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The application of unsaturated soil mechanics to the assessment of weather-related geo-hazards

L'application de la mécanique des sols non saturés pour l'évaluation des geo-risques climatiques

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ABSTRACT

The primary goal of this paper is to illustrate the manner whereby unsaturated soil mechanics can be taken from the soil property assessment level using techniques based on the soil-water characteristic curve, (SWCC), to the quantification of geo-hazards. A concise description of a weather-related geo-hazards assessment model (W-GHA model) is presented. The soil system is represented by a series of partial differential equations (PDE's) governing conservation of mass and momentum and an equation for the soil-atmosphere coupling. Slope stability conditions are quantified using a "dynamic programming" solution. Parameter uncertainty is addressed through a probabilistic and sensitivity analysis framework. An example problem is presented. It is shown that the probability of failure and factor of safety provide important, complementary information. The sensitivity of the input parameters is presented and discussed. Some soil properties are found to have time-dependent sensitivity. Several input parameters appear to have negligible sensitivity and should not be considered random variables under the soil and atmospheric conditions presented herein.

RÉSUMÉ

L'objetif de cet l'article est d'illustrer la façon par laquelle la mécanique de sols non saturés peut être basée en techniques d'estimation de propriétés de sol appuyée sur la courbe caractéristique sol-eau visant la quantification des geo-risques. Une description concise du modèle d'évaluation de geo-risques climatiques (modèle W-GHA) est présentée. Le sol est représenté par une série d'équations différentielles partielles (PDE) de conservation de la masse et d'équilibre et équations pour l'accouplement sol-atmosphère. La stabilité de talus est mesurée en utilisant une solution de programmation dynamique. L'incertitude des paramètres est considérée dans un cadre de probabilité et d'analyse de sensibilité. Un exemple est présenté. On montre que la probabilité de la rupture et le coefficient de sécurité fournissent des informations importantes et complémentaires. La sensibilité des paramètres est présentée et discutée. On a observé que la sensibilité des paramètres de sol change avec le temp. Plusieurs paramètres d'entrée semblent avoir la sensibilité négligeable et ne devraient pas être considérés des variables aléatoires pour les conditions de sol et atmosphère présentés dans cette étude.

1 INTRODUCTION

Weather-related geo-hazards are a major concern for the railway industry in Canada. The financial losses that result from derailments and delays amount millions of dollars every year. The safety exposure of employees and the public is also a concern. The railway industry faces a serious dilemma; namely, the industry operates networks with tens of thousands of kilometres, crossing several types of geographical terrain, soil, and weather conditions from coast to coast. It is extremely difficult to protect and/or remediate every site under risk. At the same time, risks must be managed in an affordable manner.

Many miles of Canadian railway subgrade are susceptible to embankment failure. According to the Transportation Safety Board (1997), the occurrence of railway subgrades composed of "moisture-sensitive" alluvial deposits reflects the limited construction capabilities and understanding of soil characteristics at the time of construction. Railway embankments were constructed as early as the turn of the century. Therefore, safety risks are found at numerous sections of Canadian railways that can be exposed to high levels of precipitation, rapid melt of snow pack, or drainage disruption.

Some aspects of the Railways geotechnical programs are generally proactive but the measures for identified problems are mostly reactive in nature (Transportation Safety Board, 1997). Steps are required in order to identify, monitor, and modify "moisture-sensitive" embankments, and build drainage systems to prevent water degradation. Beaver control, culvert inspections, and adoption of modern construction standards are acknowledged remedial measures already in place. However, there is a lack of *decision support systems* that can make use of information obtained from the monitoring of weather and embankment conditions. *Decision support systems* capable of quantifying railway embankments hazards can assist in indicating appropriate inspection frequencies, thereby providing a way to rationale railway resources.

2 ELEMENTS OF A HAZARD MANAGEMENT SYSTEM FOR RAILWAY EMBANKMENT

The development of a proactive hazards management system is a challenging task. Figure 1 presents the components of a management model for railway embankment hazards. According to Fig. 1, a *decision making system* must be established and entail: (i) a risk quantification criteria, (ii) acceptable risk levels, and (iii) a collection of managing actions available. Risks are a function of hazard levels and potential hazard consequences. Therefore, a hazard management model requires a *decision support system* that is capable of providing a measure of embankment hazard. The primary focus of this paper is on *decision support systems* capable of quantifying railway embankment hazards.

Railway hazard assessment models can be developed and implemented at various scales. It is possible to focus on specific areas that have a high potential for becoming unstable and attempt to monitor the embankment stability. It is also possible to produce a rather crude and yet appropriate hazard assessment for a whole region of the railway network with the aid of a geographic information system, GIS.

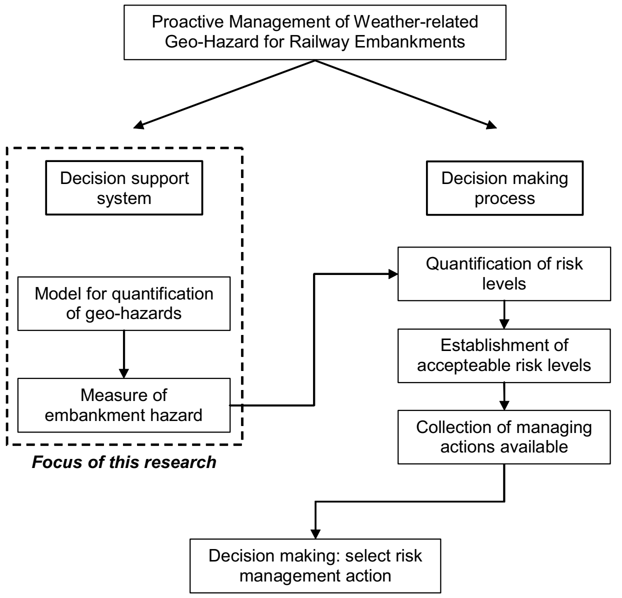

Figure 1. Components of a management model for railway embankment hazards

This paper focuses on the former scale (i.e., the assessment of hazards in a local scale). Besides being applied to specific areas, a local scale hazard assessment model can also be considered the first step towards the assessment of hazards on a regional scale. The simplifications required by GIS-based management systems must be founded on findings based on the mechanisms associate with local scale embankment failures.

3 QUANTIFICATION OF EMBANKMENT HAZARDS USING UNSATURATED SOIL MECHANICS

Unsaturated soil mechanics provides a powerful theoretical background for the assessment of weather-related geo-hazards. Embankment hazards are strongly related to the reduction in soil suction and shear strength at the near ground surface soil. Soil suction and shear strength varies according to the amount of water stored in the soil. On the other hand, the amount of water within the soil is a function of the soil properties and the antecedent weather conditions.

The near ground surface soil behaves as a dynamic system that interacts with the atmosphere and the deeper layers of soil. Several coupled phenomena take place within the near ground surface unsaturated soil. These coupled phenomena can be understood and modelled using fundamental laws of physics (e.g., conservation of momentum, mass, and energy) and constitutive relationships for unsaturated soils. Soil-atmosphere interaction can also be considered, including infiltration, evaporation, and runoff.

The relationship between the amount of water in the soil and the soil suction is given by the soil-water characteristic curve. Figure 2 explains in a qualitative manner the relationship between the amount of water stored in the soil and the embankment hazard level. The soil comprising an embankment can be viewed as a "water tank". The soil-water characteristic curve works as a gauge, indicating the water level within the "water tank". The water level presented in Fig. 2 is a function of antecedent weather conditions. The water level increases after precipitation events, and retreats during dry periods. Therefore, both precipitation events and evaporation periods must be considered for the establishment of the water level at any point in time.

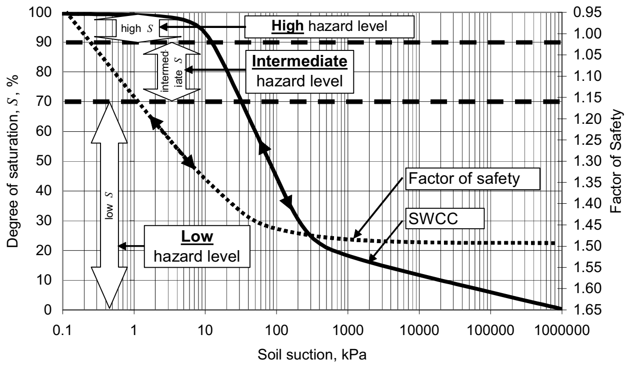

Figure 2. SWCC as a water level and embankment hazard gauge (intervals of degree of saturation and factor of safety are arbitrary)

The embankment hazard level, here represented as an arbitrary function of the *factor of safety*, F_s, varies according to that water level shown in Fig. 2. A low water level corresponds to higher suction and shear strength and results in a lower level of hazard (higher F_s). Higher water levels produce lower soil suction, lower shear strength, and a greater hazard level (lower F_s). The relatively rigorous model for assessment of embankment hazards presented herein reflects in an indirect manner, the "water tank" concept presented in Fig. 2.

The difficulties associated with the determination of unsaturated soil properties result in the need for an unsaturated soil mechanics protocol adequate for embankment hazard assessment and routine geotechnical engineering. An unsaturated soil mechanics protocol must be based on prediction techniques for soil properties and must accommodate the uncertainty of unsaturated soil properties through a formal probabilistic and sensitivity analysis framework. The probabilistic framework can be based on the analysis of formally established "case scenarios". The probabilistic and sensitivity analysis framework presented herein is a formal version of the "what-if" approach (Fredlund, 2002).

4 DETERMINISTIC MODEL FOR THE ASSESSMENT OF WEATHER-RELATED GEO-HAZARDS

The thermo-hydro-mechanical behaviour of the soil comprising a railway embankment can be represented by a system of partial differential equations (PDE's). These equations are obtained using a traditional continuum mechanics approach and appropriate state variables. The broadly accepted stress state variables; namely, net stress ($\sigma - u_a$) and matric suction ($u_a - u_w$), are used (Fredlund and Morgenstern, 1977), where σ is the total stress, u_a is the pore-air pressure, and u_w is the pore-water pressure. The displacement state variables are the horizontal and vertical displacements, u and v (x- and y-direction, respectively), and the change in volume of water and air in a referential volume. The PDE's governing (i) static force equilibrium, (ii) flow of moisture, and (iii) flow of heat are obtained from basic continuity and equilibrium laws, combined with constitutive laws that describe soil behaviour.

A series of assumptions form the backdrop for the equations for the flow of liquid water and water vapour in soils. The main assumptions are as follows:

(i) the soil phases are individually continuous and therefore can be described using a continuum mechanics approach;
(ii) the air phase is in permanent contact with the atmosphere;
(iii) osmotic pressure gradients are negligible;
(iv) local thermodynamic equilibrium between the liquid water and water vapour phases exists at all times at any point in the soil;
(v) temperature within the soil remains below the boiling point and above the freezing point of water at all times;

(vi) dissolution of air into the liquid water phase is neglected;
(vii) hysteretic behaviour of the soil-water characteristic curve can be approximated by taking the logarithmic average between the main drying and main wetting curves.

The computation of changes in water content and pore-water pressure distribution within an embankment form an essential step towards to the assessment of weather-related geo-hazards. Atmospheric forcing conditions produce internal moisture flow and changes in the pore-water pressures within the embankment. The internal flow makes moisture available for evaporation and/or allows precipitation to infiltrate the embankment. Neglecting either evaporation or infiltration may lead to unrealistic pore-water pressure predictions as both are important components of the net soil-atmosphere flux. The importance of taking into account evaporative fluxes is particularly evident in arid and semi-arid regions where the annual amount of evaporation is greater than precipitation (Fredlund and Rahardjo, 1993).

Figure 3 presents an overview of the deterministic component of the W-GHA (weather-related geo-hazard) assessment model. Partial differential equations were established for the flow and conservation of liquid water, water vapour, and heat. These transient equations must be solved taking into account soil-atmosphere forcing conditions. The stability of the railway embankment may be quantified at any time by computing the factor of safety. In order to compute the factor of safety, the pore-water pressures and total stress distribution must be used. Details about the derivation of the equations presented in Fig. 3 can be found in Gitirana Jr. (2005).

A number of variables are defined in Fig. 3; namely, k^w = hydraulic conductivity; k^v = vapour conductivity; γ_w = unit weight of water; n = porosity; S = degree of saturation; T = temperature; λ = thermal conductivity of the soil; L_V = latent heat of vaporization; ζ = volumetric specific heat of soil; NF = net soil-atmosphere flux; P = precipitation, AE = actual evaporation, α = slope of the ground surface; u_{ws} = pore-water pressure at the ground surface; D_{ij} = elastic stress-strain constitutive matrix terms; γ_{nat} = soil unit weight.

A computer code using the "dynamic programming" method (DPM) has been developed (Gitirana Jr. and Fredlund, 2003) for the analysis of embankment stability. The application of DPM to slope stability analyses has been first proposed by Baker (1980). Recent research has verified the DPM provides factors of safety similar to those using the enhanced method and conventional methods of slices (Pham, 2002). However, more flexible, no-circular slip surfaces can be obtained. Non-circular slip surfaces are a particular concern for the assessment of embankment hazards where sharp wetting fronts occur (Gitirana Jr. and Fredlund, 2003). A detailed description of the DPM analytical scheme can be found in Gitirana Jr. (2005).

5 PROBABILISTIC AND SENSITIVITY ANALYSIS FRAMEWORK FOR THE W-GHA MODEL

A formal probabilistic and sensitivity analysis framework has been established for the assessment of weather-related geo-hazards. The framework recognises the paramount importance of taking into account unsaturated soil parameter uncertainty and provides an alternative approach, compatible with approximate unsaturated soil property functions.

A brief description of the probabilistic and sensitivity analysis framework will be presented herein, but a detailed description can be found in Gitirana Jr. (2005). The probabilistic framework is based on an alternative point estimate method, APEM. The APEM is based on the combination of the Rosenblueth (1975) method for univariated functions and the Taylor series approximation. The approach is similar to that proposed by Li (1992). The equations proposed for uncorrelated variables are as follows:

$$E[F_s(X)] = F_s(E[X]) + \sum_{i=1}^{n}\left[p_i^+ F_s(x_i^+) + p_i^- F_s(x_i^-) - F_s(E[X])\right] \quad (1)$$

$$\mu_m[F_s(X)] = \left\{F_s(E[X]) - E[F_s(X)]\right\}^m + \sum_{i=1}^{n}\left[\begin{array}{l} p_i^+\left\{F_s(x_i^+) - E[F_s(X)]\right\}^m + p_i^-\left\{F_s(x_i^-) - E[F_s(X)]\right\}^m \\ -\left\{F_s(E[X]) - E[F_s(X)]\right\}^m \end{array}\right] \quad (2)$$

where F_s is the factor of safety, $E[]$ is the first central moment, μ_m is the statistical moment of order m; p_i = 0.5 for the normal distribution; $F_s(x_i^{\pm}) = F_s(E[x_1],\ldots x_i^{\pm},\ldots E[x_n])$; and n = number of input random variables.

Gitirana Jr. (2005) presents more general equations, considering skewed and correlated input variables. The first two statistical moments of F_s computed using Eqs. 1 and 2 can be used in the computation of measures of embankment stability that take into account the variability of F_s, such as the probability of failure, P_f. Equations 1 and 2 require a relatively small number of computations of F_s ($2n$+1 for uncorrelated variables), when compared to the Rosenblueth (1975) method (i.e., 2^n). For instance, the 12 input variables presented in the next section required 25 evaluations of F_s using the APEM, while 4096 evaluations would be required by Rosenblueth's equations.

Equations 1 and 2 can be used for the sensitivity analysis of the W-GHA model. The sample points used by Eqs. 1 and 2 (i.e., $F_s(x_i^{\pm})$) provide measures of F_s that can be used in the construction of probabilistic and deterministic event tornado diagrams. Tornado diagrams are commonly used in Decision Analysis and provide a formal approach for sensitivity analysis (Clemen, 1996). A deterministic tornado diagram shows the variability of F_s due to each input random variable, while a probabilistic tornado diagram show the variability of F_s when the variability of one input variable is removed at a time.

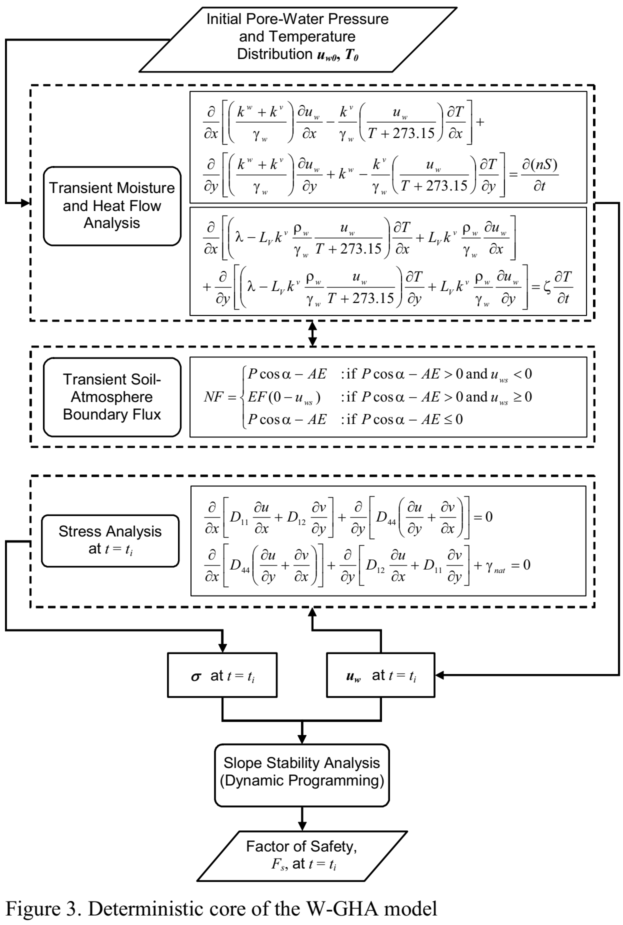

Figure 3. Deterministic core of the W-GHA model

Table 1: Soil parameters adopted for the simulations

Analysis (1)	Soil property (2)	Related parameters (3)	Expected value [(*)] (4)	CV [(**)] (5)
Moisture flow analysis	$\theta = nS$	n	0.500	17%
		$\ln(\psi_b \ln(kPa)$	0.927 (2.50)	115%
		$\ln(\lambda_d \ln(\ln(kPa)^{-1})$	-0.737 (0.50)	90%
		$\ln(\lambda_{res} \ln(\ln(kPa)^{-1})$	-2.445 (0.090)	12%
	k^w	$\ln(k^w_{sat}, \ln(m/s)$	-12.58 (3.5×10^{-6})	15%
	k^v	$\ln(D^v_{25oC}), \ln(m^2/s)$	-9.4 (8.33×10^{-5})	15%
Heat flow	λ	λ_s W/(m °C)	6.0	25%
Stress and stability analysis	D_{ij}	E, kPa	15,000	30%
		μ	0.35	22%
	τ_f	c', kPa	0.0	30%
		ϕ'	30°	10%
		$\ln(\kappa)$	0 (1)	SD = 0.5

Fixed parameters: $a = 0.050$; $G_s = 2.65$; $L_V = 2.501\times10^6 - 2.361\times10^3 T$ J/kg;
$\zeta_s = 2.23\times10^6$ W/(m³ °C); $\zeta_w = 4.15\times10^6$ W/(m³ °C);
(*) values between brackets are the exponential values;
(**) values for which the mean is close to zero may have the variability indicated in terms of standard deviation, SD.

6 APPLICATION OF THE W-GHA MODEL FOR AN HYPOTHETICAL RAILWAY EMBANKMENT

This section presents an example of the application of the weather-related geo-hazard assessment model. A hypothetical railway embankment is analyzed considering simplified atmospheric conditions corresponding to a strong rainfall. Therefore, the vapour and heat flow components of the model are expected to play a negligible role. Gitirana Jr (2005) presents additional examples, for varying conditions.

The W-GHA model was implemented using a general purpose partial differential equation solver (FlexPDE, 2003), the "dynamic programming" code Safe-DP (Gitirana Jr., 2005), and spreadsheet routines for the APEM and sensitivity analysis computations.

6.1 *Problem description*

Figure 3 presents the geometry, initial, and boundary conditions adopted in the analyses presented herein. A relatively small railway embankment, 5 meters high was analyzed. Embankment side slopes at 1.5H:1.0V were selected, based on typical embankments found along the Canadian railway network. A relatively high train load of 28 kN/m² was adopted. The initial pore-water pressure conditions are based on a 1 m deep water table and a minimum pore-water pressure of -100 kPa. A constant rainfall of 40 mm/day was applied during 14 days. This amount of precipitation is comparable to the amount of precipitation and melted water anteceding a considerable number of railway embankment failures reported by the Transportation Safety Board (1997).

6.2 *Soil properties*

Table 1 presents the soil properties required by the W-GHA model and the values adopted herein. The values adopted correspond to a lacustrine silt (or loam, according to the USDA classification system). The hydraulic conductivity function, shear strength envelope, vapour conductivity function, thermal conductivity function, and heat capacity function were estimated based on the soil-water characteristic curve. Gitirana Jr. (2005) presents a detailed description of the estimation approaches available and the estimation techniques adopted for this study. For instance, the shear strength envelope was predicted using the equation proposed by Fredlund et al. (1996):

$$\tau_f = c' + (\sigma_n - u_a)\tan\phi' + (u_a - u_w)S^{\kappa}\tan\phi' \quad (3)$$

where c' is the soil cohesion, ϕ' is the friction angle, and κ is a fitting parameter.

Typical coefficients of variation were collected from the literature for the Young modulus, E, Poisson ratio, μ, effective cohesion, c', and friction angle, ϕ'.

The remaining parameters were statistically assessed using a large database of soils (SoilVision, 2003) and a statistical assessment methodology specially developed for unsaturated soil property functions (Gitirana Jr., 2005). A soil-water characteristic curve, SWCC, with independent parameter was selected (Gitirana Jr. and Fredlund, 2004). The parameters defining the SWCC are the air-entry value, ψ_b, the primary slope, λ_d, the residual slope, λ_{res}, and a parameter controlling the sharpness of the SWCC curvatures (a = 0.05 for the lacustrine silt). The parameters presented in terms of natural logarithm (see Table 1) were determined to be log normally distributed.

The coefficient of variation of the diffusivity of water vapour, D^v_{25oC}, the thermal conductivity, λ_s, and the unsaturated shear strength parameter, κ, were assessed based on the 3-sigma rule (Duncan, 2000).

6.3 *Results and discussion*

Figure 4 presents the critical slip surfaces and factors of safety obtained for the initial conditions, and after 4, 8, and 14 days. A steady reduction in the factor of safety was observed, as expected. The slip surfaces moved toward the soil surface as the near ground surface soil approaches saturation. Non circular slip surfaces were obtained for all time steps.

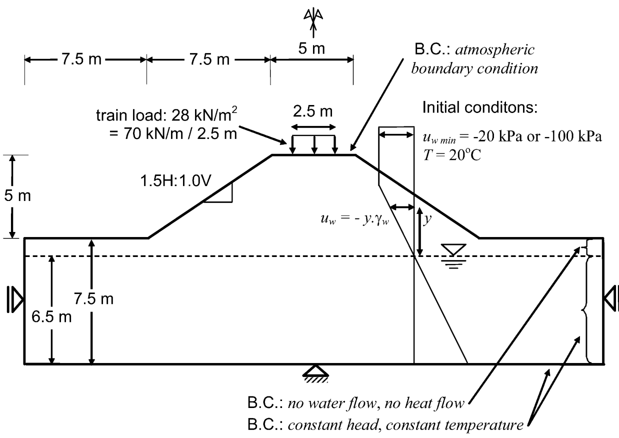

Figure 3. Embankment geometry, initial, and boundary conditions

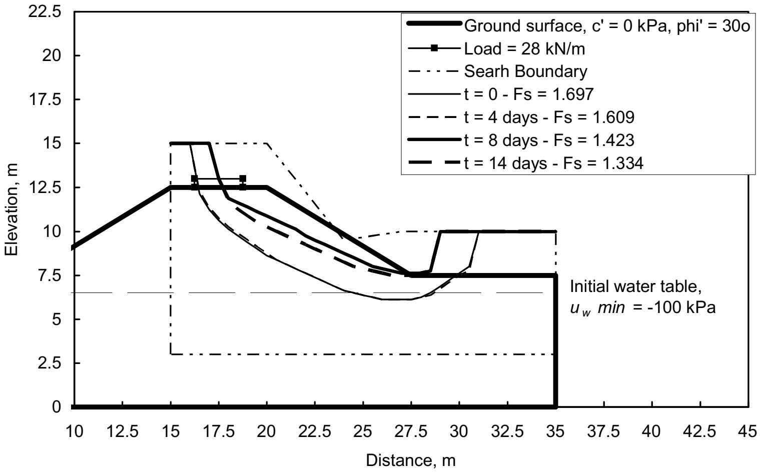

Figure 4. Slip surfaces for a Loam soil railway embankment using "dynamic programming" optimization

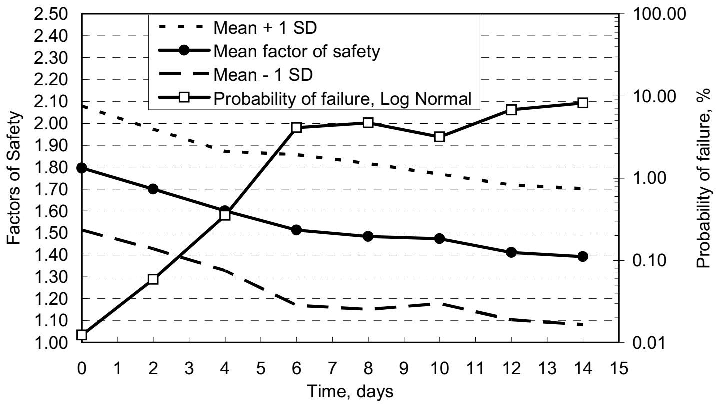

Figure 5. Factor of safety and probability of failure with time

Figure 5 presents the evolution of factor of safety with time. Based on the mean values for the factors of safety, the embankment appears stable throughout the rainfall event, with the minimum mean factor of safety slightly below 1.4.

The probability of failure was computed taking the mean and standard deviation of the factor of safety, computed using Eqs. 1 and 2, and assuming a log normal distribution for the factor of safety. Figure 5 presents values of the factor of safety plus and minus one standard deviation. The variability of the factor of safety shows that more critical combinations of input parameters can produce nearly unstable conditions. This is reflected by the probability of failure that sharply increased from approximately 1/1000 to 1/10.

Figure 6 presents a deterministic event tornado diagram constructed for t = 14 days. This tornado diagram was constructed based on the 10^{th} and 90^{th} percentiles of the factor of safety distribution obtained taking one single input variable at a time. It can be seen that the most sensitive variables are the friction angle, the saturated hydraulic conductivity, the air-entry value, and the soil cohesion. The remaining variables had small to negligible influence in the variability of the factor of safety.

Figure 7 presents important information regarding the variability of the input variables, based on a probabilistic event tornado diagram. The tornado diagram was constructed based on the 10^{th} and 90^{th} percentiles of the factor of safety distribution obtained by removing one single input variable at a time. The same "most sensitive" and "less sensitive" variables identified in Fig. 6 are also found in Fig. 7. However, Fig. 7 shows the amount of variability in the factor of safety when each input variable was considered as a certain variable. Considerably more stable conditions would be found if the saturated hydraulic conductivity, friction angle, and air-entry value uncertainties were removed from the model.

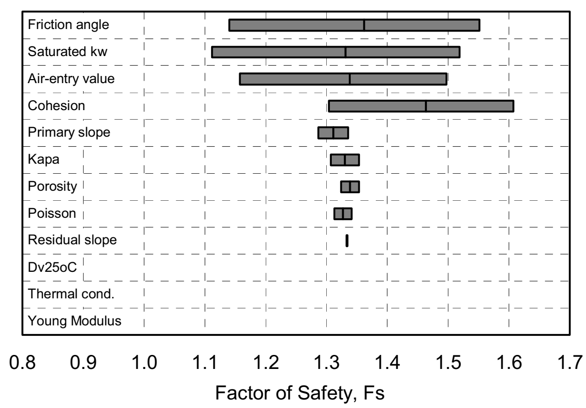

Figure 6. Deterministic event tornado diagram for t = 14 days

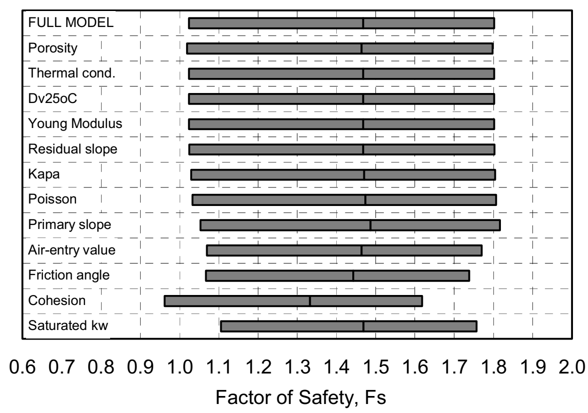

Figure 7. Probabilistic event tornado diagram for t = 14 days

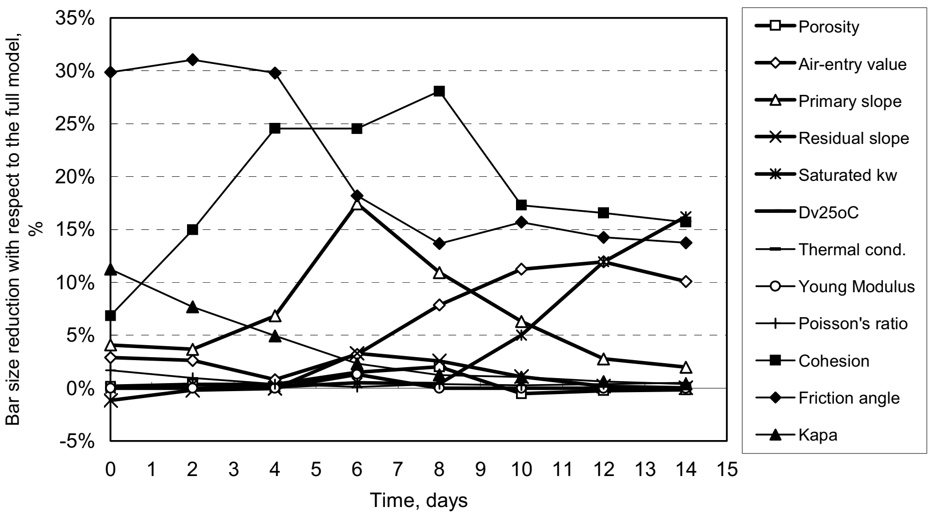

Figure 8. Bar size reduction for the probabilistic event tornado diagram over time.

Figure 7 confirms that the soil porosity, thermal conductivity, vapour diffusivity, Young modulus, residual SWCC slope, κ shear strength parameter, and Poisson ratio do not have to be considered as random variables. This information results in important simplification to the model and quicker analyses. However, these results cannot be extended to all geometry and atmospheric conditions, as shown in Gitirana Jr. (2005). For instance, long term moisture migration depends to a greater extent on the vapour diffusivity.

The relative bar size reductions shown in Fig. 7 were computed based on the original bar (i.e., for the Full Model). The values obtained for several time steps are presented in Fig. 8. Figure 8 shows that the sensitivity of the input variables changes with time. The factors of safety obtained in the initial time steps are more sensitive to the angle of internal friction and less sensitive to cohesion and hydraulic properties. As the near ground surface soil saturates, the sensitivity of the air-entry value and saturated hydraulic conductivity increase. These results were expected since the influence of the hydraulic conductivity is time-dependent and tends to accumulate over time for monotonic paths. It appears that the longer the period of the analysis, the more important is the saturated hydraulic conductivity. However, for short term analysis (less than 7-days periods), the results would not be as sensitive to k^w_{sat}.

7 CONCLUSIONS

This paper presented a model for the assessment of weather-related geo-hazards. A concise description of the weather-related geo-hazards assessment model (W-GHA model) was presented. According to the W-GHA model, the soil system is represented by a series of partial differential equations (PDE's) governing conservation of mass and momentum and equation for the soil-atmosphere coupling. Stability conditions are quantified using a "dynamic programming" solution for slope stability. Parameter uncertainty was addressed through a probabilistic and sensitivity analysis framework. The probabilistic framework is based on an efficient, alternative point estimate method.

An example problem was presented, considering a typical embankment under heavy rainfall. Slip surfaces, factors of safety, probabilities of failure were evaluated. It was shown that the probability of failure and factor of safety provide important, complementary information. The sensitivity of the input parameters was presented and discussed. Deterministic and probabilistic event tornado diagrams showed that the soil parameters sensitivities are time-dependent. A number of input parameters appear to have negligible sensitivity and should not be considered random variables under the soil and atmospheric conditions presented herein.

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