Modeling of rate dependency of dynamic strength for clay and its application

Influence de la vitesse de modélisation sur la résistance dynamique de l'argile, et ses applications

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ABSTRACT: The constitutive model with rate dependency of dynamic strength for clay was verified using a series of undrained cyclic triaxial tests at different loading rates. The importance of a rate-dependent characteristic of clay was discussed through the simulations for dynamic centrifugal model tests of the embankment on soft clay foundation. We carried out two simulation cases of prototype scale in the gravitational field and model scale in the centrifugal field. The experimental results agreed with the simulated results for model scale in the centrifugal field better than that for prototype scale in the gravitational field. This means that the direct modeling for the centrifugal model is preferable when the rate-dependent characteristics of clay is considered. Therefore, the special consideration about rate dependency of materials will be needed in the centrifugal model tests using cohesive soil.

RÉSUMÉ: La dépendance de la loi constitutive de la résistance dynamique de l'argile sur la vitesse de modélisation, a été vérifiée par des essais cycliques triaxials à plusieurs niveaux de vitesse. L'importance de cette vitesse de modélisation est discutée à l'aide d'essais sur un model dynamique-centrifuge d'un remblayage sur une fondation d'argile. Nous avons réalisé une modélisation numérique (FEM) sur deux cas ; Le premier cas c'est un prototype du remblayage à échelle naturelle avec une gravité de 1G. Le deuxième cas c'est un model réduit à la même échelle de l'essai sur la machine centrifuge. Le model numérique à échelle réduite est plus d'accord avec le résultat des essais du laboratoire que le model numérique du prototype. La vitesse de modélisation est un paramètre très important qu'on doive considérer pour les essais sur des models centrifuge-dynamiques d'un sol cohésif.

1 INTRODUCTION

The dynamic characteristics of deformation and strength for soil are significant for dynamic response analysis for a laminated ground with sand and clay layers. The dynamic deformation characteristics is manifested in the shear modulus and damping ratio which vary significantly with the amplitude of shear strain under cyclic loading. The range of shear strain amplitude is from 10^{-4} % to 1 %. On the other hand, the dynamic strength characteristics are defined in the large shear strain region of more than 5 %. For example, the liquefaction strength, which is the dynamic strength for sand, is represented as the relation between the cyclic shear stress ratio and the number of cycles to produce 7.5 % amplitude shear strain.

In conventional dynamic response analyses for a laminated ground, sandy soils were usually modeled using the dynamic deformation and strength characteristics. On the other hand, cohesive soils were usually modeled using only dynamic deformation characteristics, because the shear strain amplitude was considered to be smaller than 1 % during middle-size earthquake. However, the modeling for dynamic strength in larger strain region is also significant in the case that a large seismic load is considered. Therefore, the constitutive model for clay should reproduce the dynamic strength like the constitutive model for sand.

Cohesive soils do not experience classical liquefaction like loose sand in which the effective stress drops to zero and the shear strain rapidly increases. Undrained cyclic shear tests for cohesive soils showed that during cyclic loading the effective stress decreases and the shear strain gradually increases (Hyodo et al., 1994). In addition, cohesive soils have the rate-dependent characteristic (Procter & Khaffaf, 1984). Adachi & Oka (1982) proposed the rate-dependent elasto-viscoplastic model based on the triaxial test results at different loading rates. Oka (1992) extended the elasto-viscoplastic model to describe the cyclic behavior by using nonlinear kinematic hardening rule. However, the cyclic elasto-viscoplastic model for clay has not yet been verified for the dynamic strength at different loading rates.

In this study, a cyclic elasto-viscoplastic model that can take the rate-dependent dynamic strength characteristics of cohesive soils was used. We verified the constitutive model for clay using a series of undrained cyclic triaxial tests at different loading rates. We applied the verified constitutive model to the simulations for the dynamic centrifugal tests, in which the embankment model on soft clay foundation was excited with several acceleration amplitudes. We carried out two cases of the numerical simulation by using two FE models. One is based on prototype scale in the gravitational field, the other is model scale in the centrifugal field. The importance of a rate-dependent characteristic of clay was discussed comparing the simulated results with the results of dynamic centrifugal model tests.

2 CONSTITUTIVE MODEL

The constitutive model used for clay was a cyclic elasto-viscoplastic model (Oka, 1992). As details of this model were given in Oka (1992), a brief description of the model is given below. This constitutive model was formulated on the following assumptions:

1. The infinitesimal strain is used;
2. The overstress type of viscoplasticity;
3. The non-associated flow rule;
4. The concept of the overconsolidated boundary surface;
5. The non-linear kinematic hardening rule.

The total strain rate $\dot{\varepsilon}_{ij}$ is expressed as:

$$\begin{aligned}
\dot{\varepsilon}_{ij} &= \dot{\varepsilon}^{e}_{ij} + \dot{\varepsilon}^{vp}_{ij} \\
\dot{\varepsilon}^{e}_{ij} &= \frac{1}{2G}\dot{s}_{ij} + \frac{\kappa}{3(1+e_0)}\frac{\dot{\sigma}'_m}{\sigma'_m}\delta_{ij} \\
\dot{\varepsilon}^{vp}_{ij} &= C_{01}\Phi'(F)\frac{\partial g}{\partial s_{ij}} + C_{02}\Phi'(F)\frac{\partial g}{\partial \sigma'_m}\frac{\delta_{ij}}{3}
\end{aligned} \tag{1}$$

where $\dot{\varepsilon}^{e}_{ij}$ = elastic strain rate; $\dot{\varepsilon}^{vp}_{ij}$ = viscoplastic strain rate; G = elastic shear modulus; $\dot{s}_{ij}$ = deviatoric stress rate tensor; κ =

swelling index; σ'_m = mean effective stress; e_0 = initial void ratio; C_{01}, C_{02} = viscoplastic parameters and g = plastic potential function. The complete shape of the material function $\Phi'(F)$ is experimentally determined as follows:

$$\Phi'(F)=\sigma'_m \exp\left[m'_0\left\{(\eta_{ij}-\chi_{ij})(\eta_{ij}-\chi_{ij})\right\}^{1/2}\right] \tag{2}$$

where m'_0 = viscoplastic parameter; $\eta_{ij}=s_{ij}/\sigma'_m$ = deviatoric stress ratio and χ_{ij} = kinematic hardening parameter. Equation 2 represents the linear relation between the natural logarithm of viscoplastic strain rate and the deviatoric stress ratio.

3 VERIFICATION OF CONSTITUTIVE MODEL

The constitutive model is verified to reproduce the rate-dependent strength of a cohesive soil. The undrained cyclic triaxial tests for remolded Arakawa clay were carried out at different loading rates (Hyodo et al., 1997). The Arakawa clay is alluvial clay whose index properties are as follows: specific density G_s = 2.659; liquid limit w_L = 49.8 %; plasticity limit w_P = 32.0 % and plasticity index I_p = 17.8. The specimens were remolded and isotropically consolidated with confining stress of 100 kPa. The cyclic stress with three loading rates with frequencies of 0.02 Hz, 0.1 Hz and 1.0 Hz was loaded.

In order to verify the performance of the constitutive model, the simulated results were compared with the experimental results. Model parameters are summarized in Table 1. The following parameters, e_0, λ, κ, M_{mc}, M_{me}, M_{fc} and M_{fe} were determined by the physical property tests and the consolidated undrained triaxial tests by using the same material. The initial shear modulus of clay was estimated from the undrained shear strength by the experimental equation (Hara et al., 1974). The remaining parameters in Table 1, hardening parameter and viscoplastic parameters, can be determined directly by a monotonic shear test with different strain rates in principle. However, in this study, the parameters were determined by an adjusting technique so as to well describe the dynamic strength curve, the effective stress path and stress-strain relation of undrained cyclic triaxial tests.

Experimental and simulated results at the cyclic shear stress ratio of 0.238 at a loading rate with a frequency of 0.02 Hz are shown in Figure 1. The simulation reproduced the characteristic behavior of clay in the experiment very well. The effective stress decreases, however does not reach to zero effective stress like loose sand. The shear strain amplitude gradually increases during cyclic loading, and exceeds 10 % in double amplitude of axial strain.

The dynamic strength curves at different loading rates for experimental and simulated results are shown in Figure 2. The failure criteria for both the experiment and simulation were 5.0% in double amplitude of axial strain. In addition, the simulated dynamic strength at the loading rate of 60.0 Hz, which is the frequency of the sinusoidal input wave used in the centrifugal model tests, is also shown in Figure 2. This strength is considered later. The model quantitatively reproduced the experimental dynamic strength with different loading rates, except that the model has smaller strength than the experiment at slower loading rate. The higher the loading rate, the larger the dynamic strength.

4 SIMULATIONS FOR DYNAMIC CENTRIFUGAL TESTS

We applied the verified constitutive model to the simulations for the dynamic centrifugal tests (Matsuo et al., 1997), in which the embankment model on soft clay foundation was excited with several acceleration amplitudes. We carried out two cases of the numerical simulation for two FE models. One is prototype scale in the gravitational field, the other is model scale in the centrifugal field. The importance of a rate-dependent characteristic of clay was discussed through the comparison between the simulated results and dynamic centrifugal model tests results.

Table 1. Model parameters for triaxial tests.

Initial void ratio	e_0	0.840
Compression index	λ	1.17E-1
Swelling index	κ	2.80E-2
Initial Shear modulus	G_0 (kPa)	2.0E+04
Failure stress ratio (compression side)	M_{fc}	1.322
Phase transformation stress ratio (compression side)	M_{mc}	1.225
Failure stress ratio (extension side)	M_{fe}	1.058
Phase transformation stress ratio (extension side)	M_{me}	0.980
Hardening parameter	B_0	20.0
Viscoplastic parameter	m'_0	18.5
Viscoplastic parameter	C_{01} (1/s)	3.0E-07
Viscoplastic parameter	C_{02} (1/s)	7.5E-08

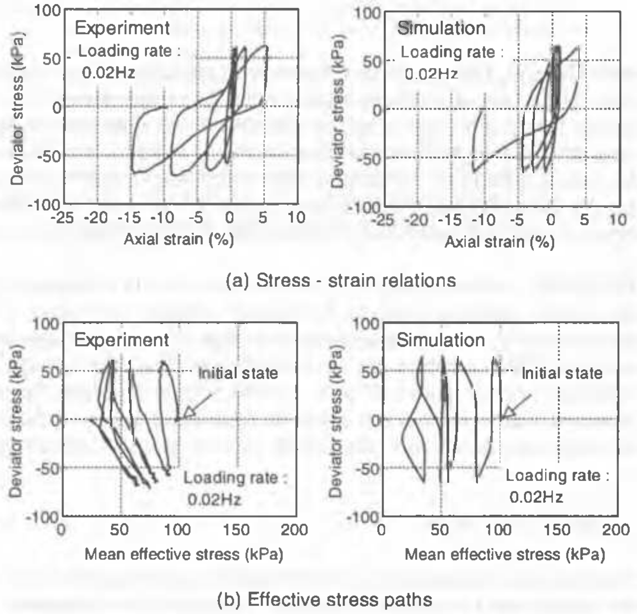

Figure 1. Cyclic triaxial tests (Experiment: Hyodo et al., 1997).

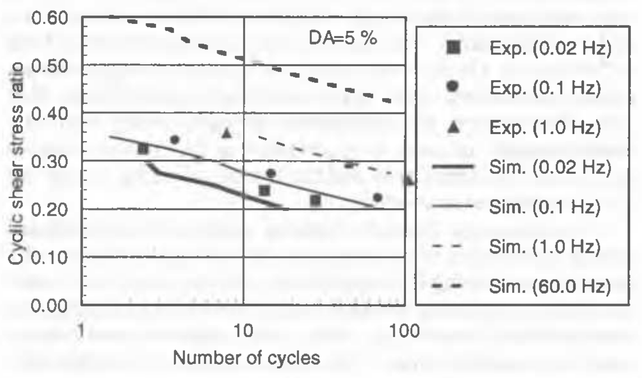

Figure 2. Dynamic strength curves (Experiment: Hyodo et al., 1997).

4.1 *Dynamic centrifugal model tests*

The dynamic centrifugal tests were performed by Matsuo et al. (1997). Figure 3 shows the cross section of test models and the location of transducers. The foundation soil was composed of upper clay layer and lower sand layer in a rigid soil container. The lower sand layer was made of well-compacted fine sand and it was not likely to liquefy during shaking. The upper clay layer was prepared by pouring Arakawa clay, which was slurry with the water content of 80 %. The clay layer was sufficiently consolidated under the centrifugal acceleration of 50 G. The embankment was made of a mixture of Keisa sand and a silty sand with a ratio of 2 : 1 in weight and a water content of 12.5 %.

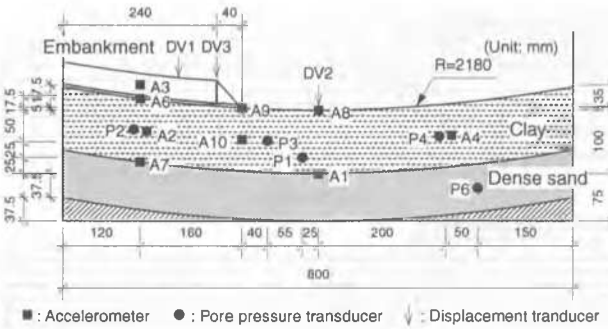

Figure 3. Centrifugal model (Matsuo et al., 1997).

Table 1. Model parameters for centrifugal tests.

Name of soil profile		Embankment	Clay	Dense sand
Density	ρ (t/m^3)	1.800	1.746	2.000
Initial void ratio	e_0	0.606	1.228	0.700
Coefficient of permeability	k (m/s)	-	1.0E-08	1.0E-04
Compression index	λ	0.200	0.117	0.015
Swelling index	κ	0.010	0.028	0.002
Initial shear modulus ratio	G_0/σ'_m	638	242	1906
Failure stress ratio	M_f	1.072	1.322	1.511
Phase transformation stress ratio	M_m	1.072	1.225	1.378
Hardening parameter	B_0	1790	20	3870
For cohesive soils				
Viscoplastic parameter	m'_0		18.5	
Viscoplastic parameter	C_{01} (1/s)		3.0E-07	
Viscoplastic parameter	C_{02} (1/s)		7.5E-08	

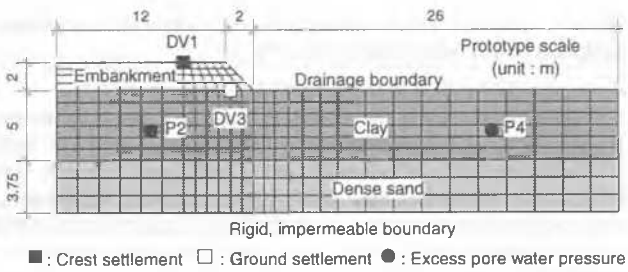

Figure 4. Numerical model in gravitational field.

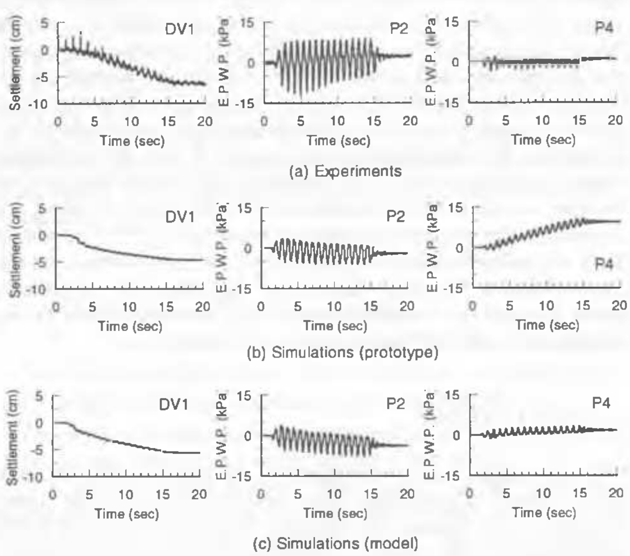

Figure 5. Time histories of responses with input acceleration of 104 Gal.

A horizontal shaking was conducted under a centrifugal acceleration of 50 G. Sinusoidal input motion with a frequency of 60 Hz and 20 cycles was applied to the model. The amplitude of the input motion was increased step by step with 4 stages for the same test model. In each shaking stage, the next shaking started after the excess pore water pressure dissipated completely.

4.2 *Analytical conditions*

The numerical method was the dynamic response FE analysis that incorporated the cyclic elasto-plastic constitutive model for sand, the cyclic elasto-viscoplastic model for clay, and Biot's two phase mixture theory (Oka et al., 1994). Two types of the FE models in gravitational and centrifugal field were used. The numerical model in gravitational field was shown in Figure 4. The numerical model in Figure 4 was 50 times larger than the model in centrifugal field in Figure 3.

The cyclic elasto-viscoplastic model for clay was used for the clay layer, and the cyclic elasto-plastic model for sand (Oka et al., 1999) was used for the dense sandy layer and the embankment. The model parameters are summarized in Table 2. The parameters of the ground were determined based on measurements during preparation of the test models and laboratory tests for Arakawa clay and Keisa sand. The parameters of the embankment and dense sand layer were determined based on the past study (Matsuo et al., 2000). The parameters of clay in Table 1 were basically used for the clay layer, because the ground was the same material as the laboratory tests. The dynamic strength with 20 cycles in the centrifugal field was about 1.4 times larger than that in the gravitational field as shown in Figure 2. It is noted that the simulated dynamic strength at the loading rate of 60.0 Hz was calculated by using Equation 2, which assumed that the linear relation between the natural logarithm of viscoplastic strain rate and the deviatoric stress ratio.

The initial stress state was computed by the static elasto-perfectly-plastic analysis, in which the Drucker-Prager type failure surface model was employed. In the dynamic analysis, a time integration step was set to be 0.0025 seconds in gravitational field and 0.00005 seconds in centrifugal filed to ensure the numerical stability. The Rayleigh damping proportional to initial stiffness was used in order to describe the damping especially in the high frequency domain. The factor of Rayleigh damping was set to be 0.001 in gravitational field and 0.00002 in centrifugal filed. The amplitude and the frequency of input motions in gravitational field were 1/50 as those in centrifugal field. Although the input motions with 4 amplitudes were continuously applied to the same test model in the experiment, the virgin model in each shaking stage was used in the simulation. To facilitate direct comparison with the numerical results, the measurements and the numerical results in centrifugal filed are hereinafter demonstrated on the prototype scale in gravitational field.

4.3 *Comparison between two simulations and experiments*

Figure 5 shows the time histories of crest settlement at DV1, excess pore water pressure at P2 and P4 in the case with the acceleration amplitude of 104 Gal in gravitational filed. The simulated crest settlement at DV1 qualitatively agreed with the measured one. Comparing with two types of simulations, the responses of crest settlement at DV1 and excess pore water pressure at P2 show the similar tendency. However, the accumulation of excess pore water pressure at P4 in gravitational field was larger than that in centrifugal field and the experimental result. The cyclic shear stress ratio at P4 in the horizontal ground was lager than that at P2 under the embankment. Therefore, the rate-dependent effect, that the dynamic strength is smaller under the lower loading rate, appeared clearly in the horizontal ground.

Figure 6 compares the relations between the input maximum acceleration and the ground settlements after shaking at DV3 as shown in Figure 4 for all shaking stages. The simulated settlement in gravitational filed was 2 – 5 times larger than that in centrifugal field. This is due to the rate-dependent dynamic strength of the constitutive model. Namely, the dynamic strength of the model in gravitational field with low loading rate was smaller than that in centrifugal filed with high loading rate as shown in Figure 2. Comparing with the measured settlement, the

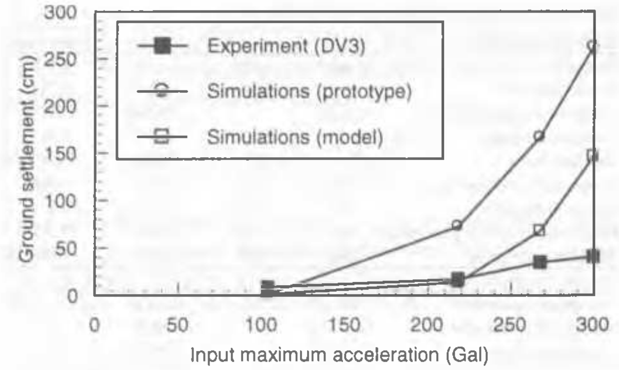

Figure 6. Relations between input maximum accelerations and ground settlement (Experiment: Matsuo et al., 1997).

simulated settlement in the centrifugal field is larger than the experimental one. This discrepancy tends to be large at larger input acceleration. This overestimation of the settlement is mainly due to that the simulation used the virgin model in each shaking stage. In the experiments, the soil stiffness after shaking and consolidation at each shaking stage might be larger than that at the initial state.

Figure 7 and Figure 8 show simulated and measured deformed mesh after shaking respectively at the second shaking stage. The deformed configuration in the experiment was drawn based on the sketch of model ground after shaking. The experimental and simulated results showed that the soil around the toe of the embankment slope deformed largely. However, the simulation in gravitational field overestimated the experimental deformation. The experimental results agreed with the simulated results for model scale in the centrifugal field better than that for prototype scale in the gravitational field. However, we must not forget that the dynamic strength of the model in the centrifugal field was calculated by using Equation 2, which assumed that the linear relation between the natural logarithm of viscoplastic strain rate and the deviatoric stress ratio. We needs to verify the assumption with high loading rate by laboratory tests.

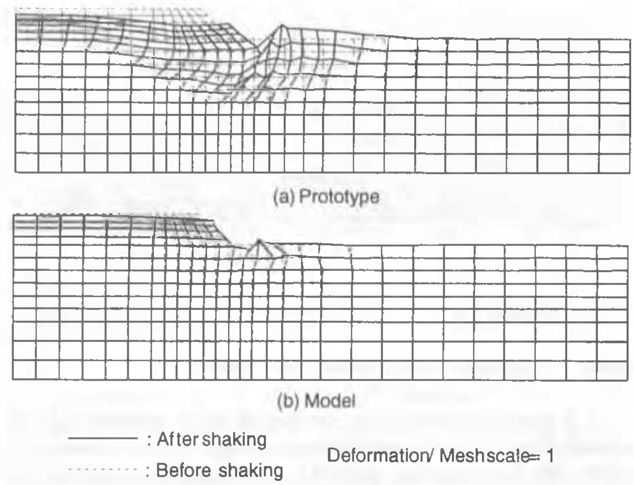

Figure 7. Simulated deformation in the second shaking stage.

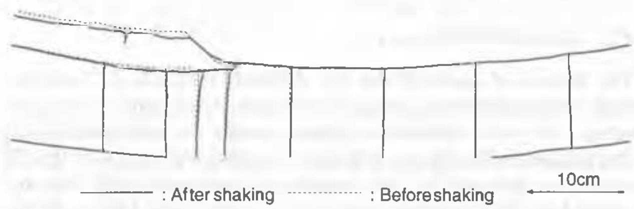

Figure 8. Measured deformation in the second shaking stage.

5 CONCLUSIONS

We verified the elasto-viscoplastic constitutive model for clay using a series of undrained cyclic triaxial tests at different loading rates, and applied the verified constitutive model to the simulations for the dynamic centrifugal tests with the embankment model on soft clay foundation. The analytical results led to the following conclusions.

- The cyclic elasto-viscoplastic model for clay quantitatively reproduced the experimental dynamic strength at different loading rates.
- The experimental results agreed with the simulated results for model scale in the centrifugal field better than that for prototype scale in the gravitational field. This means that the direct modeling for the centrifugal model is preferable when the rate-dependent characteristics of clay is considered. Therefore, the special consideration about rate dependency of materials will be needed in the centrifugal model tests using cohesive soil.

ACKNOWLEDGEMENT

The authors wish to thank Prof. Fusao Oka of Kyoto University and Prof. Atsushi Yashima of Gifu University for their worthy advice to the numerical analyses, Prof. Masayuki Hyodo of Yamaguchi University for providing the laboratory tests data of Arakawa clay and Mr. Osamu Matsuo of Public Works Research Institute for providing the experimental data of the centrifugal model tests.

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