Modelling one-dimensional compression of a fibrous peat

Modélisation le compactage unidimensionnel d'une tourbe fibreuse

C.D.O'Loughlin & B.M.Lehane – *Dept. of Civil Engineering, Trinity College, Dublin 2, Ireland*

ABSTRACT: Amongst the most widely used state-of-the art constitutive models for strain-rate dependant soils in one-dimensional compression are the *EVP* model proposed by Yin & Graham (1994, 1996) and the *abc* model developed by den Haan (1996). Although these models were developed independently, both assume an elastic visco-plastic type soil response. In the first section of this paper the fundamental differences between the models are clarified, whereas in the second section both models are applied to a poorly humified, fibrous peat from the Irish midlands. The relative merits of both approaches for modelling this peat are discussed.

RÉSUMÉ: Parmi le plus largement répandu des modèles constitutifs pour les sols dépendants de vitesse de deformation pour le compactage unidimensionnel sont les modèles *EVP* proposés par Yin et Graham (1989, 1996) et le modèle *abc* développé par den Haan (1996). Ces deux modèles ont été développés indépendamment, bien que tous les deux assument une réponse du sol de type élastique visco-plastique. Dans la première partie de cet article les différences fondamentales entre les modèles sont clarifiées, tandis que dans la deuxième partie les deux modèles sont appliqués à une tourbe fibreuse à partir des Midlands irlandais. Les mérites relatifs des deux modèles sont discutés par rapport à de tels matériaux complexes.

1 INTRODUCTION

Fibrous peat is encountered in the majority of raised bogs in central Ireland, and is an extremely compressible material with an extraordinarily high in-situ water content and initial voids ratio. Consolidation in peat occurs relatively quickly and settlement is dominated by the so-called *secondary* or *creep* compression. Conventional practice relies on methods similar to that suggested by Mesri & Choi (1985), whereby creep strains are measured relative to the end of primary (EOP) stress-strain curve. Such methods imply that EOP strains are unique irrespective of the drainage conditions, which is contrary to laboratory data reported by workers such as Edil et al. (1992), Berre & Iversen (1972) and Leroueil et al. (1986). Consequently, the focus over the past few decades has been to develop constitutive models that appropriately define the relationship between stress, strain and strain rate. The aim of this paper is to test the usefulness of the *abc* model of den Haan (1996) and the *EVP* model of Yin & Graham (1994, 1996) in describing the one-dimensional compression of an Irish fibrous peat under a variety of loading conditions.

ELASTIC VISCO-PLASTIC MODELS

Elastic visco-plastic models generally assume that, in the normally consolidated region, the creep strain rate for a given stress and strain state is unique and that, at a given creep strain rate, there is a linear relationship between strain (ε) and the logarithm of effective stress (σ') and a linear relationship between strain and the logarithm of creep strain rate ($d\varepsilon/dt$). The stress-strain-strain rate relationships assumed by the *abc* and *EVP* models are summarised in Figure 1 and particular characteristics of each of the models are now described briefly.

EVP model

The *EVP* model chooses to scale creep strain rates by introducing a term referred to as equivalent time, t_e. The relationship between strain, effective stress and creep strain rate for the *EVP* model is given by:

$$\varepsilon = \lambda/v \ln(\sigma'/\sigma'_o) + \psi/v \ln((t_o + t_e)/t_o) \tag{1}$$

where ε is the linear (or Cauchy) strain, σ'_o is the effective stress at zero strain on the *EVP* model's Reference Time Line (RTL), λ/v is the slope of lines of constant creep strain rate, ψ is the coefficient of creep compression, v is the specific volume and t_o is a constant used to define the creep strain rate on the RTL. Elastic strain rate (denoted by subscript *e*) is formulated as:

$$\dot{\varepsilon}_e = \kappa/v \frac{d \ln \sigma'}{dt} = \frac{\kappa/v}{\sigma'} \frac{d\sigma'}{dt} \tag{2}$$

The total rate of compression is taken as the sum of elastic strain rate and creep strain rate, which when equated to the rate of volume change, yields the modified consolidation equation expressed in terms of excess pore water pressure, ***u***:

$$\frac{k}{\gamma_w} \frac{\partial^2 u}{\partial z^2} = \frac{\kappa/v}{\sigma'} \frac{\partial u}{\partial t} - \frac{\psi/v}{t_o} \exp\left\{-\varepsilon \frac{v}{\psi}\right\} \left(\frac{\sigma'}{\sigma'_o}\right)^{\lambda/\psi} \tag{3}$$

Equation 3 can be used together with Equation 1 to solve problems for single-stage or multi-stage loading conditions. A more suitable formulation for constant rate of strain (CRS) conditions may be written as:

$$\dot{\varepsilon} = \frac{\kappa/v}{\sigma'} \left(\frac{\partial \sigma'}{\partial t}\right) + \left(\frac{\psi/v}{t_o + t_e}\right) \tag{4}$$

where t_e may be determined from Equation 1.

Yin & Graham (1994) propose determination of the elastic parameter, κ/v, from ε - $\ln\sigma'$ data in the overconsolidated region, and the remaining *EVP* parameters from two load increments in the normally consolidated region. Firstly, the RTL is positioned so that it passes through a hypothetical stress-strain state, where the stress level is the total stress upon application of the first load increment, and the strain level is increased by the corresponding elastic component of strain. As t_e is taken as zero on the RTL, t_e values are equal to actual loading durations for this first load increment. Secondary compression $\varepsilon - \ln t$ data for the first load increment can then be fitted to the second term on the RHS of Equation 1 to yield the parameters ψ/v and t_o. The magnitude of elastic strain due to the second load increment can be calculated either from Equation 1, or by moving along the instant line (Figure 1) from the previously known ε - σ' state. This allows the

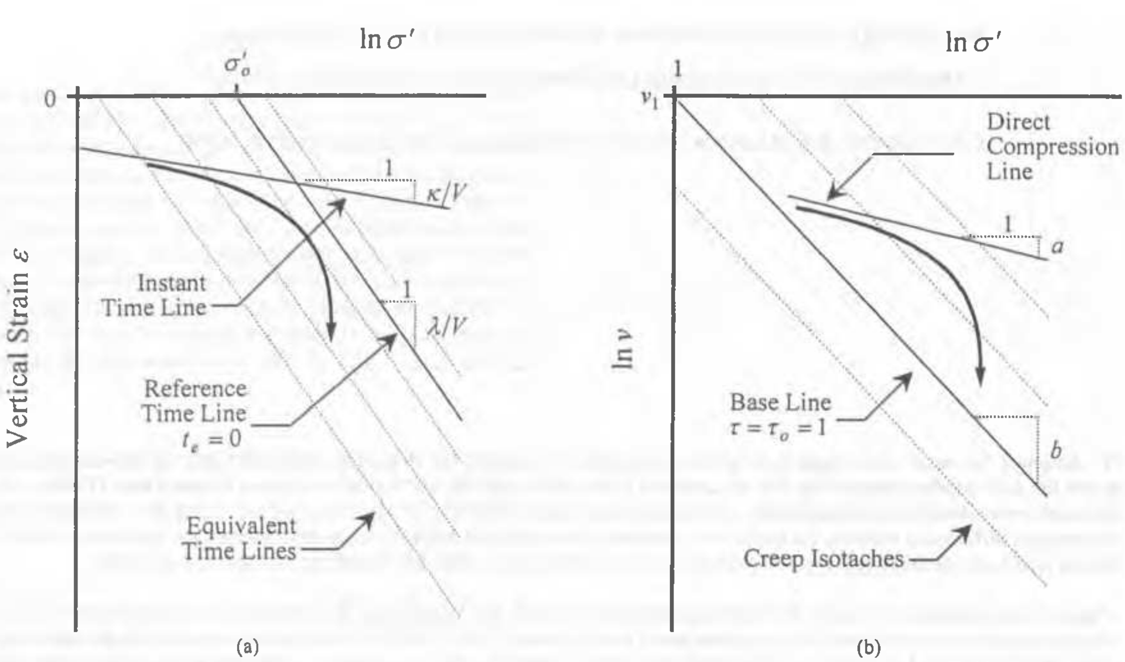

Figure 1 Stress - strain – creep strain rate relationship for (a) *EVP* model, (b) *abc* model

equivalent time for this stress-strain state to be calculated. As before, further creep straining from this point can be calculated from Equation 1, by considering that t_e at the end of loading is equal to t_e at the end of elastic straining plus the loading duration Δt. Enough information is now available to calculate the parameters λ/v and σ'_o uniquely using the end states of the two load increments.

abc model

In contrast to the *EVP* model, the *abc* model uses natural rather than linear strain as a measure of the soil compression, and adopts v_o as the specific volume at zero strain. Creep strain rates are linked to a term referred to as intrinsic time (τ) and defined as $c/\dot{\varepsilon}$, which relates $\dot{\varepsilon}$ uniquely to one value of intrinsic time. $\tau = \tau_o = 1$ day locates the base or reference line on which the strain rate is c/τ_o and v_1 is the specific volume at unit effective stress. The equivalent form of Equation 1 for the *abc* model is:

$$-\ln(v/v_1) = b\ln(\sigma') + c\ln(\tau/\tau_o) \tag{5}$$

The *abc* model introduces direct strain rate, which is defined as in Equation 2, but with the parameter *a* instead of κ/v; the model also uses the term direct as these strains are not assumed to be completely recoverable. The equivalent forms of Equations 3 and 4 for the *abc* model are:

$$\frac{d\sigma'}{dt} = \frac{\sigma'}{a}\left[-\frac{\gamma_s - \gamma_w}{\gamma_w}\frac{\partial}{\partial z}\left(\frac{k}{v}\right) + \frac{1}{\gamma_w}\frac{v_o}{v}\frac{\partial}{\partial z}\left\{k\frac{v_o}{v}\frac{\partial\sigma'}{\partial z}\right\} - \frac{c}{\tau}\right] \tag{6}$$

$$\dot{\varepsilon} = \frac{a}{\sigma_z}\frac{\partial\sigma'}{\partial t} + \frac{c}{\tau} \tag{7}$$

where γ_s and γ_w are the unit weights of soil and water respectively and the permeability, *k*, is allowed to vary with specific volume according to:

$$C_k \ln(k/k_o) = \ln(v/v_o) \tag{8}$$

Den Haan (1996) recommends that Equation 5 be fitted to the creep tails of $\varepsilon - \ln t$ data for all load increments in the normally consolidated stress range to give the creep parameter *c* and a constant equal to ($b\ln\sigma' - \ln v_1$). An average value of *c* is taken from these creep tails, and used to define the creep strain rate on the base line (c/τ_o). The interpolated strain at this creep strain rate is then determined for each normally consolidated stress increment and Equation 5 fitted to the interpolated values varying *b* and v_1 until an optimal fit is obtained. The parameter *a* may be estimated in much the same way as κ/v was for the *EVP* model. Permeability (*k*) is estimated for each load increment using Taylor's root time, and the empirical parameters C_k and k_o are derived from Equation 8.

Differences between the abc and EVP formulations

Despite differences in terminology, it is evident that the *abc* and *EVP* approaches are essentially very similar. There are, however, some notable differences in the detail published to date, which are summarised in Table 1.

Table 1 Differences in the *abc* and *EVP* models

***abc* model**	***EVP* model**
Natural strain	Linear Strain
Strain origin linked to soil's initial specific volume/stress history	Arbitrary strain origin
k dependant upon *v*	*k* assumed constant
Parameters derived from all data in normally consolidated region	Two load increments used as a basis for parameter selection
Finite strain formulation	Thickness of soil layer assumed constant throughout straining
Base line is related to 24-hour compression curves	RTL related to end of primary consolidation curves

LABORATORY TESTS

The performance of the *abc* and *EVP* models was assessed by comparing their predictions with oedometer and Constant Rate of Strain (CRS) tests carried out on a typical peat from the Irish midlands. This peat is a highly compressible, poorly humified fibrous peat with a Von Post Humification value of $H_{2\text{-}3}$, average water content of 1800%, unit weight of 10.2 kN/m^3, specific gravity of 1.41, and organic content of 98%. The specimens were retrieved from undisturbed block samples taken from a depth of approximately 0.5 m. The oedometer tests involved different Load Increment Ratios (LIR) and Load Increment Durations (LID), and the CRS tests were performed at different strain rates. Two oedometer tests and one CRS test were selected for specific examination in this paper:

- Test 1: Standard 24 hour oedometer test with LIR=1

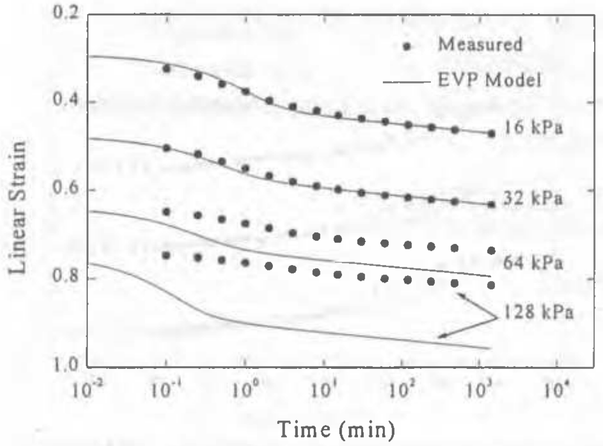

Figure 2 Comparison of measured results with those calculated using the *EVP* model for Test 1.

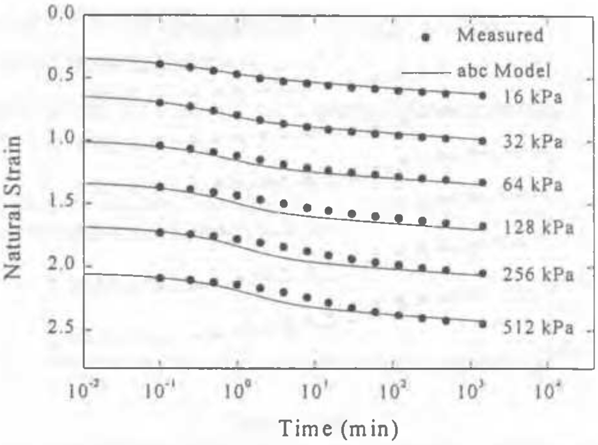

Figure 3 Comparison of measured results with those calculated using the *abc* model for Test 1.

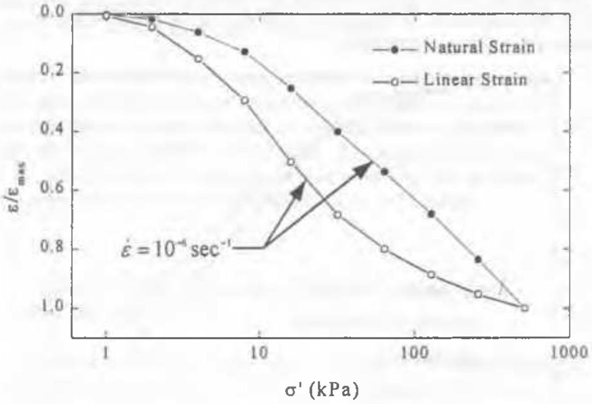

Figure 4 Stress-strain relationship for Test 1

- Test 2: Oedometer test with high LID and LIR=2
- Test 3: CRS test performed at a nominal strain rate of $7.34\text{x}10^{-5}$ min^{-1}; negligible excess pore pressures were recorded at this relatively slow rate.

As the majority of reported 1-D compression data are from standard 24-hour oedometer tests (with LIR=1), Test 1 was selected to derive the model parameters. These parameters are then used to predict the response of Test 2 and Test 3.

PREDICTIONS

The model parameters were derived from Test 1 in accordance with the procedures recommended for the *abc* and *EVP* models and are listed in Tables 2 and 3. All load increments in the virgin stress range were used to determine the *abc* model parameters, whereas the 16 kPa and 32 kPa load increments were used to determine the *EVP* model parameters.

Table 2 *EVP* model parameters determined from Test 1

κ/v	λ/v	ψ/v	t_o: min	σ'_o: kPa	k: m/sec
0.052	0.234	0.007	$1.28\text{x}10^{-6}$	4.095	$1.15\text{x}10^{-7}$

Table 3 *abc* model parameters determined from Test 1

a	b	c	v_1	v_o	k_o: m/sec	C_k
0.056	0.519	0.018	73.886	32.833	$1.77\text{x}10^{-6}$	0.215

The parameters listed in Tables 2 and 3 have been used in conjunction with programmed formulations of the *abc* and *EVP* models to predict the settlements for Test 1. Figure 2 compares measured data with that predicted using the *EVP* model for selected normally consolidated load increments in Test 1. It is obvious from Figure 2 that the *EVP* model does not capture the entire normally consolidated stress-strain response. However, it is worth noting that the increments from which the parameters were determined, namely the 16 kPa and 32 kPa stress increments are quite well predicted. Figure 3 compares the measured and calculated strains for Test 1 using the *abc* model and the parameters given in Table 3. Good agreement exists between calculated and measured values for all stress increments.

Although the *EVP* model is expressed in terms of linear strain, Yin & Graham (1989) conceded that natural strain may need to be used for very compressible soils. Figure 4 shows Test 1 data plotted in stress-strain space at a strain rate of 10^{-6} sec^{-1} using both natural and linear strain as a measure of the compression. Strain data are normalized by the maximum strain in each case to allow for ease of comparison. It is evident from Figure 4 that the use of natural strain is much more successful in linearizing ε - log σ' data for this type of soil.

The *EVP* model has been reformulated in terms of natural strain by the authors, with provision for varying permeability and changing dimensions of the soil layer during compression. Figure 5 compares the measured strains with those calculated using the re-assessed parameters given in Table 4 and the modified *EVP* model. There is an obvious marked improvement in the use of natural strain over linear strain in predicting the settlements over a wider stress range.

Both the *abc* model and the *modified EVP* model have been used in conjunction with the parameters in Tables 3 and 4 to predict settlements for Test 2. Measured and calculated data for selected load increments from Test 2 are shown in Figure 6. The 37.8 kPa increment was maintained for 40 days, whereas the 113.4 kPa and the 340.2 kPa increments were maintained for 60 days. The initial specific volume (v_o) for Test 1 was 32.8 whereas that in Test 2 was 29.8. This variation was taken into account in the *abc* prediction for Test 2 by employing v_o = 29.8 and a modified v_1 value derived from:

$$v_1' = v_o'\left(v_1/v_o\right) \qquad (9)$$

where v'_1 and v'_0 refer to the modified parameters in Test 2. In this case both models predict final settlements adequately, although settlements during consolidation are over-estimated.

Table 4 Modified *EVP* model parameters determined from Test 1

κ/v	λ/v	ψ/v	t_o: min	σ'_o: kPa	k_o: m/sec	C_k
0.056	0.515	0.014	$1.57\text{x}10^{-5}$	7.775	$1.77\text{x}10^{-6}$	0.215

The measured stress-strain relationship for the CRS test is shown in Figure 7. The apparent preconsolidation pressure for this test is approximately 6 kPa, at which point large fluctuations in the stress-strain readings were recorded. This may be attributed to the breakdown of large fibres in the peat sample, which could cause a sudden decrease in stress. Calculated stress-strain responses are also shown for each model in Figure 7. Both mod-

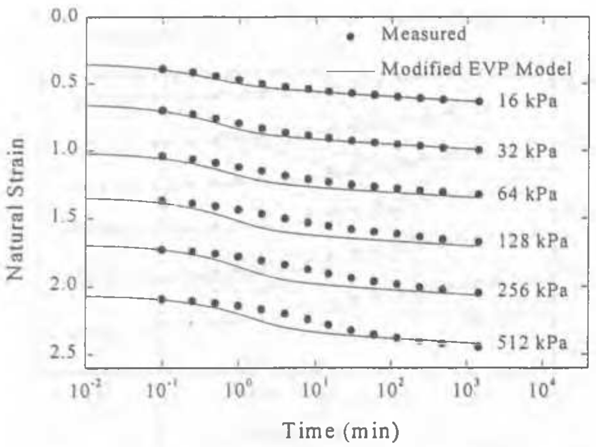

Figure 5 Comparison of measured results with those calculated using the modified *EVP* model for Test 1.

els capture the overall stress-strain relationship quite well, although the calculated stress-strain plots are shifted somewhat to the left of the measured results.

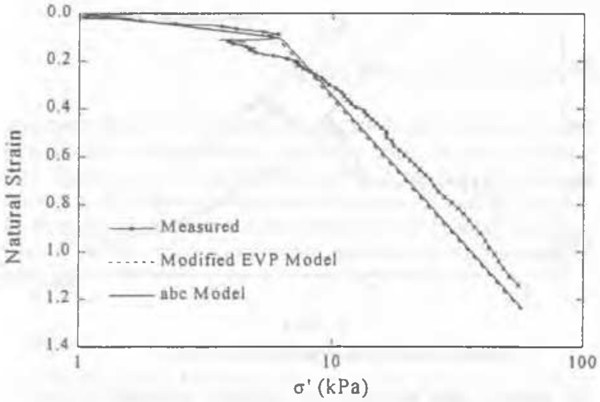

Figure 7 Comparison of measured results with those calculated using the *abc* and modified *EVP* models for Test 3.

DISCUSSION

The *abc* model is evidently adequate for predicting the overall settlement for all load increments in the virgin stress range, although certain modifications need to be made to the *EVP* model to describe the one-dimensional compression of fibrous peat. The most important of these modifications is the definition of strain. It is evident from Figures 2 and 4 that linear strain is not suitable for linearizing ε - log σ' data over a wide stress range. When the *EVP* parameters are determined from natural strain data, then the *EVP* model is much more successful in predicting settlements for all load increments (Figure 5). Good agreement has been obtained for both the *abc* and modified *EVP* models during the consolidation phase of Test 1, but strains for this phase were over-estimated for all load increments of Test 2. This is presumably due to the differences between the empirical parameters C_k and k_o (Equation 8) in the respective samples. However this is generally of little consequence for fibrous peat as the primary consolidation phase in such materials is typically short.

In Test 2, v_o was less than that in Test 1, and it was found that a best fit for the *abc* model required adjustment of the location of the base line (by modifying v_1) to keep it fixed in stress-strain space. This adjustment is consistent with the assumption of a constant σ'_o value employed in the *EVP* model.

The main difference in the models is the positioning of a reference creep isotache or time line from which to measure creep strains. The *EVP* model employs the parameter t_o, to position the RTL. Yin & Graham (1996) suggest that values determined for t_o should be of the same order as the time taken for primary consolidation (t_p). However, the value of $t_o = 1.57\text{x}10^{-5}$ min given in Table 4 is obviously several orders less than t_p. Despite this, t_o appears to be a useful parameter in scaling creep strain rates. The *abc* model positions the base line at $\tau = 1$ day, which roughly corresponds to standard 24-hour compression curves. The creep strain rate on the *abc* model's base line is $2.08\text{x}10^{-7}$ sec^{-1}, whereas the corresponding creep strain rate on the *EVP* model's RTL is 14.86 sec^{-1}, with the result that the RTL is positioned at a creep strain rate not normally encountered in the majority of one-dimensional compression tests. Regression techniques cannot, as a consequence, be employed to fit Equation 1 to measured creep strain data.

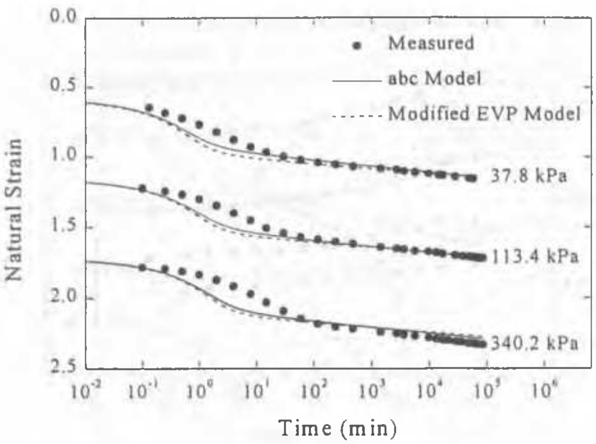

Figure 6 Comparison of measured results with those calculated using the *abc* and modified *EVP* models for Test 2.

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