General relaxation equations for soils

Equations générales pour la relaxation des sols

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ABSTRACT: The principle of natural proportionality that has already provided many general equations to describe the mechanical behaviour of geomaterials is applied to obtain general equations to describe the relaxation behaviour in the pre and postpeak regions of soils in general and to a frozen sand in particular. This particular frozen sand had an "inverted" stress-strain behaviour in the prepeak region. A general stress-strain equation for it and general equations to describe the volume change behaviour in the pre and postpeak regions are included. The strength and time of failure relationship is also included.

RÉSUMÉ: Le principe de la proportionnalité naturelle qui a déjá fourni de nombreuses équations générales permettant de décrire le comportement mécanique des géomatériaux est ultilisé pour décrire la relaxation dan les domaines antérieur et postérieur au pic de résistance des sols en général et d'un sable congelé en particulier. Ce sable congelé présente un compoirtament "inverse" avant le pic. Des équations générales décrivant les relations constraintes-déformations et les variations de volume sont présentées. Une relation entre la résitance et le temps de rupture est également incluse.

1. INTRODUCTION

The principle of natural proportionality postulated in 1985 (Juárez-Badillo 1985) is a unifying principle from which many general equations, used to describe the mechanical behaviour of geomaterials, have emerged. This time the stress-strain, volume change and relaxation behaviour in the pre and postpeak regions of a frozen sand are analyzed and described with already known as well as new general equations provided by the principle of natural proportionality. A special characteristic of this frozen sand is that it showed an "inverted" stress-strain behaviour in the prepeak region. This experimental data has already been published by (Ladanyi and Benyamina, 1995).

2. STRESS-STRAIN EQUATIONS IN THE PRE AND POSPEAK REGIONS

Figure 1 shows the experimental points and the theoretical curves of three short-term triaxial compression tests with frozen Ottawa sand carried out at –5°C and a strain rate of 0.016 min^{-1}. Figure 1(a) shows the results of the three tests. Figure 1(b) shows the result of only one test; the results of the other two tests have been deleted to gain clarity since they were very similar to the one shown (σ_3 = 100 kPa). From figure 1(a) it is obvious that we have two mechanical phases; the prepeak and the postpeak regions where the peak strength appears to occur for a failure deviatoric natural strain e_{af} = -3.0 %. (The common axial strain ε_a will be considered as being very close to the axial deviatoric natural strain e_a). For the prepeak region if we take $(\sigma_1 - \sigma_3)$ and e_a as the proper variables, the corresponding proper funtions are $(\sigma_1 - \sigma_3)$ and $1/e_a - 1/e_{af}$ and the equation given by the principle of natural proportionality is (Juárez-Badillo 1985, 1992):

$$\left(\frac{1}{e_a} - \frac{1}{e_{af}}\right)(\sigma_1 - \sigma_3)^{\nu} = \text{constant} \qquad (1)$$

If we use the characteristic stress $(\sigma_1 - \sigma_3)^* = 3.0$ MPa for the strain $e_a = \frac{1}{2} e_{af}$ we may write equation (1) as:

$$e_a = \frac{e_{af}}{1 + \left[\dfrac{\sigma_1 - \sigma_3}{(\sigma_1 - \sigma_3)^*}\right]^{-\nu}} \qquad (2)$$

where ν is the shear exponent (Juárez-Badillo 1994 b, 1995 a).

Figure 1(a) shows equation (2) that appears to describe the prepeak behaviour for the three confining pressures of 100, 200 and 300 kPa with the values noted above and with $\nu = 1$.

A common value of the shear exponent for drained tests on clays, sands and concrete is also ν = 1. For undrained tests on clays a common value is ν = 2 (Juárez-Badillo 1994 b, 1995 a, 1996 a, b).

For the postpeak region the equations are simpler. Now the proper functions are the proper variables and the equations are of the form (Juárez-Badillo 1996 b).

$$\frac{e_a}{e_{a1}} = \left[\frac{\sigma - \sigma_3}{(\sigma_1 - \sigma_3)_1}\right]^{-\nu} \qquad (3)$$

where $[(\sigma_1 - \sigma_3)_1, e_{a1}]$ is a known point and again ν = 1. The

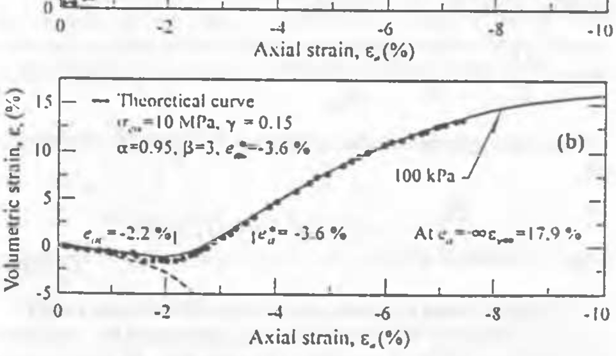

Fig 1 Results of three short-term triaxial compression tests with frozen Ottawa sand at -5° C and 0.016 min^{-1}. (a) Stress-strain curves. (b) Volume strain curves.

corresponding equations for $\sigma_3 = 100$, 200 and 300 kPa are with the known points $[(\sigma_1-\sigma_3)_1, e_{a1}] = (5, -4.0)$, $(5, -5.0)$, and $(5, -5.6)$ respectively .

3. VOLUME CHANGE EQUATIONS FOR THE PRE AND POSTPEAK REGIONS

A general volume change equation in the prepeak region for clays has already been postulated (Juárez-Badillo 1969, 1975), (Juárez-Badillo and Rico-Rodríguez 1975). A similar equation has already been postulated for sands, concrete and rocks (Juárez-Badillo 1996 a, b, 1999, 2000). It is also useful for our case. It reads:

$$\frac{V}{V_0} = \left\{ \frac{\sigma_{eo} + \Delta\sigma_i - \alpha(\sigma_{eo} - \sigma_{co})y}{\sigma_{eo}} \right\}^{-\gamma} \tag{4}$$

where V = volume, V_0 = initial volume, σ_{eo} = initial equivalent consolidation pressure, σ_{co} = initial consolidation pressure, $\Delta\sigma_i$ = isotropic pressure increment, γ = compressibility coefficient and y = sensitivity function given the equation

$$y = \frac{1}{1 + \left(\frac{e_a}{e_a^*} \right)^{-\beta}} \tag{5}$$

where e_a^* = characteristic e_a such that $y = ½$ for $e_a = e_a^*$ and β = a pore pressure coefficient.

Equation (4) has as assumption that σ_{eo} is much greater than σ_{co} and with y given by equation (5) includes the postpeak region, that is, when e_a varies from 0 to ∞.

Equation (4) may be written as a function of strains only. From equation (2) we get:

$$\sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)^* \left(\frac{e_a}{e_{af} - e_a} \right)^{1/\nu} \tag{6}$$

Introducing equation (6) into equation (4) we get ($\nu = 1$)

$$\frac{V}{V_0} = \left\{ 1 + \frac{1}{3} \frac{(\sigma_1 - \sigma_3)^*}{\sigma_{eo}} \frac{e_a}{e_{af} - e_a} - \alpha \frac{\sigma_{eo} - \sigma_{co}}{\sigma_{eo}} y \right\}^{-\gamma} \tag{7}$$

However, due to the "inverted" characteristic of the prepeak region, equation (7) is to be used only in the prepeak region for e_a from 0 to a critical strain e_{ac} corresponding to the maximum experimental value of $\sigma_1 - \sigma_3 = 8$ MPa. From equation (2) we obtain $e_{ac} = -2.2\%$.

For shear strains grater than $e_{ac} = -2.2\%$, equation (7) is to be used with this value of $e_a = e_{ac}$ in its first term. At the start of the test, say, from $\sigma_1 - \sigma_3 = 0$ to 3 MPa the change in volume is mainly due to $\Delta\sigma_1$. This fact permits calculate γ given σ_{e0} in equation (4). In this zone we have:

$$d\varepsilon_v = \frac{dV}{V} = -\frac{1}{3}\gamma \frac{d(\sigma_1 - \sigma_3)}{\sigma_{eo}} \tag{8}$$

From the experimental data in figure 1 it was found, therefore, that

$$\frac{\gamma}{\sigma_{eo}} = -3 \frac{d\varepsilon_v}{d(\sigma_1 - \sigma_3)} = 3x0.005 = 0.015 MPa^{-1} \tag{9}$$

The author found convenient to use $\sigma_{eo} = 10$ MPa and $\gamma = 0.15$ with $\sigma_{co} = 100$ kPa = 0.1 MPa. With these values and $(\sigma_1 - \sigma_3)^* = 3$MPa and $e_{af} = -0.03$ equation (7) with only the first term becomes

$$\varepsilon_v = \frac{\Delta V}{V_0} = \left\{ 1 + \frac{1}{10} \frac{e_a}{-0.03 - e_a} \right\}^{-0.15} - 1 \tag{10}$$

Figure 1(b) shows equation (10) in a discontinuous curve. We need now to determine the parameters α, β and e_a^*. They can be determined from de postpeak behaviour. For this region the final equation introducing equation (10) into equation (7) with the limiting value of $e_a = e_{ac} = -2.2\%$ we obtain ($\sigma_{e0} = 10$ MPa and $\sigma_{c0} = 0.1$MPa)

$$\varepsilon_v = \frac{\Delta V}{V_0} = \left\{ 1.275 - \alpha x 0.99 \frac{1}{1 + \left(\frac{e_a}{e_a^*} \right)^{-\beta}} \right\}^{-0.15} - 1 \tag{11}$$

From the final part of the experimental curve e_a is large, y is close to unity and the value of α was found equal to $\alpha = 0.95$ (It should always be less than 1). Using the experimental curve for $y = ½$ the value of $e_a^* = -3.6\%$ was determined and using the complete experimental curve was found the value $\beta = 3$.

For the prepeak and postpeak regions the final equation is:

$$\varepsilon_v = \frac{\Delta V}{V_0} = \left\{ 1 + \frac{1}{10} \frac{e_a}{-0.03 - e_a} - \alpha x 0.99 \frac{1}{1 + \left(\frac{e_a}{e_a^*} \right)^{-\beta}} \right\}^{-0.15} - \tag{12}$$

Figure 1 (b) shows in a continuous curve the theoretical curve given by equations (11) and (12) with the values $\alpha = 0.95$, $\beta = 3$ and $e_a^* = -3.6\%$.

It should be observed, however, that the values of the parameters α and γ are highly dependent on the value of σ_{eo}. The value of γ is directly proportional to σ_{e0} (equation 9) and the value of α is approximately inversely proportional to σ_{e0}. The correct determination of σ_{e0} needs tests at high values of σ_{c0}.

4. RELAXATION EQUATIONS FOR THE PRE AND POSTPEAK REGIONS

The principle of natural proportionality has already provided general equations for creep (Juárez-Badillo 1994 a). General equations for relaxation may be obtained proceeding in a very similar way: to attain a given shear strain at $t = 0$ it is needed a value of $\sigma_1 - \sigma_3 = \infty$. As time elapses the value of $\sigma_1 - \sigma_3$ relaxes such that at $t = \infty$ the value of $\sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)_\infty$. Assuming that these concepts are proper variables their proper functions are t and $(\sigma_1 - \sigma_3) - (\sigma_1 - \sigma_3)_\infty$. The relation between them according to the principle of natural proportionality should be:

$$\left[(\sigma_1 - \sigma_3) - (\sigma_1 - \sigma_3)_\infty \right] t^{\xi} = \text{constant} \tag{13}$$

where ξ = shear fluidity whose value should be the same than for creep tests according to the same principle .

If $(\sigma_1 - \sigma_3)_\infty = 0$ as it appears to be for ice and frozen soils (Juárez-Badillo 1993 a, b, 1995 b) then equation (13) may be writen as:

$$\sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)_1 \left(\frac{t}{t_1} \right)^{-\xi} \tag{14}$$

where $[t_1, (\sigma_1 - \sigma_3)_1]$ is a known point.

The correct application of equations (13) and (14) as well as the correct application of the general creep equations needs, however, a correct time scale . An error that we all still make is to divide strains in instantaneous or initial and differed (creep) as well as with loads: instantaneous or initial and relaxed. Nothing can happen in time zero, all strains and all loads are time consumming. This needs to be taken into account in all creep tests as well as in all relaxation tests. This fact has caused some misunderstanding in the past (Juárez-Badillo 1993 a, b).

Figure 2 shows the stress-strain curves with seven relaxation stages and figures 3 and 4 show the prepeak and the postpeak relaxation curves in log-log plot. From figure 3 it appears, that the authors of the experimental curves measured the times from the moment they arrived at the specified strain with the noted strain rate of 0.016 min^{-1}. It is necessary, therefore, to take a part (that resulted of the order of 75%) of the time needed to arrive to the specified strain to define the "theoretical origin" of the time scale. Let t_0 be these quantities. The relaxation curves, therefore, will take the form given by the equation:

$$\sigma_1 - \sigma_3 = (\sigma_1 - \sigma_3)_1 \left(\frac{t + t_0}{t_1 + t_0} \right)^{-\xi} \tag{15}$$

Taking t_1 = 10,000 sec the author found values of ξ = 0.16 and 0.18 (with one value of 0.11). The values of the different parameters are: ξ = 0.11, 0.16, 0.18, 0.16, 0.16, 0.16; $(\sigma_1 - \sigma_3)_1$ = 0.31, 0.72, 1.35, 2.00, 2.50, 2.80; t_0 = 20, 30, 50, 50, 80, 90 for the axial strains –0.5, -1, -1.5, -2, -2.5 and –3% respectively.

Figure 4 shows the postpeak relaxation experimental points and the theoretical curves given by equation (15) with ξ = 0.18, t_0 = 0 and the values of $(\sigma_1 - \sigma_3)_1$ at t = 10,000 sec given by the following equations for the axial strains ε_a = -4 and –10% respectively:

$$\sigma_1 - \sigma_3 = 2.20 \left(\frac{t}{10{,}000} \right)^{-0.18} \tag{16}$$

$$\sigma_1 = \sigma_3 = 0.95 \left(\frac{t}{10{,}000} \right)^{-0.18} \tag{17}$$

Note that for the postpeak region t_0 = 0. It remains to find out why here it is so.

For creep tests, the complementary equation to equation (14) is (Juárez-Badillo 1994 a)

$$e_a = e_a^* \left(\frac{t_f}{t} - 1 \right)^{-\xi} \tag{18}$$

where t_f = time of physical failure ($e_a = \infty$) and e_a^* = characteristic e_a for t = ½ t_f.

Figure 2 also shows the same theoretical curve given by equation (2) for the prepeak region as well as as the final postpeak theoretical curve given by the equation:

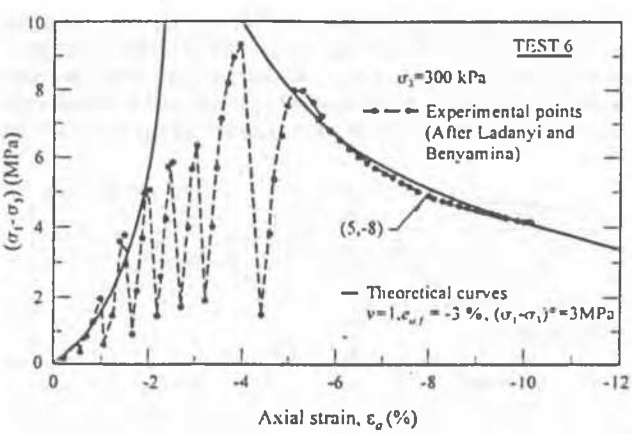

Fig 2 Test 6: frozen ottawa sand at -5° C. Stress-strain curve with 7 relaxation stages

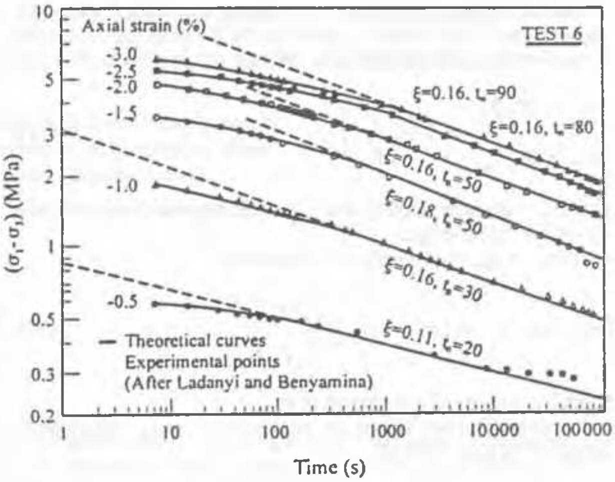

Fig 3 Test 6: prepeak relaxation curves in log-log plot

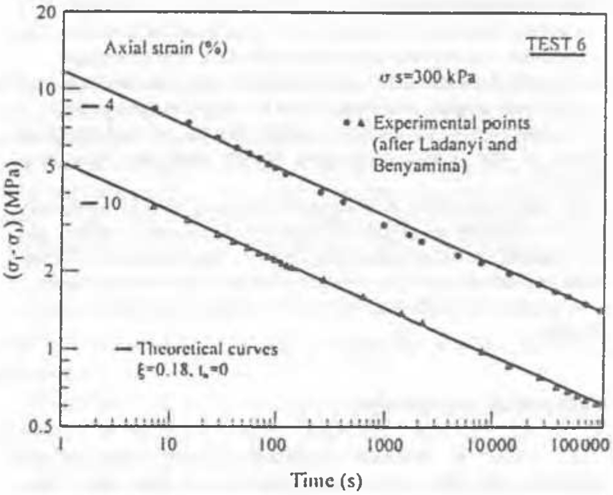

Fig 4 Test 6: post-peak relaxation curves in log-log plot

$$e_a(\%) = -8.0 \left(\frac{\sigma_1 - \sigma_3}{5} \right)^{-1.0} \tag{19}$$

Note that the strain -8.0% for the stress 5 Mpa is now higher than –5.6% in figure 1 due to the path and history dependence. Note also the influence of the early relaxation stages on the final ones in the prepeak region.

5. STRENGTH-TIME EQUATION

All geomaterials when loaded present two zones with respect to strength. If the load is sufficiently small the strains will continue as time elapses (creep) asymptotically to a final value at $t = \infty$. The general equation for creep is (Juárez-Badillo 1994 a):

$$e_a = \frac{e_{af}}{1 + \left(\frac{t}{t^*} \right)^{-\xi}} \tag{20}$$

where e_{af} = final value of e_a at $t = \infty$ and t^* = characteristic time for e_a = ½ e_{af}.

If the load is sufficiently large the strains are described by equation (18) until failure at time t_f. The time of failure t_f decreases as the load increases. There is a threshold of the load, $(\sigma_1 - \sigma_3)_{f\infty}$ such that the failure occurs at $t = \infty$. If $(\sigma_1 - \sigma_3) < (\sigma_1 - \sigma_3)_{f\infty}$ we are in the stable zone. If $(\sigma_1 - \sigma_3) > (\sigma_1 - \sigma_3)_{f\infty}$, we are in the unstable zone. The relationship between a failure load $(\sigma_1 - \sigma_3)_f$ and time of the failure t_f provided by the principle of natural proportionality is (Juárez-Badillo 1994 a):

$$\frac{(\sigma_1 - \sigma_3)_f}{(\sigma_1 - \sigma_3)_{f\infty}} = 1 + \left(\frac{t}{t^*}\right)^{-\varsigma} \tag{21}$$

where ζ = strength fluidity and t^* = characteristic time for which $(\sigma_1 - \sigma_3)_f = 2\,(\sigma_1 - \sigma_3)_{f\infty}$.

If $(\sigma_1 - \sigma_3)_{f\infty} = 0$, equation (21) becomes:

$$(\sigma_1 - \sigma_3)_f = (\sigma_{-} - \sigma_3)_{f1}\left(\frac{t}{t_1}\right)^{-\varsigma} \tag{22}$$

where $[t_1, (\sigma_1 - \sigma_3)_{f1}]$ is a known point.

It appears that for ice and frozen soils $(\sigma_1 - \sigma_3)_{f\infty} = 0$ (Juárez-Badillo 1993 a, b, 1995 b).

6. CONCLUSIONS

The main conclusions are:

1.- General equations to describe the relaxation behaviour in the pre and postpeak regions of geomaterials have been presented.

2.- A general equation for the "inverted" stress-strain behaviour in the prepeak region of a frozen sand has also been presented.

3.- A general volume change equation for the pre and postpeak regions of the above mentioned frozen sand has also been presented.

4.- An important point in creep and relaxation tests is to properly define the theoretical origin of the time scale since all strains and load changes are time consuming, that is, instantaneous or initial strains or loads do not exist. Nothing happens in time equal zero.

5.- A general strength-time of failure equation for frozen soils is included.

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8. REFERENCES

Juárez-Badillo, E. 1969. Pore pressure and compressibility theory for saturated clays. *Specialty session No. 12 on Advances in Consolidation Theories for Clays*. University of Waterloo, Canada: 99-116.

Juárez-Badillo, E. 1975. Constitutive relationships for soils. *Symposium on recent developments in the analysis of soil behaviour and their application to geotechnical structures*. University of New South Wales, Kensington, Australia: 231-257.

Juárez-Badillo, E. 1985. General volumetric constitutive equation for geomaterials. Special Volume on Constitutive Laws of Soils. Eleventh International Conference on Soils Mechanics and Foundation Engineering. Japanese Society of Soils Mechanics and Foundation Engineering, Tokyo: 131-135.

Juárez-Badillo, E. 1992. Natural laws in geomechanics. *Raúl J. Marsal Volume*. Sociedad Mexicana de Mecánica de Suelos, México, D.F.: 31-36.

Juárez-Badillo, E. 1993 a. Creep behaviour of fine-grained frozen soils: Discussion. *Canadian Geotechnical Journal*, 30: 385-387.

Juárez-Badillo, E. 1993 b.:Temperature-independent relationships for frozen soils: Discussion. *ASCE Journal of Cold Regions Engineering*, 7 (3): 92-95.

Juárez-Badillo, E.. 1994 a. General stress-train-time equation for soils. *13th. ICSMFE, New Dehli, India*, Vol. 1: 337-348.

Juárez-Badillo, E. 1994 b. Deviatoric stress-strain equation for soils. *13th. ICSMFE, New Dehli, India*, Vol. 1: 349-356.

Juárez-Badillo, E. 1995 a. General deviatoric stress-strain theory for soils. *X Panamerican Conference on Soil Mechanics and Foundation Engineering*, Guadalajara, Jal., México, Vol. 1: 204-214.

Juárez-Badillo, E. 1995 b. Shear and strength fluidities of ice. *X Panamerican Conference on Soil Mechanics and Foundation Engineering. Guadalajara*, Jal., México, Vol. 1: 215-222.

Juárez-Badillo, E. 1996 a. General equations to describe the mechanical behaviour of concret. Pending of publication.

Juárez-Badillo, E. 1996 b. General equations to describe the mechanical behaviour of granular soils. Pending of publication.

Juárez-Badillo, E. 1999. Static liquefaction of very loose sands: Discussion. *Canadian Geotechnical Journal*, Vol. 36: 967-973.

Juárez-Badillo, E. 2000. General equations to describe the mechanical behaviour of rocks. *Geotech-Year 2000, Developments in Geotechnical Engineering*, Bangkok,Thailand: 39-52.

Juárez-Badillo, E. and Rico-Rodriguez, A. 1975. Mecánica de Suelos, Tomo 1, *Fundamentos de Mecánica de Suelos*, Ed. Limusa, México.

Ladanyi B and Benyamina M. B., 1995, Triaxial relaxation testing of a frozen sand. *Canadian Geotechnical journal*, 32: 496-511.