Modelling ten years of downhill creep data

La modélisation des résultats du fluage pendant dix années

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ABSTRACT: Worldwide, shallow slopes in clays often move downhill at rates which vary as a consequence of seasonal fluctuations in ground-water level (i.e. mean porewater pressure in the sliding mass). In the paper a simple, one-dimensional, lumped-mass, three-degree-of-freedom dynamic model of this process, incorporating non-linear, velocity-dependent damping, is applied to a ten-year record of ground-water versus surface-displacement data obtained from an over-consolidated clay slope at Cortina d'Ampezzo (Italy). This very extensive record has been interpreted successfully by a version of the model in which the mean, effective friction angle on the sliding interface was assumed to decrease progressively with increasing downhill displacement along it.

RÉSUMÉ: A l'échelle mondiale, les terrains en pente glissent fréquemment à des vitesses qui varient selon les variations saisonnières du niveau phréatique. Cet article décrit l'application d'un modèle mathématique dynamique et simple, y inclus l'amortissement non-linéaire qui dépend de la vitesse de déplacement, au comportement, mesuré pendant dix ans, des niveaux phréatiques vis-à-vis le mouvement du terrain, les données prises sur une pente argileuse sur-consolidée à Cortina d'Ampezzo (Italie). Ce dossier de données, d'un étendu très important, a été interprété avec succès par moyen d'une version du modèle qui présume la diminution progressive de l'angle de frottement interne moyenne du talus selon son déplacement le long d'une surface de discontinuité.

1 INTRODUCTION

In many areas of the world, shallow slopes in clays undergo progressive downhill motion, which accelerates and decelerates in response to fluctuations in pore-water pressure brought about by seasonal changes in ground-water levels. Typical slope movements are a few mm/year, relative to a plane of sliding at a specific depth in the hillside. Such motion is conventionally described as 'slope creep', although a true creep process takes place under constant effective stresses which is not the case when the porewater pressures in the hillside change over time. The analysis in the paper uses a simple, one-dimensional, lumped-mass, dynamic model of the "creep" process (Butterfield, 2000). The empirical parameters used in the model can be back-calculated from in-situ data comprising time-series records of piezometric levels and downhill displacements of the sliding soil mass. Although these can be determined directly from a few records, an extensive data-set provides a more rigorous test of the model, in effect checking its predictive capacity over a number of monthly, or even annual, cycles of ground-water level change. In the paper, a ten-year record has been used for this purpose.

2 SUMMARY OF THE MODEL

Figure 1a shows a unit-length, free-body diagram of a slab-sliding mass M, of thickness D, on a slope inclined at an angle θ, necessarily less than the mobilised effective friction angle φ' on the interface. If a non-linear damping force $C\dot{x}^n$ is mobilised ($\dot{x}$ being the downslope velocity of M) then the relevant equation of motion is,

$$M\ddot{x} + C\dot{x}^n = (P - F) = N(\tan\theta - \beta\tan\varphi') \quad (1)$$

Where $P = Mg\sin\theta$, $F = N'\tan\varphi'$, $N = Mg\cos\theta$, $N' = N\text{-}U$ and $\beta = N'/N$ with U the uplift force due to the piezometric level in the hillside.

Figure 1b shows the phreatic surface at height d above and parallel to the surface of the sliding mass, whence,

$$N' = \gamma_s D\cos\theta - \gamma_w d\cos\theta \quad (2)$$

$$\beta = (N - U)/N = 1 - (\gamma_w/\gamma_s)(d/D) \quad (3)$$

in which γ_w and γ_s are, respectively, the unit weights of water and the soil.

Field records of (x,d) are easily transformed into ($\dot{x}$,β) pairs. From such a set, a unique, and usually recurring pair (v_0,β_0) can be selected which identify the step change from a static slope to a sliding one (i.e. v_0 is the onset of detectable motion). Substituting these values into Equation 1, together with $\ddot{x} = \theta$, provides the value of $\varphi' = \varphi'_0$ mobilised at the onset of sliding, i.e.

$$\beta_0\tan\varphi'_0 = \tan\theta - C(v_0)^n/N \quad (4)$$

Substituting $\varphi' = \varphi'_0$ into Equation 1 leads to,

$$(\ddot{x}/g) + C\dot{x}^n/Mg = \sin\theta(1 - \beta/\beta_0) + (\beta/\beta_0)C(v_0)^n/Mg \quad (5)$$

By introducing the following dimensionless variables into Equation 5,

$$T = t/v_0;\; X = xg/v_0^2;\; \dot{X} = \left(dX/dT\right) = \dot{x}/v_0;\; \ddot{X} = \left(d\dot{X}/dT\right) = \ddot{x}/g$$

it can be re-written as,

$$\ddot{X} + G\dot{X}^n = (B + (\beta/\beta_0)G) = H \quad (6)$$

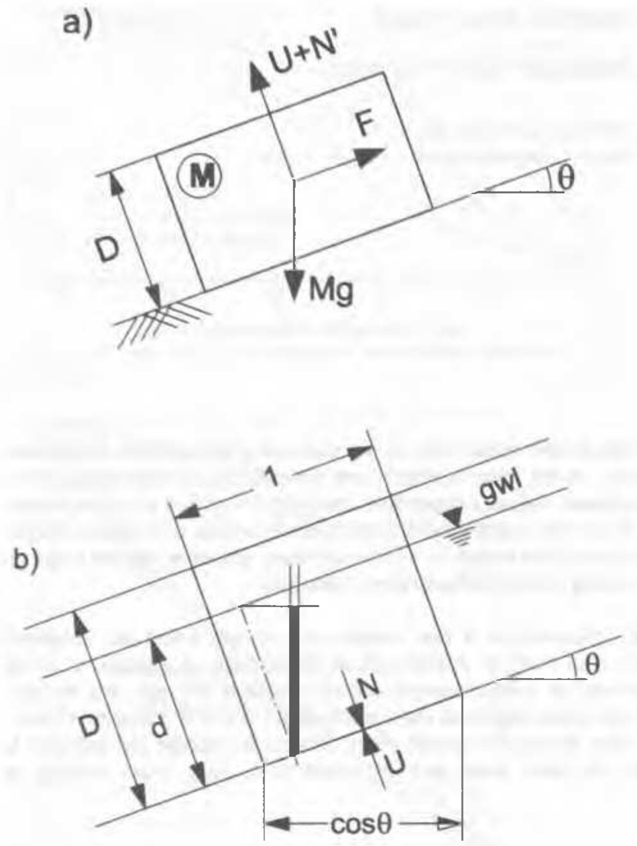

Figure 1. (a) Free-body diagram; (b) ground-water level and uplift forces.

Wherein, $B = \sin\theta(1-\beta/\beta_0)$ and $G = C(v_0)^n/Mg$. G is therefore an empirical constant, which subsumes the two unknown quantities (C,M). The values of G and n, which best fit the ($\dot{x}$,β) data via Equation 6, are to be found. Since $(\beta/\beta_0) \le 1$ and G turns out to be small, H = B is an acceptable approximation. This enables the ($\dot{x}$,β) data to be converted directly into ($\dot{X}$,H) form.

Since $v = v_0$ when $\beta = \beta_0$, sliding stops when $\beta > \beta_0$. Therefore Equation 6 only has meaning when $\beta \le \beta_0$, the downhill velocity then responds to changes in the piezometric level. The solution to the equation consists of a transient complementary function, which will be damped-out over time, and a particular integral, which will provide the "long term" solution required. The particular integral used hereafter, together with the initial conditions $\dot{X}(0) = v_0$ and $X(0) = 0$, is,

$$X = T(H/G)^{1/n};\quad \dot{X} = (H/G)^{1/n};\quad \ddot{X} = 0 \tag{7}$$

Although the acceleration phases are not captured by the model, the step changes in velocity fit very well with the form of the field data.

If the second term in Equation 7 is re-written as,

$$n\log(\dot{X}) + \log(G) = \log(H) \tag{8}$$

"best fit" (n,G) values to the ($\dot{X}$,H) data and the selected (v_0,β_0) pair, can be obtained by a conventional linear regression procedure. This process has been used to analyse the data from Cortina d'Ampezzo (Italy) – named after their location, Alverà - and provide the following interpretation of it.

3 THE ALVERA' FIELD DATA

The "mudslide" at Cortina D'Ampezzo (Alverá), in the eastern Dolomites (Italy), was described by Deganutti & Gasparetto (1992). Extensive data on its downhill motion and the ambient piezometric levels has been published by Angeli *et al* (1989, 1996, 1998 and 2000). The material within which the movements were recorded "consists of irregular, poorly sorted blocks of the original rock dispersed in an argillaceous matrix" (Angeli *et al*, 1996). They reported tests on the clay fraction of two samples recovered at the slip surface (well defined at D = 5m), which gave $w_L = 0.95$, $I_p = 0.48$, $\gamma_{sat} = 18.73$ kN/m^3 and $\varphi'_r = 15.9°$ (ring shear). The latter being rather high according to the classification data and Lupini *et al* (1981).

Angeli *et al* have recorded downhill displacements (measured by 11 steel-wire extensometers and 4 inclinometers) and piezometric levels (from 11 transducers), in this hillside, since 1989. The complete slide is about 1700 m long, with a central, more extensively studied section, some 800 m long by 80 m wide with an average slope angle of about 8°. The dynamic model described above was first published by Butterfield (2000), who successfully fitted it to the Angeli *et al* 1990-1993 data.

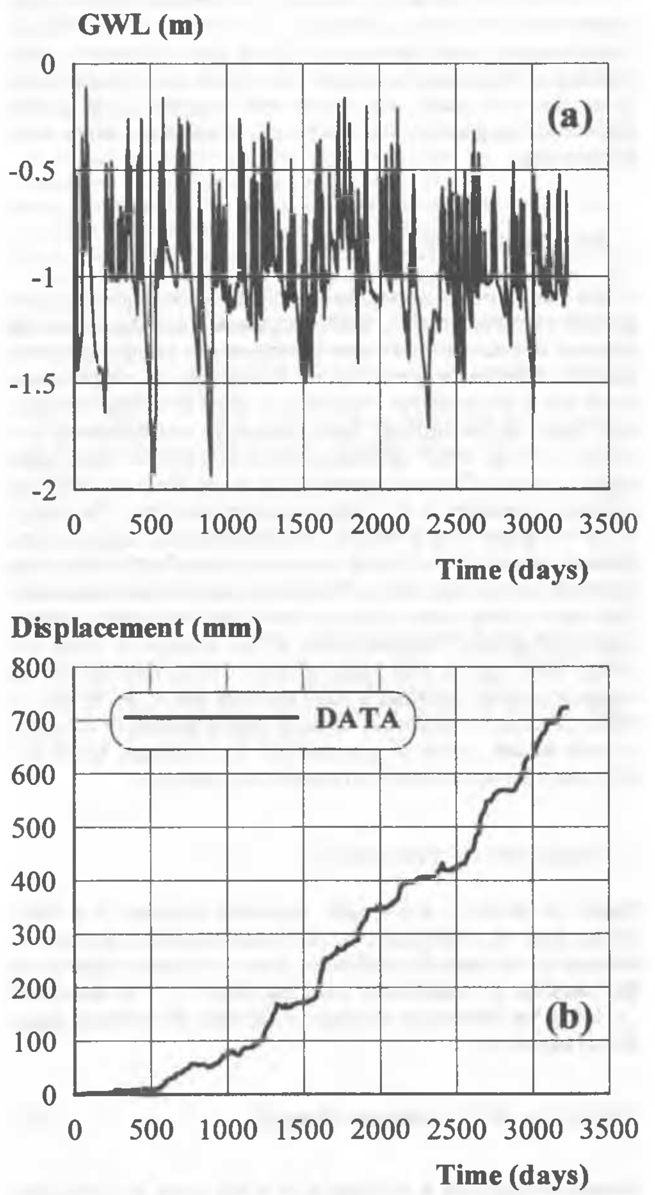

Figure 2. (a) ground-water level and (b) surface displacement data from Alverá.

Since then they have provided measurements taken over a period of almost 10 years and this is the data analysed below. (Incidentally, they quoted displacements previously which were too big by a factor of almost x 3. This does not, of course, invalidate the principle of the fit previously achieved with the model.)

The unprocessed data is shown in Figures 2a, b. The salient features of the records are; (a) ground water levels which oscillate quite rapidly over a range of about 1.5 m and (b) increasing downhill ground-surface velocities at times of high groundwater level coupled to a general trend of the velocity increasing over time, even though recent average ground-water levels been slightly lower than those in previous years. A value of v_0 = 15 mm/year, coupled with β_0 in the range 0.61 to 0.62 can be established quite easily from the displacement versus time record. However, it proved to be quite impossible to correlate the displacement measurements and the raw piezometric level data satisfactorily using the model.

Mathematica code was written to enable; (i) the recorded water-levels to be averaged over an arbitrary period, (ii) an arbitrary time lag to be imposed on the displacement response to changes in piezometric levels and the best-fit procedure to be automated. It is self-evident that, as the period of the averaging process is extended in (i), a better fit of the 3° of freedom model (n, G, β_0) will be achieved, ultimately destroying the time-variability of the record. Fortunately, a good correlation could be obtained using piezometric levels (and displacements) averaged over only 14 days, a period which was short enough to preserve the detailed fluctuations in the displacement-time response. No benefit was evident from introducing a time-lag (ii) and this was set at zero.

4 OUTPUT FROM THE ANALYSIS

Figure 3 shows the fit achieved by the model to the totality of the field displacement data using β_0 = 0.617, G = 0.0071 and n = 0.315 (i.e. φ'_0 = 12.20°, from substitution of β_0 and G in Equation 4). The standard deviation between the predicted and measured displacements was minimised by adjusting the value of β_0 iteratively whilst repeatedly applying the regression analysis to Equation 8.

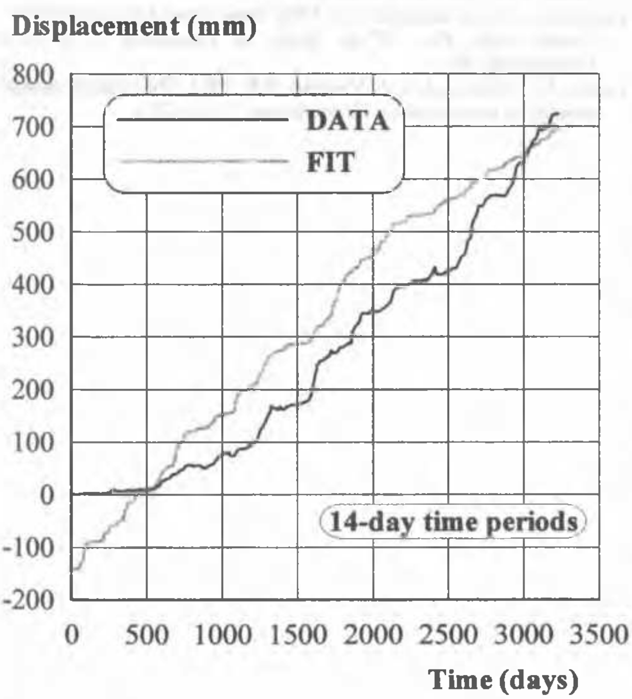

Figure 3. Comparison of displacement prediction (φ'_0 = constant) with field data.

Nevertheless, although most of the undulations in the field data are reflected in the output from the model the overall fit is not satisfactory.

Since the model had previously successfully interpreted a three-year portion of the same data (Butterfield, 2000) the data was divided into four, approximately 900 day, sections to each of which the model was fitted independently. The hypothesis being tested here was that the value of φ'_0 might be decreasing with increasing downslope movement (i.e. with increasing displacement along the 5 m deep discontinuity). If G and n remained sensibly constant during this process then a progressively decreasing value of φ'_0 would be reflected in a progressively increasing value of β_0. Figures 4, 5 and 6 show the results of this investigation. In each case G = 0.0071 and n = 0.315 were fixed and the iteration was solely on β_0, to which the output is quite sensitive, until the standard deviation between the measured and predicted displacements were minimised. The quality of the fit to the shorter time series is now very much better.

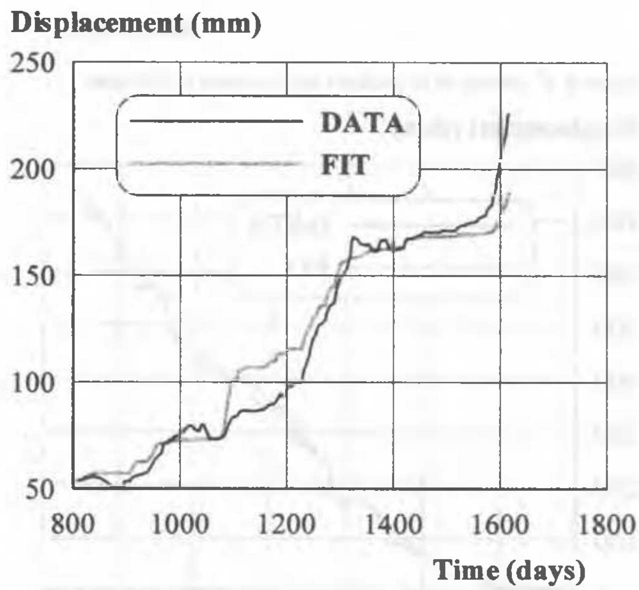

Figure 4. 2nd quartile fit of predicted displacements to field data.

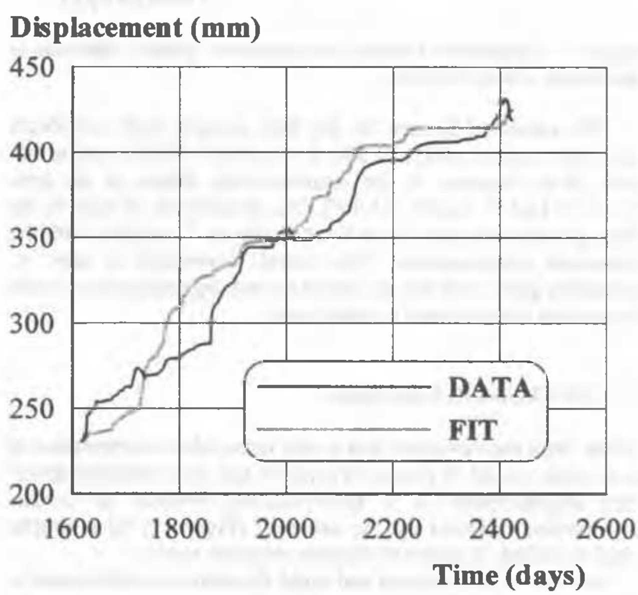

Figure 5. 3rd quartile fit of predicted displacements to field data.

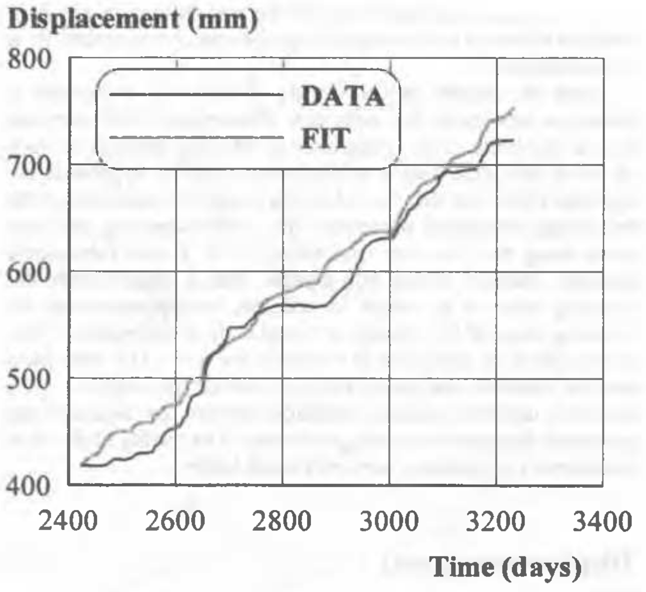

Figure 6. 4th quartile fit of predicted displacements to field data.

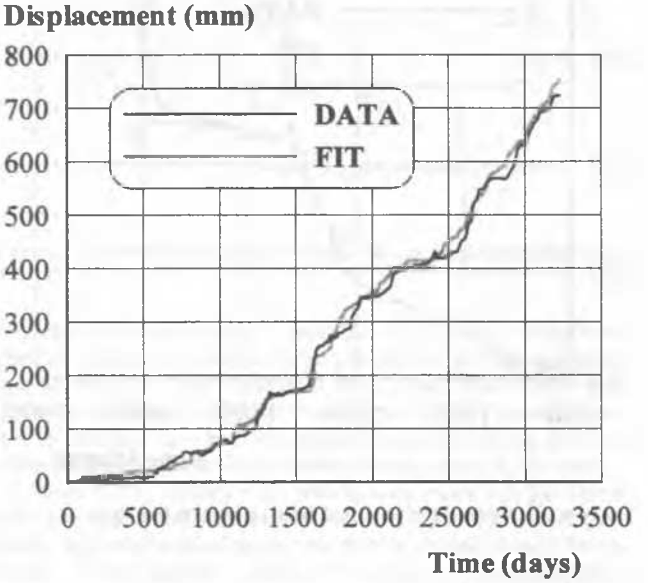

Figure 7. Comparison between the cumulative quartile displacement predictions and the field data.

The values of β_0 used for the first, second, third and fourth quartiles were, in order, (0.568, 0.605, 0.609, 0.635) and, as before, from Equation 4, the corresponding values of φ'_0 were (13.21°, 12.43°, 12.35°, 11.86°). The predictions of each of the four quartiles are shown combined in Figure 7, together with the measured displacements. The overall agreement is now remarkably good, with the φ'_0 values decreasing progressively with increasing displacement as anticipated.

5 CONCLUDING REMARKS

It has been demonstrated that a very reasonable interpretation of a ten-year record of piezometric levels and the associated downhill displacements in a slowly-moving hillside at Cortina d'Ampezzo (Alverá) can be achieved (Figure 7) by a simple, well specified, 3-degree of freedom dynamic model.

In view of the frequent and rapid fluctuations which occur in the groundwater data (Figure 1a) it is remarkable that it only needs to be averaged over 14 day periods to obtain a satisfactory correlation with the displacement response.

Underlying this result is an assumption that the mobilised effective friction angle on the sliding-interface, at 5 m depth in the slope, is decreasing with increasing relative displacement on it. The interpretation presented is that this angle has decreased by about 1.5 degrees (from 13.21° to 11.86°) during the 0.75 m slip that has occurred over a 10 year period. These are plausible values. It is clearly feasible to include an analytical expression in the analysis coupling β_0 with slope displacement.

It is interesting to note that when ground-water level is at the ground surface, $d/D = 1$, β falls to 0.466 (Equation 3). Were this to occur, the model predicts (Equation 7) that the value of x (currently $\approx$ 160 mm/yr) would increase to almost 2 m/yr (for $\beta_0 = 0.635$) and double again if φ'_0 were to fall by a further 1°.

The dimensionless parameters (n, G) are entirely empirical and it remains to be investigated whether or not the model can be applied equally successfully to "creeping" slopes in different soils, under different ground-water regimes and, if so, what values of (β_0, n, G) might be relevant.

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