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Modeling of soil moisture profile during infiltration into vadose zone

Modélisation de la dynamique de l'humidité du sol s'infiltrant dans la zone vadose

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ABSTRACT

A model of dynamic soil moisture distribution in unsaturated soil is proposed in order to solve Klute's equation simply. This model can represent relationships among soil moisture, depth and elapsed time by using a sigmoid function. By substituting this model into Klute's equation (the governing equation of one-dimensional vertical seepage flow in the unsteady state), Klute's equation can be solved easily and the hydraulic conductivity of unsaturated soil can be obtained.

RÉSUMÉ

Un modèle de distribution de l'humidité du sol dynamique dans le sol non saturé est proposé pour faciliter la résolution de l'équation de Klute. Ce modèle peut représenter la relation entre l'humidité du sol, la profondeur et le temps passé en utilisant une fonction sigmoïde. En remplaçant l'équation de Klute par ce modèle (cette équation est une équation gouvernante du débit d'infiltration vertical à une dimension dans un état instable), elle peut être facilement résolue et le coefficient de perméabilité du sol non saturé peut être obtenu.

1 INTRODUCTION

Unsteady-state methods such as the instantaneous profile method and disk permeameter method (e.g. White et al.,1992) are used to test the permeability of unsaturated soil in the field because they can quickly measure hydraulic conductivity. However, a large hydraulic gradient occurs near the wetting front and many sensors need to be set up, so the methods are not practical for measuring in deep ground. Therefore, a practical, simple and economical method of measuring the hydraulic conductivity of unsaturated soil needs to be developed.

The Boltzmann transformation method (Klute, 1952) is an unsteady-state method that measures instantaneous moisture distribution only. In this method, the hydraulic conductivity of unsaturated soil can be calculated with a specific water capacity. The specific water capacity means the gradient of the soil water characteristic curve, and can be measured using a soil water retentivity test in the laboratory. Although this can be done in a laboratory with horizontal infiltration, it is impossible in the field where large vertical infiltration exists. For developing a method of measuring the hydraulic conductivity of unsaturated soil in the field, the soil moisture profile model (SMPM) is proposed, which can represent dynamic moisture distributions during infiltration into the vadose zone. The author has succeeded in solving Klute's equation (e.g. Klute, 1972) easily with this model and measuring the hydraulic conductivity of unsaturated soil in vertical seepage flow.

2 VERTICAL SEEPAGE FLOW THROUGH UNSATURATED SOIL

2.1 *Richards's Equation and Klute's Equation*

Richards's governing equation (Richards, 1931) of one-dimensional vertical seepage flow is written as:

$$C\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(k_{wu}\frac{\partial\psi}{\partial z}\right)-\frac{\partial k_{wu}}{\partial z} \tag{1}$$

where ϕ : pore water pressure, θ : volumetric water content, t: time, z: depth , k_{wu}: water conductivity of unsaturated soil, and C: specific water capacity ($=\partial\psi/\partial\theta$).

Klute substitutes the volumetric water content for the pore water pressure to derive the governing equation of one-dimensional vertical seepage flow as follows:

$$\frac{\partial\theta}{\partial t}=\frac{\partial}{\partial z}\left(D\frac{\partial\theta}{\partial z}\right)-\frac{\partial k_{wu}}{\partial z} \tag{2}$$

where, D : soil water diffusivity.

It is difficult to solve these equations because they are nonlinear, so they must be solved by using Philip's solution (Philip, 1957) or a numerical analysis method.

2.2 *One-dimensional seepage test*

A seepage test was performed with Toyoura sand in a water

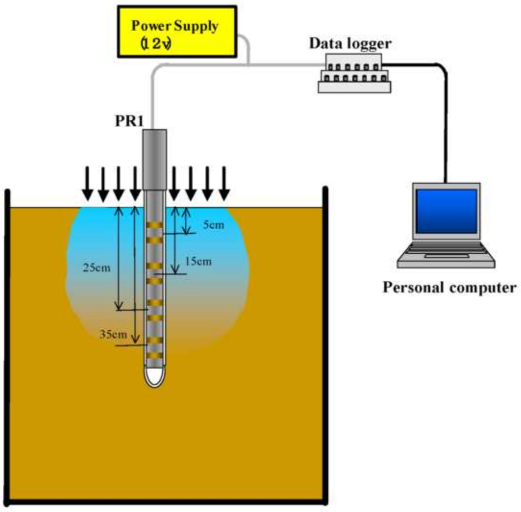

Figure 1. Apparatus used for laboratory test.

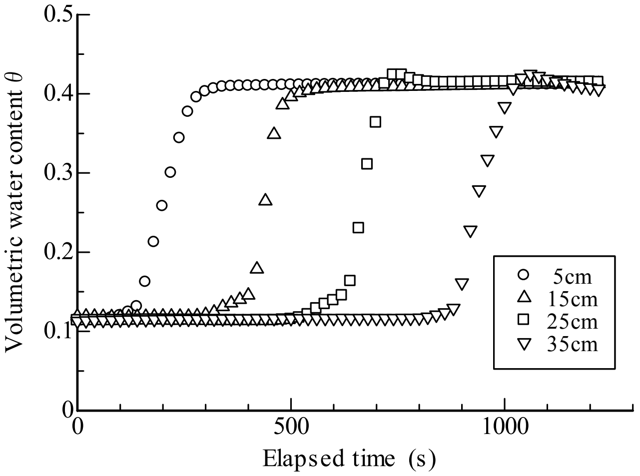

Figure 2. Measured volumetric water content during seepage into soils.

tank as shown in Figure 1. Only one sensor (PR1) is inserted into the ground, which can measure the dynamic data of volumetric water content at four depths. Water is sprinkled on the ground surface of which the initial water content is dry. In order not to prevent air from getting out, the intensity of sprinkling is less than 1.25 cm/s. Figure 2 shows the temporal response of the amount of moisture. It is found that soil moisture begins to increase sequentially from a shallow position, and the moisture of all points finally reaches a certain level. In the case of homogeneous soil, the four rates of advance of the wetting front are similar at different depths.

3 APPLICATION OF SIGMOID FUNCTION

A sigmoid function (Eq. (3)) is used to express the behavior of moisture in unsaturated soils (Sugii and Uno, 1996). The sigmoid function can express some value from 1 to 0 continuously. Therefore, volumetric water content is converted to relative volumetric water content with final and initial volumetric water content. Various "S curve" lines can be drawn merely by changing the following two parameters:

$$\frac{\theta-\theta_{in}}{\theta_f-\theta_{in}}=\frac{1}{1+\exp(b_0+b_1t)} \tag{3}$$

$$b_0+b_1t=\log_e\left(\frac{\theta-\theta_{in}}{\theta_f-\theta_{in}}-1\right) \tag{4}$$

where, θ_{in}: initial volumetric water content, θ_f: final volumetric water content, t: elapsed time, b_0 : fitting parameter concerning depth, b_1: fitting parameter concerning time (it has a negative sign).

By rearranging Eq. (3) as Eq. (4), it becomes easy to estimate these fitting parameters because the sigmoid function becomes a linear function with time. These parameters can also be estimated by using a solver of MS Excel. In this paper, these parameters were estimated by the latter method. The estimated parameters in the four depths are shown in Figure 3 and Table 1. Although b_0 is a constant, b_1 is proportional to distance from source. Therefore, parameter b_1 is estimated by regression analysis:

$$b_0=a_0+a_1z \tag{5}$$

where, a_0, a_1: fitting parameters concerning depth.

The observed data and the estimated results by using the estimated parameters (Table 2) are shown in Figures 4 and 5. A small error is identified in the increasing velocity of moisture at

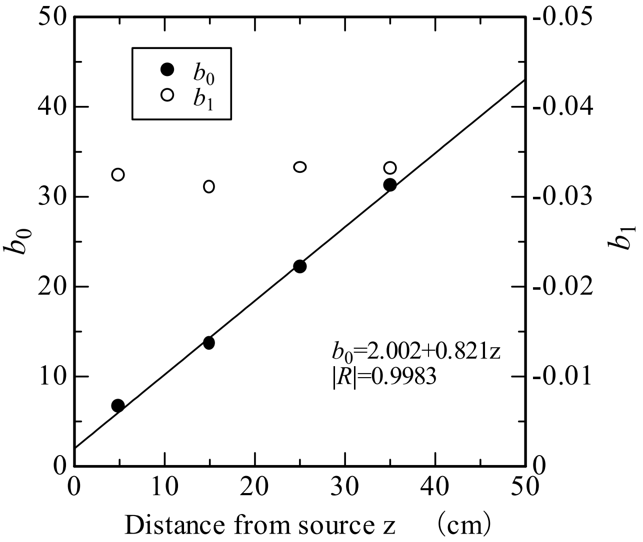

Figure 3. Fitting results of parameters.

Table 1. Estimated parameters

z (cm)	b_0	b_1	θ_f	θ_{in}
5	6.666	-0.0323	0.412	0.114
15	13.708	-0.0310	0.413	0.118
25	22.091	-0.0332	0.424	0.113
35	31.248	-0.0332	0.425	0.113
parameter	a_0	a_1	$b_1(ave)$	
	2.002	0.821	-0.0324	

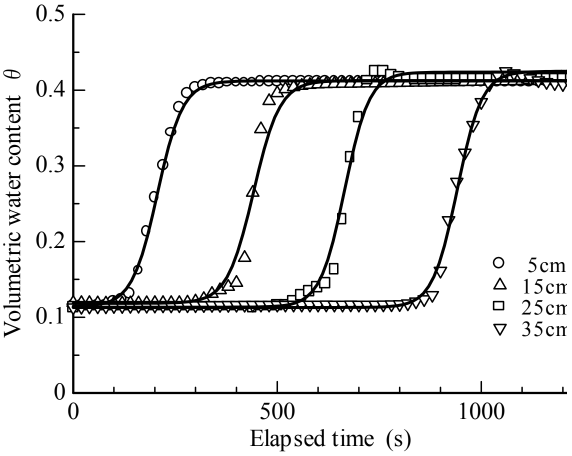

Figure 4. Measured and estimated volumetric water content (Solid lines: estimated lines, 5, 15, 25 and 35cm in depth).

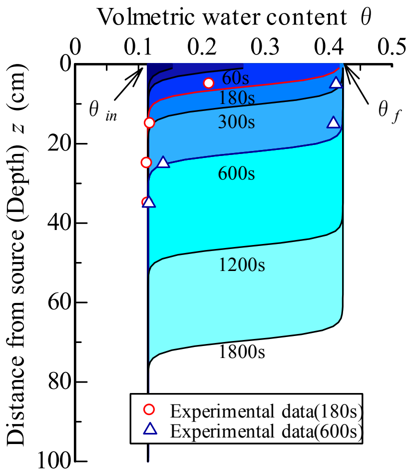

Figure 5 Measured and estimated soil moisture profile.

the depth of 15 cm, but the behavior of soil moisture can be simulated in time and depth. In this paper, Eq. (6) is called the soil moisture profile model (SMPM).

$$\theta = \theta(z,t) = \frac{\theta_f - \theta_{in}}{1+\exp(a_0 + a_1 z + b_1 t)} + \theta_{in} \tag{6}$$

The lower the initial moisture, the more accurately Eq. (6) can estimate the hydraulic conductivity of unsaturated soil. If θ and t are considered to be independent variables, Eq. (7) can be derived from substituting Eq. (6) for Eq. (2), and likewise with Eq. (8) and Eq. (7). Therefore, Eq. (9) can be derived by comparing Eqs. (7) and (8). This equation is an important characteristic of the proposed model.

$$\begin{aligned}\frac{\partial\theta}{\partial t} &= \frac{\partial}{\partial t}\left(\frac{\theta_f - \theta_{in}}{1+\exp(a_0 + a_1 z + b_1 t)} + \theta_{in}\right)\\ &= -(\theta_f - \theta_{in})\frac{b_1 \exp(a_0 + a_1 z + b_1 t)}{(1+\exp(a_0 + a_1 z + b_1 t))^2}\end{aligned} \tag{7}$$

$$\begin{aligned}\frac{\partial\theta}{\partial z} &= \frac{\partial}{\partial z}\left(\frac{\theta_f - \theta_{in}}{1+\exp(a_0 + a_1 z + b_1 t)} + \theta_{in}\right)\\ &= -(\theta_f - \theta_{in})\frac{a_1 \exp(a_0 + a_1 z + b_1 t)}{(1+\exp(a_0 + a_1 z + b_1 t))^2}\end{aligned} \tag{8}$$

$$\frac{\partial\theta}{\partial t} = \frac{b_1}{a_1}\frac{\partial\theta}{\partial z} \tag{9}$$

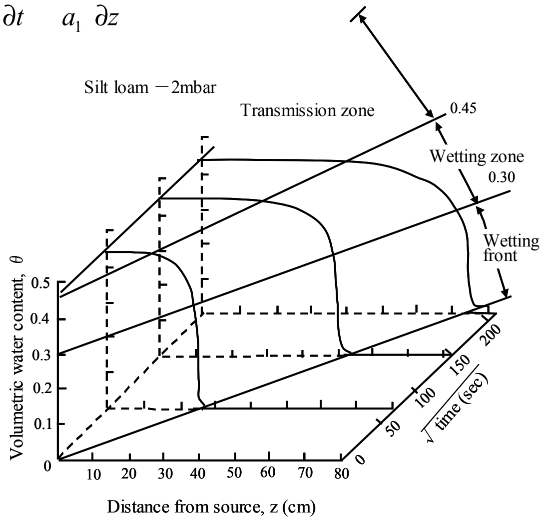

Figure 6. Relationship between wetting front and time (Young and Warkentin, 1975).

The left-side expression of Eq. (2) can be rewritten as $\partial\theta/\partial t = -\partial\theta/\partial z \cdot \partial z/\partial t$, consequently b_1/a_1 in Eq. (9) is equal to $-\partial z/\partial t$ or the rate of advance of the wetting front. According to Table 1, the rate of advance of the wetting front is estimated at about 0.039 cm/s.

Figure 6 shows the relationship between the wetting front and time, which Young and Warkentin, (1975) obtained in a horizontal seepage flow by the Boltzmann transformation method. It is found that Figure 7 drawn using SMPM and Figure 6 have the same shape even though the kinds of soil are different.

4 APPLICATION OF SMPM TO PERMEABLE TEST FOR UNSATURATED SOILS

The Boltzmann transformation method can estimate hydraulic conductivity from Klute's equation. There are drying and wetting types in this method. However, neither type is applicable to vertical flow in which the gravity term dominates such as Eq. (2). Therefore, this method can be used to measure the hydraulic conductivity of unsaturated soil in the laboratory only.

This paper suggests that the unsaturated hydraulic conductivity in vertical seepage flow can be determined simply by measuring the moisture during infiltration into unsaturated soil. Although the specific water capacity of soils is needed, the possibility of measuring unsaturated hydraulic conductivity in vertical seepage flow is useful for field tests.

4.1 *Calculation of hydraulic conductivity of unsaturated soil*

$$\frac{b_1}{a_1}\frac{\partial\theta}{\partial z} = \frac{\partial}{\partial z}\left(-\frac{k_{wu}}{C}(\theta_f - \theta_{in})\frac{a_1 \exp(a_0 + a_1 z + b_1 t)}{(1+\exp(a_0 + a_1 z + b_1 t))^2}\right) - \frac{\partial k}{\partial z}$$

Equation (10) can be derived from the relationship among Eqs. (2), (6) and (9).

(10)

Equation (10) is the second partial derivative for only z, so it is easy to solve for θ :

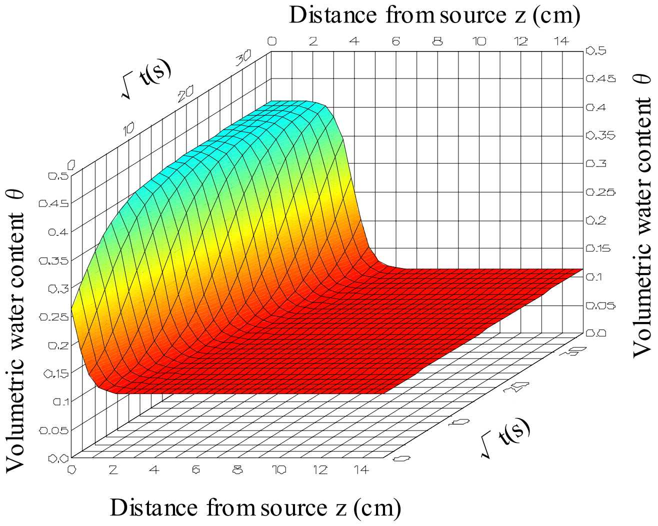

Figure 7. Relationship between wetting front and time (SMPM).

$$\theta = -\frac{k_{wu}}{C}\left(\theta_f - \theta_{in}\right)\frac{a_1^{\,2}\exp\left(a_0 + a_1 z + b_1 t\right)}{b_1\left(1+\exp\left(a_0 + a_1 z + b_1 t\right)\right)^2} - k_{wu}\frac{a_1}{b_1} + c_1 \tag{11}$$

where, c is an integral constant.

θ becomes θ_{in} for any depth when t equals 0. In addition, the first term of the right side of the expression is equal to zero because $\partial\theta/\partial t = 0$, therefore, $c_1 = \theta_r + k_{wu}a_1/b_1$ is given.

The solution of Klute equation is given as follows:

$$\theta = k_{wu}\left(-\frac{\left(\theta_f - \theta_{in}\right)}{C}\frac{a_1^{\,2}\exp\left(a_0 + a_1 z + b_1 t\right)}{b_1\left(1+\exp\left(a_0 + a_1 z + b_1 t\right)\right)^2}\right) + \theta_{in} \tag{12}$$

from which soil water diffusivity is given by:

$$D = \frac{k_{wu}}{C} = -\frac{\left(\theta - \theta_{in}\right)}{\left(\theta_f - \theta_{in}\right)}\frac{b_1\left(1+\exp\left(a_0 + a_1 z + b_1 t\right)\right)^2}{a_1^{\,2}\exp\left(a_0 + a_1 z + b_1 t\right)} \tag{13}$$

If a specific water capacity, *C*, is measured from a soil water retentivity test in the laboratory, then the hydraulic conductivity of unsaturated soil can be obtained.

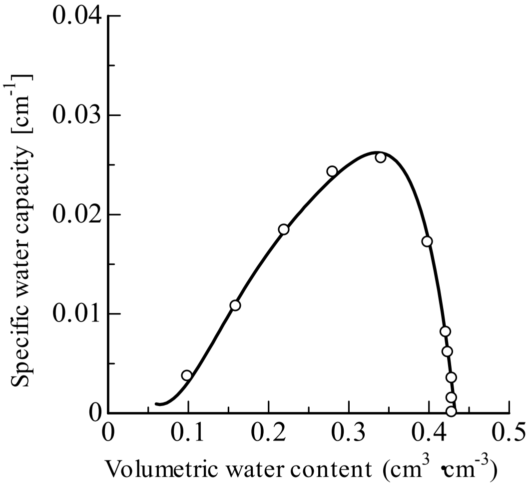

Figure 8. Experimental results of specific water capacity.

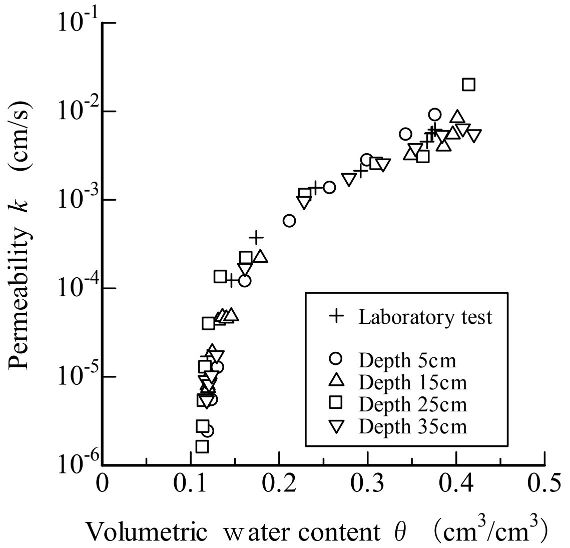

Figure 9. Comparison of SMPM and laboratory experiment results.

Figure 8 shows the experimental results of the specific water capacity of Toyoura sand in the laboratory. Consequently the hydraulic conductivity of unsaturated soil can be derived as follows:

$$k_{wu} = -C\frac{\left(\theta - \theta_{in}\right)}{\left(\theta_f - \theta_{in}\right)}\frac{b_1\left(1+\exp\left(a_0 + a_1 z + b_1 t\right)\right)^2}{a_1^{\,2}\exp\left(a_0 + a_1 z + b_1 t\right)} \tag{14}$$

Figure 9 shows the results of hydraulic conductivity obtained by Eq. (14) and specific water capacity. The results of laboratory tests (pressure method, Sugii et al., 2000) are shown in Figure 9 together for comparison. Although there are differences in the weight density and accuracy of fitting of the moisture profile, it is clear that the results of the proposed method are in good agreement with the results of other laboratory tests. Field tests have some problems regarding accurate burying of sensors and lack of de-aired water. Therefore, the proposal method is effective for measuring hydraulic conductivity in the field.

5 CONCLUSIONS

The results of this study may be summarized as follows:

1) The moisture profile, which changes with time and depth in vertical seepage flow, can be simulated with a sigmoid function.
2) According to the soil moisture profile model, the derivative of volumetric water content with respect to distance from the water supply or elapsed time is equal to the rate of advance of the wetting front and to the quotient of parameters.
3) The proposal model (SMPM) expresses the relationship among elapsed time, depth and volumetric water content during infiltration into the vadose zone.
4) Without using numerical analysis, the Klute equation can be simply solved by the soil moisture profile model.

The hydraulic conductivity of unsaturated soil determined by using the soil moisture profile model is as accurate as that determined by laboratory tests.

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