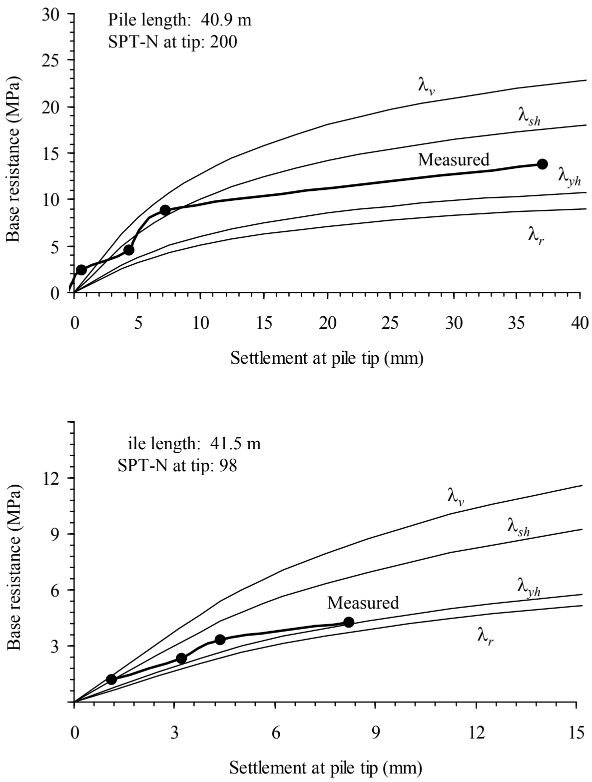

Figure 3. Predicted and measured load-settlement curves

where G_0 is the initial shear modulus, p_a is the reference pressure (100 kPa), and μ is a parameter accounting for the particle distribution. For many sandy soils the value of μ can be approximately taken as 75.

5 VERIFICATION USING LOAD TEST DATA

The proposed method takes into account the dependency of the soil strength and stiffness on the stress and relative density, which will vary during the loading process. An iterative scheme is therefore needed to tackle the complex inter-relations in the calculation. The accuracy of the proposed approach in estimating the end-bearing capacity of piles in sandy soils is demonstrated in Table 1 where a number of pile load tests are summarized.

Table 1: Predicted and measured end-bearing capacity of piles

No.	Data source	Pile dimensions		Soil description		Measured q_p (MPa)	Predicted q_p (MPa)
		D (m)	L (m)	Soil type	SPT-N		
1	Vesic (1970)	0.46	12.0	Ogeechee River sand	66	12.6	13.31: λ_{yh}
2			15.0		51	14.6	12.86: λ_{yh}
3	BCP Committee (1971)	0.20	4.0	Fine sand	20	5.8	5.94: λ_{yh}
4			11.0	Dense sand	48	25.0	26.03: λ_v
5	Coyle & Castello (1981) Arkansas #1, #2, #3	0.37	16.2	Silty sand with clay	42	7.2	8.42: λ_{sh}
6		0.46	16.1		31	7.7	7.57: λ_{sh}
7		0.52	16.2		42	7.0	8.42: λ_{sh}
8	Coyle & Castello (1981) Jonesville #1, #2, #3	0.52	11.6	Silty sand with clay	84	13.1	14.14: λ_v
9			13.7		90	8.9	7.19: λ_{yh}
10			16.5		97	12.2	13.04: λ_{sh}
11	Coyle & Castello (1981) Low-Sill #2, #4, #6	0.53	19.8	Fine to medium sand	97	7.3	7.88: λ_{yh}
12		0.43	20.1		97	13.1	13.11: λ_{sh}
13		0.46	19.8		97	8.8	7.87: λ_{yh}
14	JGS (1993)	1.50	22.4	Sand	30	2.9	4.76: λ_r
15			26.5		30	4.2	5.20: λ_r
16			32.0		30	5.7	5.78: λ_r

For each field case, using Eqs. (12)-(15) will give rise to four different predictions. The predicted value shown in Table 1 for each case is the one that is closest to the measured base resistance. It appears that the proposed method can provide fairly good prediction of the end-bearing capacity. Note that the major input required for the calculation includes only the SPT-N value and overburden pressure at pile tip as well as the pile dimensions.

The load-settlement behavior predicted by the present model for two test piles (Yang et al., 2004) is shown in Fig. 3. Both piles are steel H-piles jacked into the ground comprised mainly of completely decomposed granite (CDG), which is a type of residual soil with its engineering property close to silty sand (Lumb, 1965). It is of interest to note from Fig. 3 that the use of λ by Vesic (1972) will lead to an upper bound prediction while the factor due to Randolph et al. (1994) gives a lower bound. In general, the use of λ by Yasufuku and Hyde (1995) and by Sayed and Hamed (1987) may produce a reasonably good fit to the test data.

6 CONCLUSIONS

A framework has been proposed for estimating both the end bearing capacity and the load-settlement behavior of piles in sandy deposits. The framework is developed based on the theory of cavity expansion combined with the finite strain theory and the energy conservation principle. Using a number of pile load tests, it has been demonstrated that the proposed model can provide fairly accurate prediction of the end-bearing capacity and the settlement of piles in sandy soils. While it contains many aspects that require further research, the new framework is linked closely with the physical process involved and requires very simple input for calculation. Moreover, it has sufficient flexibility to allow refinement and modification.

ACKNOWLEDGMENTS

This work is supported by the Research Grants Council of the HKSAR Government (HKU7131/03E) and The University of Hong Kong.

REFERENCES

BCP Committee 1971. Field tests on piles in sand. *Soils Founds*, 11(2), 29-50.

Bolton, M. D. 1986. The strength and dilatancy of sands. *Géotechnique*, 36(1), 65-78.

Coyle, H. M. and Castello, R. R. 1981. New design correlations for piles in sand. *J. Geotech. Engrg. Div.*, ASCE, 107(GT7), 965-986.

Cubrinovski, M. and Ishihara, K. 1999. Empirical correlation between SPT *N*-value and relative density for sandy soils. *Soils Founds*, 39(5), 61-71.

Gupta, R. C. 1991. Finite strain analysis for deep cone penetration. *J. Geotech. Engrg.*, ASCE, 117(10), 1610-1630.

Hirayama, H. 1990. Load-settlement analysis for bored piles using hyperbolic transfer functions. *Soils Founds*, 30(1), 55-64.

JGS 1993. *Standard for Vertical Load Tests of Piles*. Japanese Geotechnical Society.

Lo Presti, D. 1987. *Mechanical behaviour of Ticino sand from resonant column tests*. Ph.D. Thesis, Politecnico di Torino.

Lumb, P. 1965. The residual soils in Hong Kong. *Geotechnique*, 15, 180-194.

Meyerhof, G. G. 1976. Bearing capacity and settlements of pile foundations. *J. Geotech. Engrg. Div*., ASCE, 102(GT3), 197-228.

Poulos, H. G. 1989. Pile behaviour: theory and application. *Géotechnique*, 39(3), 365-415.

Randolph, M. F. 2003. Science and empiricism in pile foundation design. *Géotechnique*, 53(10), 847-875.

Randolph, M. F., Dolwin, J. and Beck, R. 1994. Design of driven piles in sand. *Géotechnique*, 44(3), 427-448.

Sayed, S. M. and Hamed, M. A. 1987. Expansion of cavities in layered elastic system. *Int. J. Numer. Analyt. Methods Geomech*., 11, 203-213.

Vesic, A. S. 1970. Tests on instrumented piles, Ogeechee River site. *J. Soil Mech. Founds. Div*., ASCE, 96(SM2), 561-584.

Vesic, A. S. 1972. Expansion of cavities in infinite soil mass. *J. Soil Mech. Founds. Div*., ASCE, 98(SM3), 265-290.

Vesic, A. S. and Clough, G. W. 1968. Behavior of granular materials under high stresses. *J. Soil Mech. Founds. Div.*, **94**(SM3), 661-688.

Yang, J. and Li, X. S. 2004. State-dependent strength of sands from the perspective of unified modeling. *J. Geotech. Geoenvir. Engrg*., ASCE, 130(2), 186-198.

Yang, J., Tham, L. G., Lee, P. K. K. and Yu, F. 2004. Observed performance of long steel H-piles jacked into sandy soils. *Research Report*, Dept. of Civil Engineering, The Univ. of Hong Kong.

Yasufuku, N. and Hyde, A. 1995. Pile end-bearing capacity in crushable sands. *Géotechnique*, 45(4), 663-676.