Proceedings of the 16th International Conference on Soil Mechanics and Geotechnical Engineering

doi:10.3233/978-1-61499-656-9-2775

Use of Finite Element Methods in Geotechnical Ultimate Limit State Design

Utilisation de la Méthode aux Eléments finis pour les calculs aux Etats Limites Ultimes

C. Bauduin
Besix, 1200 Brussels, Belgium
University of Brussels, Belgium

K.J. Bakker
Plaxis, 2628 XJ Delft, The Netherlands
Delft University of Technology, The Netherlands

R. Frank
CERMES (ENPC-LCPC), 77455 Marne-La-Vallée Cedex 2, France

ABSTRACT

The paper introduces to the use of FEM in Ultimate Limit State Design. The accuracy of FEM to predict failure load is briefly discussed. Procedures for FEM in MFA (Material Factoring Approach) and LRFA (Load and Resistance Factoring Approach) formats are proposed. In MFA it is advised to simulate the stress and loading history using unfactored values of the material parameters and, at each stage where the ULS requirements have to be checked, to reduce stepwise the shear strength parameters to their design values (φ'-c' reduction, usually performed using a simple elastic-purely plastic model). This allows to check for ULS in the ground and provides design values of forces in structural members. In LRFA, the design values of forces in structural members are obtained by multiplying their characteristic values by the load factor. Checking for geotechnical failure in LRFA is in some cases not always straightforward. The links between the proposed procedures and the three Design Approaches introduced by Eurocode 7 for persistent and transient design situations are explained. The advantages and limitations involved by the use of a simple elastic-purely plastic model for the φ'-c' reduction are discussed.

RÉSUMÉ

La présentation introduit à l'utilisation des éléments finis dans les calculs aux ELU. L'exactitude des éléments finis pour la prédiction des charges ultimes est brièvement discutée. Des procédures pour l'application de E.F. dans des formats MFA (coefficients partiels appliqués aux matériaux) et LRFA (coefficients partiale appliqués aux effets des actions et aux résistances) sont proposées. En MFA, il est proposé de simuler l'histoire de chargement en utilisant des valeurs caractéristiques (non factorisées) des paramètres des matériaux et d'effectuer une réduction pas à pas des paramètres de résistance jusqu'à leur valeur de calcul en ELU à chaque situation où il est nécessaire de vérifier les états ultimes (réduction φ'-c', avec modèle élastique-purement plastique). Ceci permet de vérifier s'il y a rupture dans le sol et donne des valeurs de calcul des efforts dans les éléments structuraux. En LRFA, la vérification de la rupture dans le sol n'est pas toujours directe ; les valeurs de calcul des efforts dans les éléments structuraux sont obtenues en multipliant leurs valeurs caractéristiques par les coefficients partiels des actions. Les liens entre les procédures proposées et les trois approches introduites dans Eurocode 7 sont expliqués. Les avantages et les limitation de la réduction φ'-c' sont discutés.

1 INTRODUCTION

Limit state design requires that both ultimate and serviceability limit states shall be checked. The FEM was initially developed to model as accurately as possible stresses and deformations at service state and is therefore a powerful tool to check the geotechnical structure for serviceability limit states. Application of FEM to ultimate limit state design (ULS), i.e. for checking against failure in the ground and for providing design values of internal forces in the structural members of the geotechnical structure may differ from the more traditional use of analytical and semi-empirical models for ULS and hence requires careful consideration of some aspects. The scope of this paper is to highlight some main features and to indicate possible procedures for the use of FEM in ULS design, with special emphasis to Eurocode 7.

2 THE ACCURACY OF THE FINITE ELEMENT METHOD TO PREDICT FAILURE LOADS

For a material that fulfils Drucker's postulate, assuming an associated flow rule, lowerbound and upperbound solutions for the Ultimate Limit State exist. Lowerbound solutions can be found for a statically admissible stress field, approximating the kinematics, whereas upper bound solutions can be found for a kinematically admissible deformation field, approximating the statics, see Verruijt (2004).

What does this mean for Limit State Analysis with the Finite Element method? The functions used to interpolate the displacement field are continuous and compatible and therefore kinematically admissible, whereas the statics in the stress field are only evaluated in integration points, meaning that the statics in the stress field are "only" approximated. Meaning that FE analysis gives an upperbound for the ultimate limit load. However, different studies have indicated that with mesh refinement, the solution tends towards a statically admissible stress field, which means that a Limit State analysis with FEM approximates the exact solution, the accuracy of which can be checked with mesh-refinement.

Attention has to be paid when using special elements with adapted strain integration or in case of under-integration. For that situation FE solutions that underestimate the Ultimate Limit load bound are known. Again with mesh refinement one can check the accuracy of the solution. Further, for frictional materials, i.e. exhibiting volumetric strain at failure, non-locking elements should be advocated; i.e. with at least quadratic integration in plane strain, and cubic integration in axi-symmetry (i.e. six nodded elements in plane-strain and fifteen nodded elements in axi-symmetry).

According to Drucker's postulate the argumentation that FEM approaches the exact solution from above is restricted to materials with an associated flow rule, such as clay in undrained shearing. However in addition to that, De Borst and Vermeer (1984) has shown that for frictional materials with $\phi = \psi = 40º$, simulating Prandtl's strip footing problem, that with mesh refinement the Finite Element result also comes down to the exact solution for the Ultimate Limit load.

However, frictional materials do not exhibit the high values of the dilatancy angle that correspond to associated flow, and thus may be expected to yield at a lower strength than found with associated flow. It appears however that the failure mechanisms are not significantly influenced by the value of the angle of dilatancy, and it is found that also for non-associated flow, with mesh refinement the FE analysis approximates the exact solution.

For associated flow, i.e. if $\phi = \psi$, uniqueness of the solution is assured. With the introduction of non associated flow rules, uniqueness of the limit load found in numerical analysis is not guaranteed, but depends on the initial stress and the sequence of loading; see de Borst & Vermeer 1984. Apart from the non-uniqueness due to differences in stress history or differences in loading history, non-uniqueness might also be triggered due to bifurcation behaviour, e.g. shear band development. Up to now, common FE codes do not satisfactorily treat the development of shear bands. Theoretical and numerical research has been going on for some time to solve this issue, see de Borst (1986), and more recently, Wells et al (2002).

3 HOW TO INTRODUCE THE PARTIAL FACTORS OF ULS IN FE CALCULATIONS

Modern design codes are based on Limit State design. The formats to check the Ultimate Limit States when ground strength plays a significant role in providing resistance may be of the following types:
- MFA: the design values of action effects and of resistances are obtained by applying partial factors at the sources of the uncertainty, i.e. on the actions and on the ground strength parameters c' and φ' or c_u (material properties).
- LRFA: the design values of action effects and resistances are obtained by applying partial factors on the actions or the action effects and on the characteristic value of the resistances;

3.1 *Material factoring approach (MFA)*

When using FEM, and especially when the equilibrium of soil masses is involved (e.g. embankments, excavations, retaining structures...), MFA formats are generally applicable to all types of problems for checking against failure in the ground and for obtaining design values of internal forces in structural members at design value of the ground strength parameters.

From a conceptual point of view, partial factors of a MFA can be introduced in ULS FE calculations through two different procedures (Bauduin et al, 2000):
- by performing the calculations using design values of actions and of ground strength parameters right from the start throughout the complete loading history. The ULS requirement is checked if no ultimate limit state is reached for the introduced design values of actions and material properties; the obtained values of action effects in structural members (bending moments, shear forces) are design values;
- by performing the calculations using characteristic values of the ground parameters and of the actions throughout the simulation of the complete load history. The calculations simulating the loading history deliver thus a "characteristic stress field" at each stage, reflecting as exactly as possible the real stress field and the soil-structure interaction phenomena. At each step where the ULS requirements are intended to be checked, the values of the (structural) actions and the ground strength parameters are stepwise increased respectively decreased, from their characteristic value to their design values. Thus for each construction stage, the ULS stress field is reached by increasing stepwise the actions and decreasing stepwise the shear resistance parameters from the "characteristic" stress field to its design values in parallel calculations. The stepwise reduction of the shear strength parameters is performed with an ideally elastic-perfectly plastic Mohr-Coulomb model. The ULS requirements for each stage are fulfilled if no ultimate limit state is reached when all the parameters reach their design value at that stage without failure of the ground. The values of action effects in structural members obtained are design values.

When using the first procedure, the design is readily checked for design values of the parameters. It might be explained as an analysis of the geotechnical structure for the hypothetical situation that the ground strength parameter values would be worse than expected. When using the second procedure, the "distance" from the characteristic stress situation towards the stress situation obtained by decreasing the parameter values is compared to the required values of the partial factors. The two different procedures lead to somewhat different calculation result. (Bauduin et al, 2000). The second procedure is preferred especially when the construction sequences and loading history influence the calculation results and when ground-structure interaction phenomena play an important role.

3.2 *Load and resistance factoring approach (LRFA)*

In a LRFA procedure, the loading history is also simulated using characteristic values of ground strength parameters and of actions. Design values of member forces are obtained by multiplying the values obtained using characteristic values of actions and ground strength parameters by the load factors. In this procedure, the load factors act more as model factors. This procedure is restricted to linear behaviour of the structural members. Extension towards highly non-linear behaviour of structural members, e.g. by the development of plastic hinges, is out of the scope of this paper. An LRFA framework for checking failure in the ground is less straightforward than MFA. For situations where a "load settlement curve" of any kind can be established the ULS requirement can be checked by comparing the design value of the resistance and of the action. Typical examples are problems governed by external loads, such as foundations subjected to structural actions. In cases where actions from the ground are involved for providing resistance (e.g. passive resistance), the use of LRFA for checking against failure in the ground is not straightforward: ULS in the ground could only be checked by comparing the mobilised ground resistance to the maximum available resistances; this requires however some parallel evaluation of the maximum available resistance which might be not straightforward and inaccurate for soil-structure interaction problems or for problems involving hardening behaviour.

Figure 1 illustrates the calculation scheme for a staged construction where MFA and LRFA are applied. The stress history is simulated for all consequent loading stages by performing the calculations using characteristic values of the action and ground strength properties. For each stage, an MFA and/or an LRFA calculation can be performed, starting from the characteristic stresses and deformations obtained in the staged construction. After these ULS checks, the calculation is started again from the characteristic stress field to simulate the next loading step.

3.3 *Discussion*

In cases where no failure in the ground develops, or when very stiff structures associated with small ground displacements are involved, MFA may underestimate the design value of internal forces in the structural elements: for stiff ground retaining structures for example, the displacement may be not sufficient to allow the earth pressure at rest to drop to the active pressure.

LRFA can then be applied to obtain design values of action effects in structural members

It is important to observe that numerical methods, of which FE are part of, cannot accommodate factoring geotechnical actions at their source because they intervene as well at the action side as at the resistance side and as it introduces artificial yielding (Frank et al, 2004). Thus ULS formats requiring factors larger than 1.0 on geotechnical actions, such as earth pressures, are not appropriate for ULS design using FEM.

Limit State Design involves ULS as well as SLS checks. The simulation of the stress history using characteristic values of all parameters is usually very close (but not necessarily equal) to SLS checks.

In design situations where ground strength does not play a role, or has a minor role, in providing actions or resistance, the above mentioned methods do not allow to investigate Ultimate Limit States. Uplift problems and other static equilibrium problems are typical examples of such situations. The ULS should be then investigated using alternative procedures, e.g. by reducing the weight density of the masses providing resistance. It should be noted that increasing the piezometric height for fully submerged structures will not trigger failure and is thus not an effective method to investigate for uplift failure in such situations.

4 IMPLEMENTATION IN THE DESIGN APPROACHES OF EUROCODE 7 (EN 1997-1)

Eurocode 7 (EN 1997-1) introduces three Design Approaches DA1, DA2 and DA3 for the ULS checks for geotechnical and structural failure in persistent and transient design situations where the ground strength plays a significant role in providing resistance. (Frank et al, 2004). Table 1 summarises the recommended values of the partial factors proposed in Annex A (normative) of EN 1997-1.

When applying the partial factors of table 1 to the general procedure on figure 1, one finds the following implementations for the three DA in FEM:

Table 1. Recommended values of partial factors in persistent and transient situations for the three Design Approaches according to EN1997-1
(1) design to be based on most severe of both calculations
(2) favourable permanent action: $\gamma_G = 1.00$ (3) when unfavourable; for favourable variable action: $\gamma_Q = 0.00$
(4) geotechnical action : action transmitted to the wall through the ground
(5) structural action: action from a supported structure applied directly to the wall

	Actions or action effects γ_F		Ground parameters γ_M				Resistances	
	Permanent unfavourable (2) γ_G	Variable(3) γ_Q	Weight density	tan φ' γ_φ	c' γ_c	c_u γ_{cu}		Piles
DA1/1(1)	1.35	1.50	1.00	1.00	1.00	1.00	1.00	1.3-1.6
DA1/2(1)	1.00	1.30	1.00	1.25	1.25	1.40	1.00	1.1
DA 2	1.35	1.50	1.00	1.00	1.00	1.00	> 1.00	1.1
DA 3	geotechnical action(4): 1.00 structural action(5) : 1.35	1.30 1.50	1.00	1.25	1.25	1.40	1.00	NA

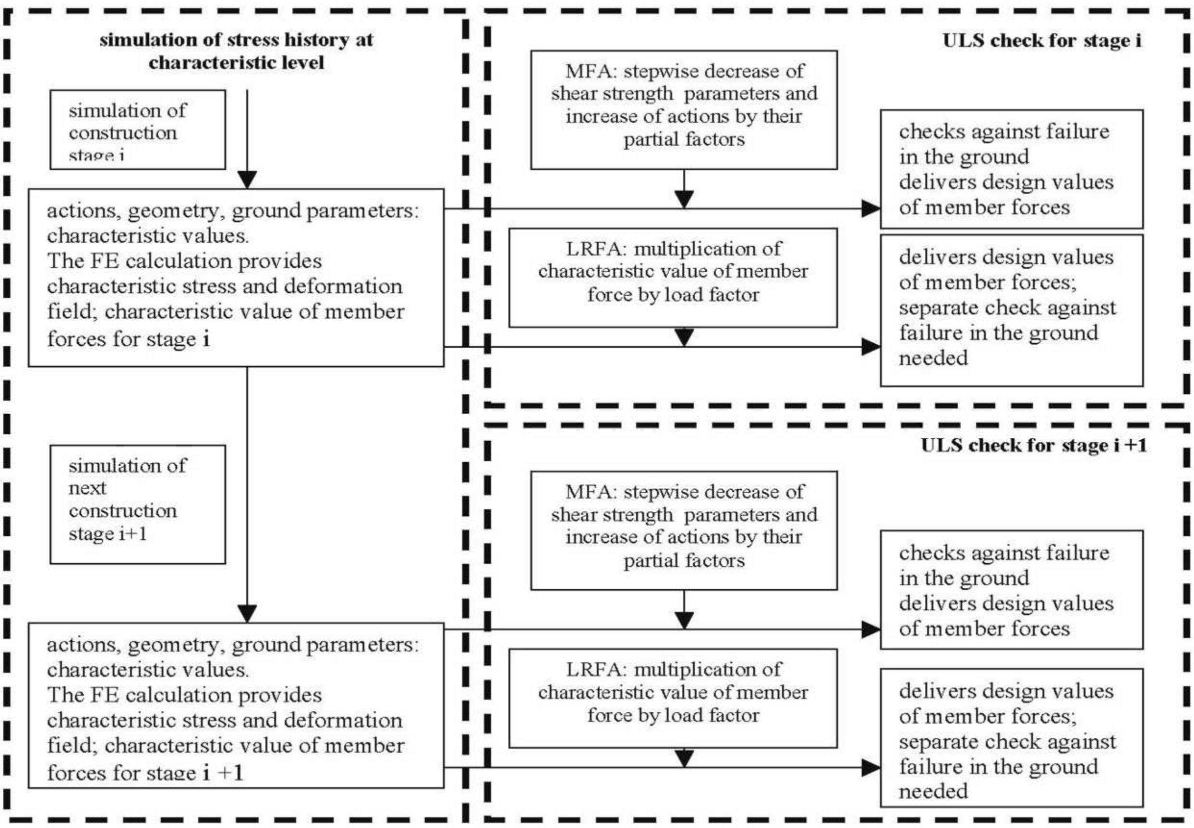

Figure 1. Calculation scheme for staged construction in MFA and LRFA

- Design Approach (DA1): both procedures "MFA" (for DA1/2) and "LRFA" (for DA1/1) have to be applied independently, starting from the characteristic stresses; the MFA checks against geotechnical failure; structural members must be checked for most severe of both "MFA" and "LRFA" calculation results;
- Design Approach (DA2): the "LRFA" procedure should be applied. This provides design values of forces in structural members. For situations where external actions and ground resistance do not interact, load-settlement curves can be established to check against failure in the ground. In some situations, such as passive resistance, the check against geotechnical failure requires supplementary considerations, e.g. mobilised resistance to maximum available resistance;
- Design Approach (DA3): the MFA procedure should be used, where the partial factors 1.35 and 1.50 are applied to the structural actions only. Attention should be paid to the definition of "structural action" in this respect: the action of a column is a structural action for the footing, but the action exerted by the footing on a nearby retaining wall is a geotechnical action. The calculation checks against failure in the ground and provides design values of forces in structural members.

The procedures of the three design approaches are easy to implement in analytical or semi-empirical models where only shear strength parameters c'-φ' or c_u play a role. EN 1997-1 gives no strict guidance concerning the design values of initial stress ratio and deformation parameters when they intervene in ULS calculations as is the case in the FEM. According to the spirit of EN 1990 and EN 1997-1, characteristic values should be advocated. In case of doubt on the values of the initial stress ratio and of the deformation parameters, the calculations should be performed with (combinations of) high and with low estimates of these parameters, instead of expecting some fallacious or misleading idea of safety by introducing "design values" of them.

5 THE PROCEDURE OF MFA SHEAR STRENGTH REDUCTION (φ'-c' REDUCTION OR c_u REDUCTION)

MFA procedures as illustrated in figure 1 are characterised by a stepwise reduction of the shear strength parameters φ' – c' or c_u up to the point that unlimited deformations are developed or at least to a reduction of strength corresponding to the value of the partial factor on the ground strength. The analysis is usually performed assuming both following assumptions:
- the strength reduction is performed using an ideally elastic purely plastic model with Mohr-Coulomb (MC) failure criterion (parameters describing the elastic behaviour: characteristic values of G and K; parameters describing failure and plastic flow: c' and φ' or c_u; dilatancy angle ψ). Whatever the material model used in the simulation of the stress history, the φ' – c' or c_u reduction analysis is then done reducing an equivalent MC yield cone;
- during the strength reduction, the ground is usually considered as non-dilatant material (dilatancy angle ψ = 0).

5.1 *Strength Reduction using linear elastic-purely plastic model*

When the stress history is simulated using an ideally elastic-purely plastic soil model, it is rather obvious to use the same model to perform the φ'-c' (or c_u) reduction. In many cases (e.g. loading in drained situation or loading in undrained situation where the undrained shear strength c_u is obtained from laboratory or in situ tests) the failure load is not significantly different when obtained by an elastic-plastic model with MC failure criterion or by a hardening model with MC failure criterion. The simple elastic-perfectly plastic soil model is usually sufficient to simulate the stress history in such cases.

Difficulties may however occur for consolidation problems when the shear resistance of the ground is calculated starting from the effective shear strength parameters and the excess pore pressures. In the elastic-perfectly plastic model, the effective stress path is not influenced by hardening: in fact within the MC yield cone elastic stresses are calculated whatever the isotropic stress. For soils exhibiting a hardening behaviour, the effective stress path and the corresponding undrained shear resistance depend on the hardening behaviour. The use of the elastic-perfectly plastic model could then overestimate the undrained shear resistance. Applying φ'-c' reduction with a soil model including a hardening law seems not to be sensible, as the failure parameters describing the MC failure envelope (which are stepwise reduced) and the parameters describing the hardening law interact in describing the ground behaviour. A more sound approach is to evaluate as exactly as possible the (undrained) shear resistance c_u available just before undrained loading or unloading using the effective stresses (characteristic values from the loading history) and the unfactored shear strength in the hardening model with parameters, and then to perform a stepwise reduction of these c_u using the elastic-purely plastic soil model. In such cases, the loading history should be simulated using a hardening model.

5.2 *Dilatancy equal to zero*

A φ'-c' reduction, should dilatancy angle ψ > 0 be used, might lead to an overprediction of the ground resistance which is not at the side of safety, at least for failure in the ground: if ψ > 0 a volumetric effect is introduced that, combined with the effect of the Poisson's ratio and restraints in the boundary condition, might lead to locking in the mesh. In some cases, it should be noted that restrained dilatancy might lead to increased member forces. A φ'-c' reduction with ψ = 0 is then not at the side of safety. Such cases involve however very small displacements and very stiff (even brittle) structures and are not well treated using MFA. LRFA using an appropriate hardening model is more suitable for such situations.

6 CONCLUDING REMARKS

Procedures have been proposed and discussed for the implementation of MFA and LRFA frameworks in FE for checking ULS. Nevertheless, some aspects need further development:
- in MFA, the φ'-c' reduction, starting from a characteristic stress field, include some conventions on the definition of failure
- in many LRFA problems, the check against failure in the ground is not at all obvious, especially for situations with complicated soil-structure interaction or with consolidation and hardening behaviour;
- the values of partial factors in ULS codes have been established bearing the analytical or semi-empirical models in mind; they have not yet been fully calibrated for design methods based on FE; sufficient probabilistic background to calibrate partial factors for FE has not been provided yet;
- non-linear behaviour of structural members.

Progress in the use of FE will need better theoretical understanding and comparative FE calculations on successful structures designed using limit state methods.

REFERENCES

EN 1997-1 (2004) Eurocode 7 part 1, General Rules CEN/TC 250/SC7. European Committee for Standardization, Brussels

Bauduin, C., Simpson, B., De Vos, M. (2000) : Some considerations on the use of Finite Element Methods in Ultimate Limit State Design, Int Workshop on Limit State Design, LSD 2000, Melbourne

De Borst, R. Vermeer P.A. (1984) Possibilities and limitations of finite elements for limit analysis, Geotechnique 34, No 2, 199-210

De Borst, R. (1986): Non-linear analysis of frictional materials PhD Thesis Delft University of technology

Frank, R., Bauduin, C., Driscoll, R., Kavvaddas, M., Krebs Ovesen, N, Orr, T., Schuppener, B. (2004) Designer's Guide to EN 1997-7 Thomas Telford ISBN 07277-3154-8

Vermeer, P.A. and R. de Borst, (1984) Non-associated plasticity for soils, concrete and rock, Heron 29 no 3

Verruijt, A. (2001,2004), Soil Mechanics, geo.verruijt.net

Wells, G.N., Sluys, L.J. & De Borst, R. (2002). Simulating the propagation of displacement discontinuities in a regularised strain-softening medium, Int. J. for Num. Methods in Eng. 53(5) 1235-1256