Multistage compression test – in a shorter way

E.Imre – *Geotechnical Research Group for the Hungarian Academy of Sciences and Budapest University of Technology and Economics, Budapest, Hungary*
L.Aradi – *Szent István University Ybl Miklós School, Budapest, Hungary*
K.Vértes – *Budapest University of Technology and Economics, Budapest, Hungary*
P.Menyhárt – *Budapest University of Technology and Economics, Budapest, Hungary*
G.Telekes – *Szent István University Ybl Miklós School, and Budapest University of Technology and Economics, Budapest, Hungary*

ABSTRACT: The duration of the multistage compression test is generally one week since the stages generally take about 24 h. The test can be performed during a day if shorter stages are applied. Some preliminary results are presented concerning two cases: (i) κ>99 % is attained during the 1-2 h stages; (ii) κ>99 % can be attained during the last stage within about 14 h, several short stages (κ<99 %) are applied which are extrapolated by inverse problem solution.

RÉSUMÉ: La durée d'essais traditionnel á l'oedomètre est une semaine étant la durée des étapes en general un jour. Appliquant des étapes plus courtes l'essais peux ètre realisé dans un jour. Quelques resultats préliminaire sont presenté.concernant deux cas: (i) la durée du consolidation primaire est moins que 2 h; (ii) la durée du consolidation primaire est moins que ~ 14 h, une étape derniere longue est appliquées parmi plusieurs étapes brèves et, des étapes brèves sont estimé avec la solution d'un inverse problème.

1 INTRODUCTION

The conventional multistage compression test can be used for the determination of the compression curve and the coefficient of consolidation. According to the standard ASTM D2435-90, the load increment is less than or equal to the previous load, the duration of the stages is longer than the 99 % consolidation time. Applying 24 h long stages - which is recommended by the standard - the test is "too" long.

Several faster procedures have been elaborated in the last few decades. These "continuous" tests need more complicated equipment and the evaluation is based on simpler models (e.g. the effect of creep is generally neglected).

Some preliminary results are presented in this paper concerning the idea that applying shorter stages can accelerate the conventional multistage compression test. The primary consolidation time is determined for some Hungarian plastic soils and, it is examined whether the final displacement can be prognosticated on the basis of the data measured during such stages that are shorter than the 99 % consolidation time.

2 METHODS

2.1 *Laboratory tests*

Seven soils from the Southern part of Hungary were tested in the Geotechnical Laboratory of the Budapest University of Technology and Economics. Undisturbed samples were taken below the groundwater level. Some soil physical properties are shown in Table 1. Rétháti & Ungár (1978) and Réthát (1988) present a statistical treatment of these freshwater soils.

Two to four oedometric tests were made from each soil sample. The load regime was as follows.

Table 1. Soils

Sample	City	Depth [m]	Soil type	I_p [%]	w_L [%]	w [%]	e
1	Algyő	10.5	silt	12.8	32.4	27.0	0.574
2	Szeged	6.5	lean clay	17.3	37.6	25.0	0.750
3	Algyő	6.0	lean clay	18.3	36.2	30.0	0.790
4	Szeged	5.0	lean clay	18.5	38.5	30.0	0.790
5	Algyő	12.0	medium clay	23.6	42.0	27.0	0.720
6	Szeged	4.5	medium clay	24.8	45.5	28.8	0.620
7	Szeged	12.0	fat clay	45.0	69.0	49.0	0.740

Table 2. Load regimes above the preconsolidation pressure p_c

Sample	1	2	3	4	5	6	7
Doubled, long	+	+	+	+	+	++	+
Doubled /2h				+		++	
Doubled /1h			+				
Doubled /0.5h			+				
100kPa/2h				+	+		+
100kPa/1h	+						
50kPa/1.5h		+					
50kPa/1h							+

Below the preconsolidation pressure (~75-100kPa) the load steps of 20kPa, 50kPa, 75kP were applied for 60min.

The stages from 100kPa entailed either equal load increment (50kPa to 100kPa) or doubled load increment according to Table 2. The last stage was "long" in the sense that its duration covered the primary consolidation time in every case. The 0.5h, 1h, 1.5h, 2h long stages are called short stages in the following.

2.2 *Modelling*

The modified Terzaghi's (A) and, the modified Bjerrum's (AC) models were used for the evaluation of the measured data. The displacement of the sample top for model A:

$$v(t) = v_0 + v_1(t) \tag{1}$$

and, model AC:

$$v(t) = v_0 + v_1(t) + v_2(t) \tag{2}$$

where v_0 is the immediate compression settlement, v_1 is the primary consolidation settlement:

$$v_1(c_v, t) = v_{1,\infty}\left[1 - \int_0^{2H} \frac{u(c_v, t, x)dx}{2H\sigma}\right] \tag{3}$$

$$v_{1,\infty} = 2H\sigma / E_s \tag{4}$$

$2H$ is the height of the sample, H is model constant, σ is the total stress increment, E_s is the oedometric modulus, u is the pore water pressure solution of the Terzaghi's model depending on the coefficient of consolidation c_v, t is time, x is the space variable, v_2 is the creep settlement:

Table 3. Identified parameters

	dependence	parameter
v_0	linear	immediate compression
$v_{1,\infty}$	linear	primary consolidation settlement
c_v	non-linear	coefficient of consolidation
C_α	linear	coefficient of creep

$$v_2(t) = C_\alpha \frac{2H}{1+e_0} \log \frac{t+t_0}{t_0} \tag{5}$$

where C_α is the coefficient of creep, t_0 is time parameter.

2.3 *Inverse problem solution*

2.3.1 *The definition of the inverse problem*

The inverse problem is an unconditional minimisation:

$$F(p) = \|h(p)\| = \min! \tag{6}$$

where the minimising parameter vector $p_{min} \in R^M$ of merit function F (the norm of the error vector $h(p) = v_m - v(p)$) is to be determined, $v_m \in R^N$ is the measured data vector containing data for v at elapsed times of $t^1 ... t^N$, $v(p): R^M \rightarrow R^N$ is the model response the elements which are generated in such a way that the sampling times $t^1 ... t^N$ are successively substituted into $v(t,p)$ which is the solution of the model for the sample top displacement. The applied merit function:

$$F(p) = \frac{\sqrt{\sum_i \left[(v_m(t_i) - v(t_i, p)\right]^2}}{v_{m,\max}\sqrt{N}} 100[\%] \tag{7}$$

where $v_{m,max}$ indicates the largest measured data.

2.3.2 *Principle of inverse problem solution*

The parameters can be categorised as "linearly" and "non--linearly" dependent as follows. Let us consider the following factorised form of the solution of the model $v(t,p)$:

$$v(t,p) = \langle p_1, f(t, p_2) \rangle \tag{8}$$

where the parameter vector is split into two parts:

$$p = [p_1, p_2], \quad p_1 \in R^J, p_2 \in R^{M-J}. \tag{9}$$

If in this form p_1 is maximal then the solution $v(t,p)$ depends linearly on J and non-linearly on M-J parameters.

Based on this definition the principle of the minimisation was as follows. The minimisation was made in the subspace of the non-linearly dependent parameters in such a way that every merit function evaluation was preceded by a subminimisation in the subspace of the linearly dependent parameters as follows.

Every non-linear parameter vector part (p_2) uniquely determines a corresponding linear parameter vector part (p_1) through the following conditional minimisation if condition (11) is met:

$$F(p_1, p_2) = \min!, \quad p_2 = p_2^* \tag{10}$$

where * means prescribed value.

The set of these pairs of p_2 and p_1 constitutes an M-J dimensional (so called minimal) section of the merit function. It follows from (10) that this section and the merit function have the same global minima. In the applied procedure the merit function is minimised along this section (Imre, 1996).

2.3.3 *Reliability of the inverse problem solution*

The solution of the inverse problem is considered as reliable if it is unique and the confidence interval of the parameters does not exceed the range of the parameters.

The solution is unique if (i) the global minimum of the merit function is not degenerated (ii) map $v(p)$ is locally one to one or:

$$\det[A^T A] \neq 0 \tag{11}$$

where A is the derivative of $v(p)$ $N>M$. In this preliminary work only the uniqueness was tested and, some earlier results are presented concerning the standard deviation of the parameters determined from long stages (according to the conventional procedure using the linearised model (Press et al 1986)).

2.3.4 *Applied procedure*

It was found that condition (11) is not met simultaneously for every parameter, not every parameter is identifiable. The merit function is practically not sensitive to the variation of parameter t_0. Therefore, not every parameter was identified (Table 3). From the identified ones the following parameters were computed:

$$E_s = \frac{2H\sigma}{v_{1,\infty}} \tag{12}$$

$$k = \frac{c_v \gamma}{E_s} \tag{13}$$

Out of the 3-4 identified parameters only one (the coefficient of consolidation c_v) was non-linearly dependent. The merit function was explored in 50 values of c_v. The corresponding linearly dependent parameters were determined by the use of the SVD algorithm (Press et al 1986) in the frame of the subminimisation. Then, a function value evaluation was made in each point.

The geometry of the merit function was tested in the case of both models by the representation of the minimal section of the merit function concerning parameter c_v.

It was found that the merit function related to model A has one minimum while the the merit function related to model AC has three minima (Fig. 1). Being more robust, the evaluation was made with model A.

It was found that the shape of merit function A is distorted if the degree of consolidation κ at the end of the stage is less than about 90%. As a result, the solution of the inverse problem is not reliable in this case (Fig. 2). Therefore, the following procedure was applied.

If κ was larger than 90 % (i.e. the solution of the inverse problem was reliable) the parameters were identified from the data of the short stage.

If κ was less than about 90 % (i.e. the solution of the inverse problem was not necessarily reliable) then only the linearly dependent parameters were identified from the short stages,

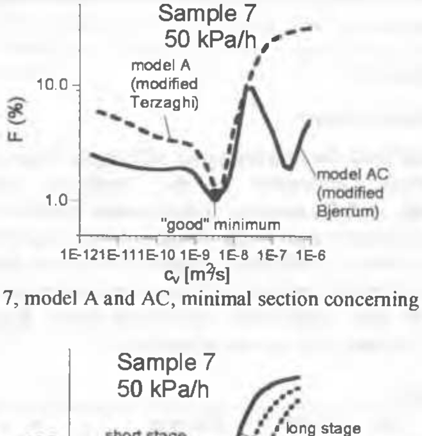

Figure 1. Soil 7, model A and AC, minimal section concerning c_v

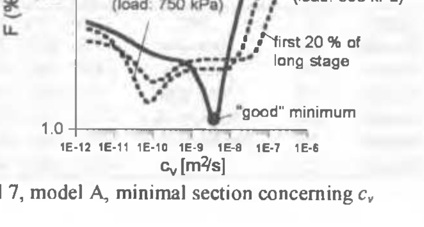

Figure 2. Soil 7, model A, minimal section concerning c_v

Table 4. Results of 7th sample *50kPa/1h*

Load [kPa]	c_v [m²/s]	E_s [MPa]	k [m/s]	v_0 [%]	F [%]	t_{max} [min]	κ at t= =60min
200	.4E-7	.547E+01	.73E-7	7	3.4	60	98
250	.2E-7	.824E+01	.24E-7	13	3.3	60	86
300	.2E-7	.100E+02	.20E-7	11	1.2	60	86
350	.2E-7	.970E+01	.20E-7	10	1.1	60	86
400	.2E-7	.877E+01	.22E-7	10	1.6	60	86
450	.2E-7	.937E+01	.21E-7	6	1.8	60	86
500	.2E-7	.719E+01	.28E-7	16	1.4	60	86
550	.2E-7	.105E+02	.19E-7	-3	3.13	60	86
600*	.1E-7	.854E+01	.12E-7	3	1.8	60	67
650	.8E-8	.726E+01	.11E-7	1	1.9	60	60
700*	.6E-8	.716E+01	.84E-8	4	2.3	60	52
750*	.6E-8	.711E+01	.84E-8	-1	2.8	60	52
800	.4E-8	.645E+01	.62E-8	1	1.1	820	42

* a priori c_v value was used for the inverse problem solution

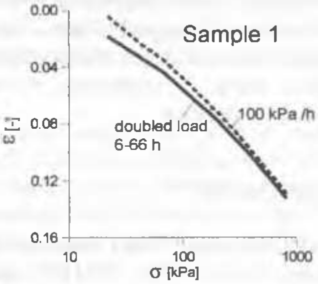

Figure 3. Compression curves for soil 1.

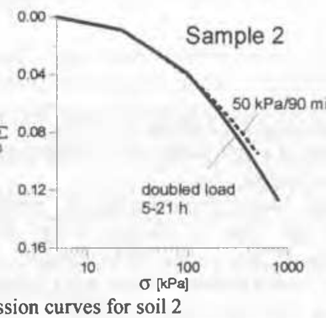

Figure 4. Compression curves for soil 2

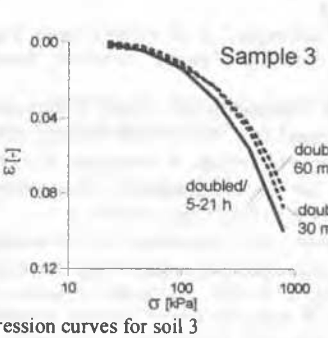

Figure 5. Compression curves for soil 3

value of c_v was estimated on the basis of the value determined from the last, long stage.

3 RESULTS

Measured data were evaluated and, on the basis of the identified parameters the displacement at 24 h elapsed time was computed. These values were plotted as compression curves.

The compression curves composed from 24 h elapsed time displacement values are shown in Figure 3-9. The parameters, the degrees of consolidation at the end of the stage are shown for the case of sample 7 in Table 4.

According to the results, κ was larger than about 94% for soils 1 to 6, at the end of every stage. For soil 7, κ was 40-99%.

The compression curves related to the short stages are situated "above" the ones related to the long stages in case of

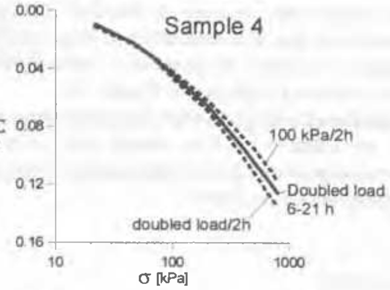

Figure 6. Compression curves for soil 4.

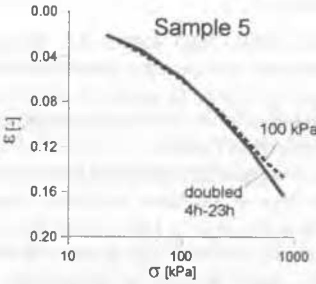

Figure 7. Compression curves for soil 5.

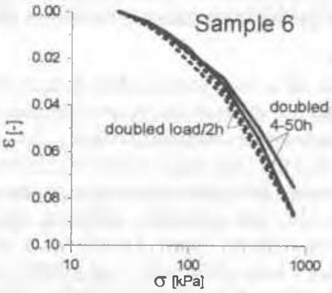

Figure 8. Compression curves for soil 6.

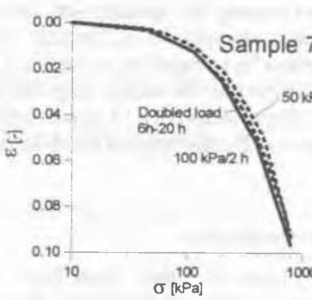

Figure 9. Compression curves for soil 7.

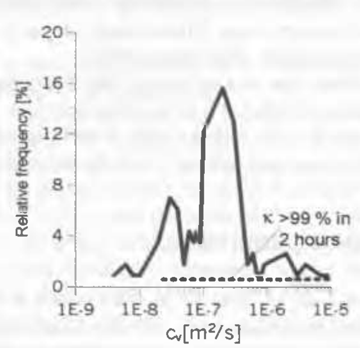

Figure 10. The coefficients of consolidations c_v identified

Table 5. Average relative standard deviations $D(p^i)/p^i$ [%] for long stages (Imre, 1995)

model	A	AC
p^i	$D(p^i)/p^i$ [%]	$D(p^i)/p^i$ [%]
v_0	11.73	7.29
v_1	6.06	3.78
c_v	26.70	15.71
C_a	-	4.15

equal load increments. In case of doubled load increments the opposite tendency was also encountered (e.g. samples 4, 6).

The relative frequency diagram of c_v identified with model A for the whole data set is shown in Figure 10.

First experiences indicate that the parameters identified with model AC are reliable for long stages only. It is very probable that the parameter error for the short stages is considerably larger than the values shown in Table 5.

4 DISCUSSION

4.1 *Inverse problem*

Model A is more robust than model AC since only the most important phenomenon (the primary consolidation) is modelled.

The merit function related to model A may be distorted as the degree of consolidation at the end of the stage is less than about 90 %, as it can be seen in Figure 2.

Further study is suggested about this phenomenon and, on the reliability of the inverse problem solution related to the short stages. In particular the case is interesting where the degrees of consolidation at the end of the stage is about 40-60 %.

The reliability tests are to be completed with the error estimation part. The parameter error can probably be explained by the concept of generalised standard deviation (Imre, 1996).

4.2 *Time effects*

Model A is based on a time independent constitutive law. Being the constitutive law of the soils in the reality time dependent, the 24 h compression curves computed from the data measured during the short and the long stages are not identical.

According to some literature data, the time dependency of the constitutive law of the soils is different for "low rates" (sedimentary compression curve, Leonards et al (1964)), for "normal" laboratory rates (Leroueil et al (1985)) and, for "large rates" (see e.g. Kondner & Stallkencht (1961), Whitman (1957)).

The differences among the compression curves encountered in this research are generally in accordance with the general relationship presented by Leroueil et al (1985). This relationship entails more rigid behaviour for shorter stage duration.

However, the slight sensitivity of some soils in case when doubled load steps are shortly applied need further experimental work.

4.3 *Degrees of consolidation*

According to Figure 10, the identified coefficient of consolidation c_v is larger than 2E-8 m^2/s for about the 95% of the cases and, the identified coefficient of consolidation c_v is larger than 3E-9m^2/s in every case. These results seem to be typical for these kind of freshwater soils (Imre,1995).

It follows from the model law of the Terzaghi's model that κ=99% is achieved within 2 h in the first case and, within 14 h in the second case. It also follows that κ=60% is achieved within 0.4 h in the first case and, within 2 h in the second case.

5 SUMMARY, CONCLUSION

Seven soils wit I_P of 12 % to 45 % were tested in this work with the conventional procedure and, with the following one day long procedure: besides several 0.5 - 2 h long stages a last ~ 14 h long stage was applied.

According to the results, κ>99 % was attained during the short stages for soils 1 to 6 (I_p=12-25%), κ>99 % was attained during the last long stage only for soil 7 (I_p=45%).

The data measured during each stage was evaluated with inverse problem solution using the modified Terzaghi's model (model A). The displacements at 24 h elapsed time were computed and plotted. The 24 h compression curves were negligibly influenced by the load regime.

Table 6. Elapsed time in the case of different degrees of consolidation

c_v [m^2/s]	κ=99% T=2	κ=90% T=0.87	κ=80% T=0.57	κ=60% T=0.28
1E-9	2.3days	24 h	15.8h	7.8h
1E-8	5.6h	2.4h	1.6h	0.8h
1E-7	33min	15min	10min	5min
1E-6	3.3min	1.5min	1min	0.5min

In case of soil 7 the inverse problem related to the short stages was generally not solvable without an a priori estimation for the coefficient of consolidation c_v which was implemented on the basis of the value identified for the last long stage.

Results can be summarised as follows: the multistage oedometric test can be performed within a day for soils (i) with ~ c_v>2E-8 m^2/s where κ>99 % can be attained within 1-2 h, (ii) with ~2E-8 m^2/s >c_v> 2E-9 m^2/s where κ>99 % is attained within about 14 h and, inverse problem is to be solved.

Additional research is suggested about the inverse problem solution related to the short stages. Further laboratory tests could clarify the time effect in connection to the doubled load imposition with short duration.

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