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## Laboratory testing

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### ABSTRACT

Brief descriptions are given of the most important, recent accomplishments in some select areas of experimental research. While the list of topics is not exhaustive, it includes soil stiffness at small strains, time effects, cross-anisotropy, strain localization and shear banding, and instability and liquefaction of silty sands.

### 1. INTRODUCTION

Laboratory testing of soil and rock specimens plays a central role in all endeavors within geotechnical engineering. Element tests are performed to study the material behavior and to provide material properties for use in design of geotechnical structures. While the list is not exhaustive, the topics reviewed below have received attention in recent years and are at the top of the list of current research interests in geotechnical engineering. Each of these topics requires special attention to the laboratory testing equipment and techniques to be used for their investigation.

### 2. SOIL STIFFNESS AT SMALL STRAINS

Determination of elastic properties such as Young's modulus, Poisson's ratio and the shear modulus and their variation with stress state for various soils plays an important role for the accuracy with which the deformation of geotechnical structures can be predicted. Under normal working loads the level of strain occurring in the ground rarely increases above 0.5%, and the accuracy of the material properties employed in the calculation of deformations therefore has a direct influence on the reliability of the predicted deflections. Small strains are also encountered in soil creep, and the deformations involved in control of stress relaxation experiments quickly become extremely small. Reliable measurements of such small quantities are therefore important for accurate description of time effects.

Recent research has indicated that measurements of small deformations due to static loading in conventional laboratory soil testing equipment may be affected by system compliance due to compression of interfaces, bedding errors, compression of lubricated ends, deflection of equipment components, etc., and that small deformations are best measured directly on the specimen inside the triaxial chamber. Comparison of methods of measurements often produces greater soil stiffness on the basis of locally measured deformations than from conventional, external measurements made by dial gages and LVDTs. Because the local measurements are free of system compliance, a number of devices have been developed for local measurements of small deformations. Recent techniques include proximity transducers, Hall effect transducer (Clayton and Khatrush 1986), inclinometers (Burland and Symes 1982), local deformation transducers (Goto et al., 1991), internal LVDTs (Lade and Abelev, 2005), and telescopic observations (Lade and Liu,

1998). Such devices allow direct measurements on the specimen to obtain accurate and stable vertical and lateral displacements lower than 0.5  $\mu\text{m}$ . Employment of such techniques allows investigation of soil behavior at strain levels smaller than 0.001 %.

Values of shear moduli and Young's moduli obtained from experiments with the more accurate, local measurement techniques are comparable with those from dynamic and wave propagation techniques such as resonant column and bender element tests, thus indicating the high quality of the local measurement techniques.

### 3. TIME EFFECTS

The behavior of soils with time is of great importance for prediction of performance of many field structures. For this reason the time rate dependency should be captured by an appropriate model. Recent reviews of the time-dependent behavior and characterization of soils (Augustesen et al. 2004, Liingaard et al. 2004) have indicated that sand and clay exhibit different types of behavior and therefore require different types of models to capture this behavior.

Fig. 1 shows schematic diagrams of the two types of behavior. The fact that the phenomena of creep, relaxation, and strain-rate effects are governed by the same basic time-mechanism is denoted "isotach" behavior, i.e. there is a unique stress-strain-strain rate relation for a given soil, as shown to the left in Fig. 1. The isotach behavior corresponds to some extent to the observed behavior of clay. This means that creep and relaxation properties can be obtained by means of constant rate of strain tests and vice versa. This kind of mechanism where creep, relaxation, and rate dependency are considered to be due to the same mechanism is also indicated by their adherence to the "Correspondence Principle."

Sands exhibit noticeable amounts of creep and relaxation but no strain rate effects (Tatsuoka et al. 2000). This leads to the conclusion that the phenomena of creep and relaxation cannot be predicted from results obtained in constant rate of strain loading tests on sand. This is because effects of changes of strain rate are temporary, as shown in Fig. 2. This behavior of sand does not correspond to the observed rate effects of clay. For sand this behavior is labeled "non-isotach" behavior. The non-isotach behavior is also illustrated to the right in Fig. 1.

It is clear that the viscous nature of time effects embodied in existing models developed so far cannot handle all the observed

time effects. The same fundamental equations are used to predict creep, stress relaxation, and constant rate of strain behavior by imposing appropriate boundary conditions on the soil element. Only the boundary conditions are changed when modeling the different aspects of time dependent behavior. Whether models are empirical, rheological, or general in nature, they all make use of the correspondence principle. Models for non-isotach materials are still under development and not available for other than the most simplistic one-dimensional cases.

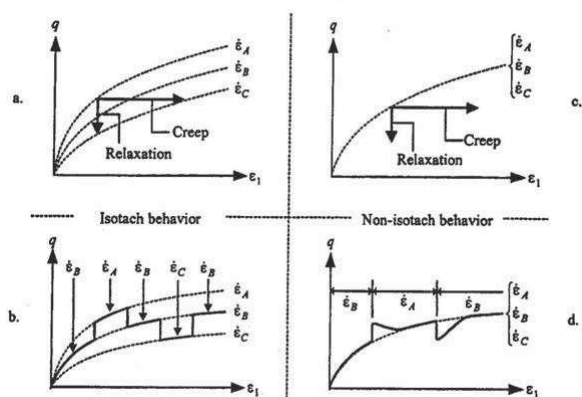


Fig. 1. Isotach behavior is observed in clay for (a) creep and relaxation and (b) stepwise change in rate. Non-isotach behavior is observed in sand for (c) creep and relaxation and (d) stepwise change in rate.

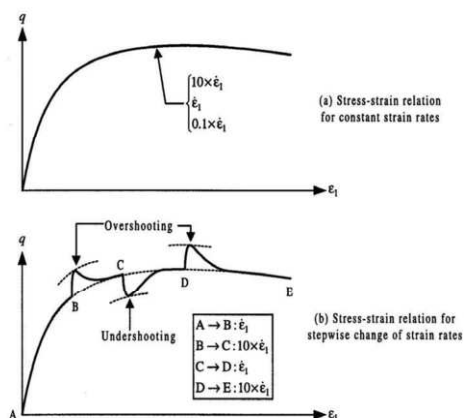


Fig. 2. Schematic diagrams illustrating the rate dependency observed for sand: The stress-strain relation for different constant strain rates coincide for the three strain rates, and (b) temporary over- and under-shooting due to stepwise change in strain rate.

#### 4. CROSS-ANISOTROPY

Geological materials exhibits behavior that is inherently anisotropic. This is due in large part to the internal directional structure of these materials, formed during various natural depositional processes. Most soils possess an axial or transverse isotropic structure, often referred to as cross-anisotropic, in which the properties are identical in all directions within the horizontal plane, and different from those in the vertical direction, which is the direction of deposition.

Failure in soils under three-dimensional conditions has been studied on many occasions. Most of the soils have been assumed to be isotropic in behavior, and several isotropic failure criteria have been proposed. However, the importance of the anisotropy was demonstrated by Oda et al. (1978), who compared the bearing capacity of two model strip footings on sand: One footing was loaded parallel to the sedimentation direction

and another was loaded normal to it. The two values of bearing capacity differed by as much as 34%. Thus, the evidence for cross-anisotropic behavior at failure is mounting, and this evidence is briefly reviewed.

Two types of anisotropy may be distinguished. The first is attributed to the initial fabric of the particle assembly in its virgin state before any loading occurs. This type of anisotropy is referred to as inherent. The other type is a result of loading and plastic deformation, and it can develop in originally isotropic material or change any preexisting (inherent) condition. Casagrande and Carillo (1944) first made this observation in relation to strength of soils.

Clays.- Cross-anisotropy was first studied in detail in clays, in which the undrained behavior stands out as possessing variable strength with direction, as indicated in a study by Duncan and Seed (1966). Whether the clays were normally consolidated or overconsolidated, there did not appear to be any consistent and systematic higher or lower strengths in the vertical than in the horizontal direction. It is difficult to study the fabric of clays, but the variation in undrained strength was taken as an expression of the orientation of the fabric of clay particles. Kirkgard and Lade (1991, 1993) and Lade and Kirkgard (2000) studied the three-dimensional, cross-anisotropic strength of intact specimens of San Francisco Bay Mud. The specimens were trimmed, isotropically consolidated, and tested in vertical and horizontal directions, and they showed strong cross-anisotropic, effective strength variation.

Sands.- The fabric anisotropy of sand deposits is easier to characterize, because the larger particles are visible to the naked eye. A number of studies were performed in the late sixties and early seventies of the fabric characteristics of random assemblies of 2-D and 3-D particles as well as natural sands. Parkin et al. (1968), Arthur and Menzies (1972), Oda (1972 a, b, c), El-Sohby and Andrawes (1973) were first to systematically study the anisotropic nature of granular materials. It was found that when natural sand or even perfectly round spheres are deposited under force of gravity, the material structure results in cross-anisotropic fabric. The major reason for this structural anisotropy is the preferred orientation of interparticle contacts that favor the direction of deposition. The interparticle contact normals are not so easy to determine, but the sand fabric may alternatively be characterized by other measures, such as the preferred orientation of particle long axes. This was done for sands sedimented through both air and water (Oda et al. 1978, Ochiai and Lade, 1983).

A number of studies of the influence of the initial anisotropy on failure of sands under three-dimensional loading conditions have been performed (e.g. Yamada and Ishihara 1979; Matsuoka and Ishizaki 1981; Ochiai and Lade 1983; Abelev and Lade 2003; Lade and Abelev 2003). The accumulated evidence show that under monotonic conditions, when loading and deposition directions coincide, and when no rotation of principal stresses occurs, then the initial anisotropic fabric largely controls the deformation process and the peak shear resistance, especially in sands with elongated particles. This fact has been utilized in testing programs to study the influence of inherent cross-anisotropy on the failure criterion for such soils.

In the study of effects of cross-anisotropy presented by Ochiai and Lade (1983), cubical specimens of dense ( $D_r = 90\%$ ) Cambria sand consisting of relatively long, flat particles were prepared with strong preferred particle orientations in vertical sections and almost completely random orientations in horizontal sections. The effective strength envelope was expected to be symmetric around the vertical axis in the octahedral plane, because the cross-anisotropic specimens have a vertical axis of symmetry with regard to their properties. Their results, as well as those obtained by Yamada and Ishihara (1979), clearly showed cross-anisotropic stress-strain behavior, while the failure surface indicated some, but was less clearly influenced by cross-anisotropy. It was noticed that the strain-to-failure in all the cubical triaxial tests on Cambria sand was

rather large, in the order of 5-8% in the midrange of  $b$ -values, and it was concluded that sufficient changes in the fabric had occurred at large strains to produce failure conditions that resembled those observed for isotropic sands.

A large number of drained, true triaxial tests were performed on cubical specimens of Santa Monica Beach sand with constant effective cell pressure,  $\sigma_3 = 50$  kPa, and with constant values of  $b = (\sigma_2 - \sigma_3)/(\sigma_1 - \sigma_3)$  to explore the entire range of Lode angle  $\theta$  from  $0^\circ$  to  $180^\circ$ . Experience with this sand showed much smaller strain-to-failure, in the order of 1-2% in the midrange of  $b$ -values, than observed for Cambria sand. The results of this study (Abelev and Lade, 2003; by Lade and Abelev, 2003) indicated pronounced effects of cross-anisotropy on the effective strength. Fig. 3(a) shows the  $\phi - b$  diagram with the friction angles obtained in each of the three sectors in the octahedral plane, and Fig. 3(b) shows the octahedral plane with the strength results plotted on one side of the vertical axis. The Mohr-Coulomb failure criterion and Lade's failure criterion (1977) are also indicated on Fig. 3(b). Both isotropic criteria have been fitted to the strengths in conventional triaxial compression located at the top of sector I.

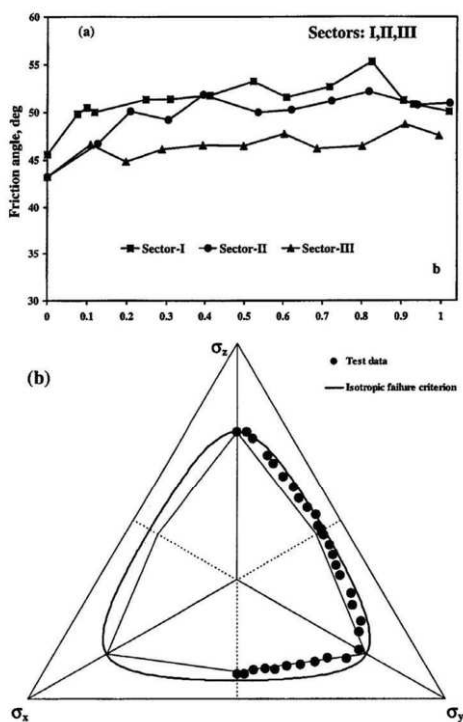


Fig. 3. Experimental results of true triaxial tests on dense Santa Monica Beach sand with cross-anisotropic fabric shown in (a)  $\phi - b$  diagram and (b) octahedral plane.

It is clear that the strength of the sand is affected by cross-anisotropy. Thus, the friction angles in sector III are of magnitudes that fit the friction angle in conventional triaxial compression, as indicated by their proximity to the Mohr-Coulomb failure surface. In particular, the friction angle in conventional triaxial extension, located at the bottom of sector III, is similar in magnitude to the friction angle in conventional triaxial compression. Thus, failure in sector III corresponds to lower friction angles.

## 5. STRAIN LOCALIZATION AND SHEAR BANDING

Experimental results with respect to the formation of shear bands under various stress conditions are available from previous investigations (e.g. Arthur et al. 1977; Vardoulakis 1980;

Desrues et al. 1985, 1996; Tatsuoka et al. 1990; Yoshida et al. 1993; Finno et al. 1996, 1997; Nemat-Nasser and Okada 2001), while theoretical developments and predictions have been attempted in other studies (e.g. Hansen 1958; Rudnicki and Rice 1975; Rice 1976; Vardoulakis et al. 1978; Vardoulakis 1980; Vermeer 1982; Molenkamp 1985; Muhlhaus and Vardoulakis 1987; Vardoulakis 1996 a,b).

Experimental studies of shear banding in true triaxial tests have been performed (Wang and Lade 2001). The three-dimensional strength characteristics and the influence of strain localization and shear banding on failure were studied and the peak strengths were compared with the failure criterion proposed by Lade (1977). Quantitative comparisons were made between the critical conditions for shear band formation obtained from experiments (Lade and Wang, 2001) and from theoretical considerations and expressed by the dimensionless hardening parameter  $H_c/E$  immediately prior to the onset of shear banding (Lade, 2003). Fig. 4 presents a comparison between measured and predicted friction angles for the true triaxial tests on dense Santa Monica Beach sand. Shear banding controls the strength in the middle range of  $b$ -values, both according to the measured and to the predicted results. Thus, the failure surface for granular material is not a smooth surface that can be described by a single expression, and a smooth peak failure is obtained only for the extreme values of  $b$ , while shear banding causes failure in the midrange of  $b$ -values, including plane strain.

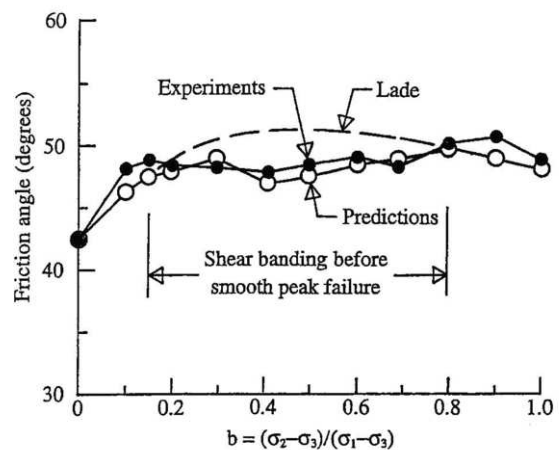


Fig. 4. Comparison of measured and predicted friction angles for true triaxial tests on dense Santa Monica Beach sand. Shear banding controls strength in the middle range of  $b$ -values.

## 6. INSTABILITY AND LIQUEFACTION OF SILTY SANDS

Most historic cases of both static and earthquake-induced liquefaction have occurred in alluvial deposits of loose silty sands (Yamamuro and Lade 1999). While most of the research conducted to study the mechanics of liquefaction has been performed on clean sands, results from recent experiments involving loosely deposited silty sands appear to dispute the assumption that clean sands always behave similarly to silty sands. There appears to be a strong correlation between the fines content and the liquefaction potential of the soil (Yamamuro and Lade 1997, Lade and Yamamuro 1997). The tests also indicate a 'reverse' behavior pattern with respect to confining pressure violating the basic assumption that loose silty sands behave similarly to loose clean sands.

Fundamental experiments were originally performed to demonstrate the conditions for which granular soils become un-

stable inside the failure surface, and it was shown that this phenomenon was a consequence of the non-associated plastic flow observed in frictional materials (Lade et al. 1987, 1988). Although both instability and failure involve reduction in shear strength and consequently may lead to catastrophic events such as gross collapse of earth structures, they are not synonymous. In fact, instability and failure are different in nature, and they require different types of analyses. Failure analysis involves comparison of current shear stresses on a potential failure surface with the shear strength available on that surface. Analysis of instability requires determination of the current effective stress states in the slope and recognition of their magnitudes relative to the instability line. Instability can potentially be triggered in any region of the slope in which the stress states are above the instability line.

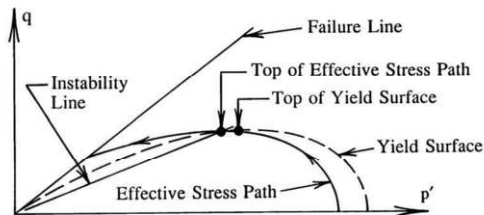


Fig. 5. Schematic diagram of location of instability line determined from consolidated-undrained test on loose sand.

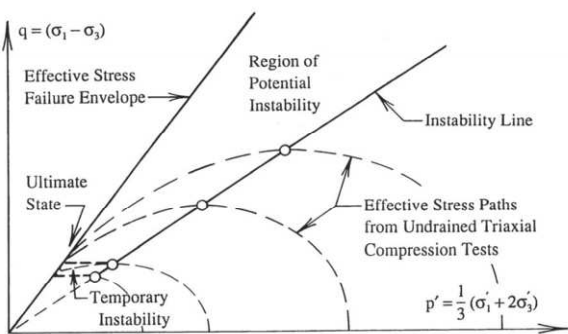


Fig. 6. Schematic diagram of location of instability line in  $p'$ - $q$  diagram.

It is a fact that loading of a compressible soil (resulting in large strains) can occur under decreasing stresses that lead to unstable behavior under undrained conditions. Loose, fine sands and silts have sufficiently low permeabilities that small disturbances in load or even small amounts of volumetric creep may temporarily produce undrained conditions in such soils, and instability of the soil mass follows. A review of mechanisms that may act as a trigger for slope instability (Lade 1993) indicated that as long as the soil remains drained, it will remain stable in the region of potential instability.

When the condition of instability is reached, the soil may not be able to sustain the current stress state. This state corresponds to the top of the tear-drop shaped yield surface, as shown schematically on the  $p'$ - $q$  diagram in Fig. 5. Following this top point, the soil can deform plastically under decreasing stresses. The top of the undrained effective stress path, corresponding to  $(\sigma_1 - \sigma_3)_{max}$ , occurs slightly after but very close to the top of the yield surface. Fig. 6 shows a schematic  $p'$ - $q$  diagram in which the line connecting the tops of a series of effective stress paths from undrained tests provides the lower limit of the region of potential instability. Experiments show that this line is straight. Since it goes through the top points of the yield surfaces which evolve from the origin of the stress diagram, the instability line also intersects the stress origin. A region of temporary instability is located in the upper part of the dilatancy

zone, as shown in Fig. 6. For very loose soils, the total strength envelope intersects the stress origin, and the region of potential instability reaches down to the origin of the stress diagram.

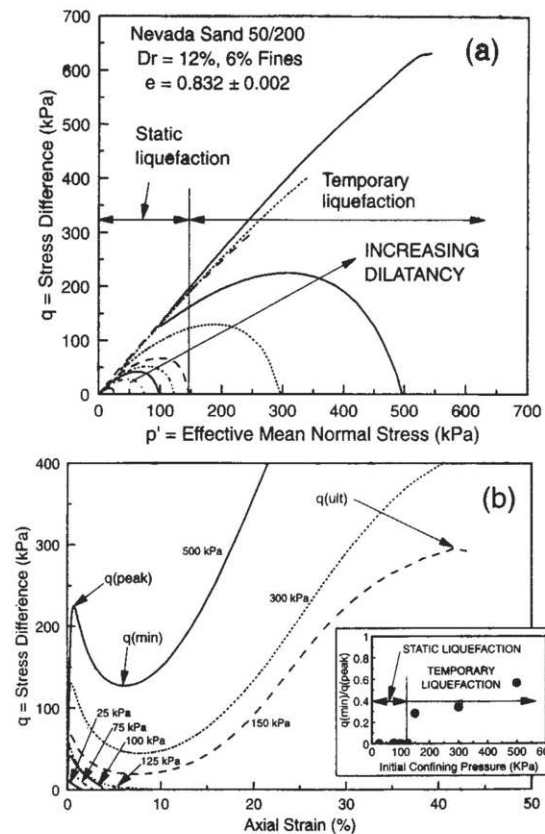


Fig. 7. Undrained triaxial compression tests with confining pressure variations; (a) effective stress paths; (b) stress-strain relationships.

However, silty sands behave differently from clean sands. Figs. 7a and 7b show effective stress paths and stress-strain curves of a series of undrained triaxial compression tests performed on 50/200 Nevada sand containing six percent non-plastic fines. The initial relative density was 12 percent and the initial confining pressures ranged from 25 to 500 kPa. The 'reverse' pattern of behavior is readily apparent. Complete static liquefaction (zero effective stress) was observed in undrained tests at low initial confining pressures (between 25 and 125 kPa), while at higher initial confining pressures the soil exhibits more dilatant behavior resulting in temporary liquefaction. Temporary liquefaction behavior occurs when the stress difference achieves an initial peak, then declines to a minimum value due to decreasing effective stresses caused by rising pore pressures, and finally increases from the minimum value because suppressed dilation (decreasing pore pressures) causes increasing effective stresses. The inset diagram in Fig. 7b shows the ratio of minimum stress difference ( $q_{min}$ ) divided by stress difference at the initial peak ( $q_{peak}$ ) plotted against the initial confining pressure. The trend indicates a rising  $q_{min}/q_{peak}$  ratio with increasing confining pressure. This indicates that the soil is exhibiting increasing stability as the confining pressure increases. This is opposite the trend observed for most granular soils. This leads to the significant conclusion that loose silty sands are most susceptible to static liquefaction at low confining pressures.

Since loose silty sands are most vulnerable to static liquefaction at low pressures, a series of undrained triaxial compression tests was performed at 25 kPa. Nevada 50/80 sand was used as the base sand, and the fines content was varied from zero to 50 percent by weight. Figs. 8a and 8b show effective

stress paths and stress-strain curves for this series of undrained tests. It is observed that the effect of increasing fines content is to increase the liquefaction potential. The test with no fines does not liquefy under static conditions, but the addition of fines results in liquefaction. Adding silt causes the effective stress paths to be depressed with smaller initial peak stress differences being exhibited. The axial strain at liquefaction decreases with increasing fines content. Thus, increasing liquefaction potential is observed despite the fact that the absolute and relative densities increase as shown in the legend (void ratio is shown in parentheses). Again, this is opposite the normally observed trends in granular soils. These results tend to indicate that void ratio or density does not correlate well with liquefaction potential of loose silty sands.

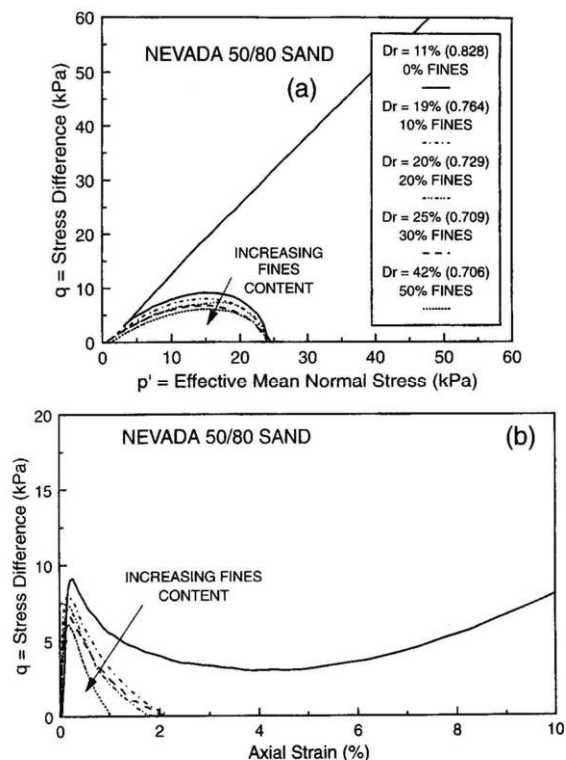


Fig. 8. Undrained triaxial compression tests with variation of fines content; (a) effective stress paths; (b) stress-strain relationships.

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