Geostatistical interpretation of soil exploration

Interprétation géostatistique des travaux de reconnaissance géotechnique

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ABSTRACT: Geostatistical techniques are increasingly used for interpreting results of soil exploration. These methods provide an objective appraisal of the spatial variation of soil properties that is extremely valuable for designers of foundations and earth structures. The paper includes a brief review of the basic theoretical concepts of Geostatistics as adapted to the field of Soil Mechanics. The paper then presents a simple example dealing with the configuration of the first hard layer found in the lacustrine zone of Mexico City.

RÉSUMÉ: Les techniques de la géostatistique sont de plus en plus utilisées pour interpréter les résultats des travaux de reconnaissance géotechnique. Ces méthodes fournissent une évaluation objective des variations spatiales des propriétés des sols qui s'avère d'une grande valeur pour les concepteurs de fondations et de structures en terre. Cette communication contient un bref rappel des concepts de la Géostatistique adaptés à la mécanique des sols. On présente ensuite un exemple simple portant sur la configuration de la première couche dure de la zone lacustre de la ville de Mexico.

1 INTRODUCTION

Geostatistics can be defined as the application of random functions theory to the description of the spatial distribution of properties of geological materials.

This technique provides valuable tools for estimating data such as the thickness of a specific stratum, or the value of a certain property of the soil at a given point where no information is available, taking into account the correlation structure of the medium. Additionally, it makes it possible to quantify the uncertainty associated to these estimates. Geostatistics can also be used to simulate possible configurations of the subsoil and to define an optimum sampling strategy.

In this paper, the main theoretical principles of geostatistics are briefly reviewed. The application of the method is illustrated by a simple example dealing with the spatial variability of the depth of the first hard layer in the lacustrine zone of Mexico City.

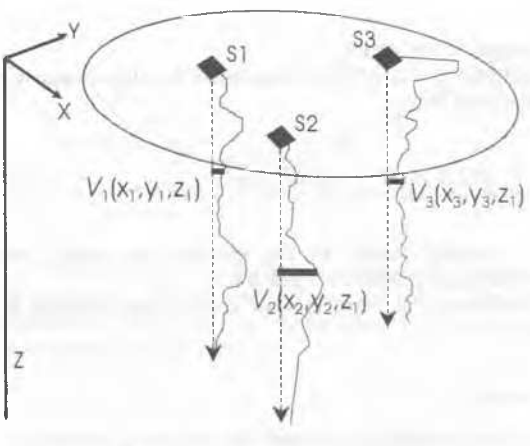

Figure 1. Random field

2 THEORETICAL CONCEPTS (Auvinet et al. 2000)

2.1 *Random field*

Let us consider a geotechnical variable $V(X)$, either of physical (i.e. water content), mechanical (i.e. shear strength) or geometrical nature (i.e. depth or thickness of some stratum) defined at points X of a given domain R^p (p = 1, 2, or 3). In each point of the domain, this variable can be considered as random due to the range of possible values that it can take (Fig. 1). The set of these random variables constitutes a random field (Vanmarcke 1983).

To describe this field the following parameters and functions are used:

- Expected value:

$$\mu_V(X) = E\{V(X)\} \tag{1}$$

- Variance:

$$\sigma_V^2(X) = Var[V(X)] \tag{2}$$

- Autocovariance function:

$$C_V(X_1,X_2) = E\{[V(X_1)-\mu_V(X_1)][V(X_2)-\mu_V(X_2)]\} \tag{3}$$

This function represents the degree of linear dependence between the values of the property of interest in two different points. It can be written in the form of a coefficient of autocorrelation without dimension whose value is always between -1 and +1:

- Correlation coefficient:

$$\rho_V(X_1,X_2) = \frac{C_V(X_1,X_2)}{\sigma_{V(X_1)}\sigma_{V(X_2)}} \tag{4}$$

- Variogram:

$$2\gamma(h) = E\{[V(X)-V(X+h)]^2\} \tag{5}$$

The variogram is the second order statistical moment of the increment $V(X)$-$V(X+h)$. Generally, it can be considered as equivalent to the autocovariance function.

The autocovariance and the correlation coefficient are not intrinsic properties of the two points X_1 and X_2, they also depend on the population, that is to say, on the domain in which the field is defined.

Frequently, it is accepted for the sake of simplicity that the expected value is constant in the considered domain and that the correlation coefficient depends only on the vectorial distance between points X_1 and X_2, that is to say that the random field is *wide sense stationary*. However, when a drift is detected in the variation of the properties of the soil with depth, it is convenient to remove this trend from the data and to work with the residual random field.

2.2 *Statistical estimation of the parameters of a random field*

The parameters and descriptive functions defined in the previous equations can be determined from the "discrete" (isolated samples) or "continuous" (borings) results of the exploration, using statistical estimators. This is commonly referred to as *structural analysis*. In the case of borings, accepting that the field is statistically homogeneous, stationary and ergodic (that is to say that its parameters can be obtained from a single realization of the field), the expected value can be evaluated using the approximation:

$$\mu_V \cong \mu^* = \frac{1}{L}\int_0^L V(X)\mathrm{d}X \tag{8}$$

where L = length of the boring.

In the same form, it is possible to estimate the autocovariance along the direction **u** as:

$$C_V(\lambda\mathbf{u}) \cong \frac{1}{L}\int_0^L V(X)V(X+h\mathbf{u})\mathrm{d}X - \mu^{*2} \tag{9}$$

Where **u** = unitary vector in the direction in which the covariance is evaluated; and h = a scalar.

Similar techniques are available for discrete data (Deutch & Journel 1992)

2.3 *Estimation*

Geostatistics can be used to estimate the value of a property of interest at points of the medium where no measurement has been made. It is then possible to interpolate between available data and to define *virtual* borings, cross-sections or configurations of a given stratum within the soil. The problem can be generalized to the estimation of the average value of a property in any sub-domain of the studied medium, for example, in a given volume or along a certain potentially critical surface.

To reach this objective, linear statistical estimators without bias and with a minimum variance can be used (Best linear Unbiased Estimation or "BLUE"). This technique, also known as kriging (Krige 1962, Matheron 1965) is also widely used in mining engineering.

2.4 *Simulation of random fields*

Simulation is the process by which a possible configuration of the random field is generated in a way compatible with its descriptive parameters (unconditional simulation) or with these parameters and, also, with the available data (conditional simulation). A plausible image of the field is obtained.

To perform data analyses, estimations and simulations in typical geotechnical problems, a group of computer programs have been developed (Juárez 2001, Medina 2001). This approach has been used successfully to describe the spatial variation of soil properties in different sites (Auvinet & Medina 1999). The simple example presented below deals only with the geometrical characteristics of a soil layer.

3 APPLICATION TO THE ANALYSIS OF THE CONFIGURATION OF THE FIRST HARD LAYER IN MEXICO CITY

3.1 *Definition of the random field*

The typical surficial stratigraphical sequence in Mexico City's lacustrine zone includes a thin drying crust, a thick clay layer and the first bearing stratum. The depth to this hard layer can be considered as a random field $V(X)$, distributed in space R^p, with p = 2 (study area). The set of values measured inside the domain R^p, constitutes a sample of that random field. The analysis is carried out in an approximate area of 1.7 km^2 of downtown Mexico City shown in Figure 2. In that same figure, the location of the borings used as support of data is indicated.

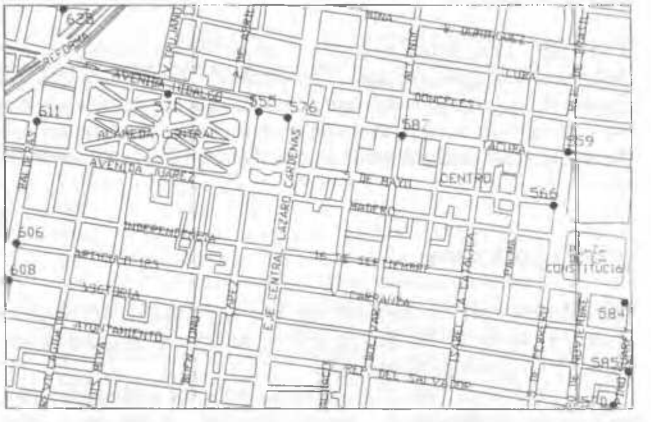

Figure 2. Study area

The experimental data used in the analysis are presented in Table 1, where coordinates (x, y) define the position of the borings in the geographical reference system UTM of Mercator.

Table 1. Data

No. boring	Coord. x (m)	Coord. y(m)	Prof. CD (m)
570	486097	2148112	-37.00
585	486137	2148200	-35.00
584	486128	2148378	-36.85
608	484514	2148437	-30.25
606	484533	2148534	-30.40
566	485942	2148637	-36.45
559	485981	2148779	-36.80
587	485544	2148822	-34.40
611	484587	2148859	-31.35
576	485247	2148870	-33.30
555	485170	2148884	-32.95
571	484935	2148930	-32.00
625	484656	2149155	-30.15

3.2 *Statistical description*

The main statistical parameters of the field estimated from the data are as follows:

Mean: -33.60m
Standard deviation: 2.65m
Variation coefficient: -0.079

3.3 *Structural analysis*

- Trend analysis

A linear function $V = ax+by+c$ can be adjusted to the data:

a = -0.0004
b = -0.004
c = 8805.7622

The presence of a trend is acknowledged and taken into account in the structural analysis and in the estimations.

- Estimation of the autocorrelation coefficient

The functions describing the correlation structure of the random field in two main directions α_1 = Az 0° (y axis); α_1 = Az 90° (x axis), with a calculation step Δh = 50 m, are shown in Figure 3. It can be seen that influence distances are of the order of several hundred meters and that some anisotropy exists.

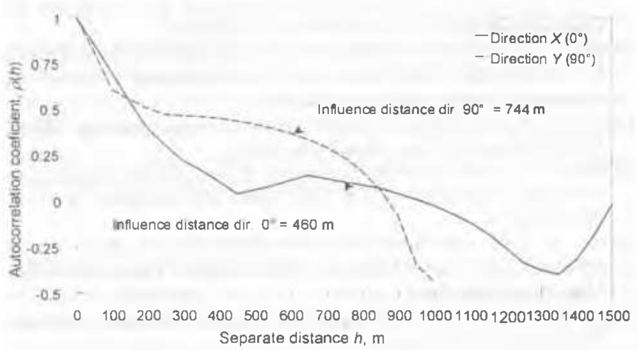

Figure 3. Correlograms of the hard layer depth

3.4 *Estimation*

From the experimental data and using the results of the structural analysis, the expected value and standard deviation of the depth of the hard layer can be obtained at all nodes of any given regular mesh using the ordinary kriging technique (Deutch & Journel 1992). A contour map can then be built to appreciate visually the spatial distribution of the depth of this layer within the studied area (Fig. 4).

Figure 4. Contour map of the hard layer depth

In the same way, a contour map of the standard deviation of the depth can be built (Fig. 5)

The map in Figure 5 shows that the standard deviation vanishes near the location of the borings where the actual depth is known. On the other hand, the uncertainty increases with the distance to those borings.

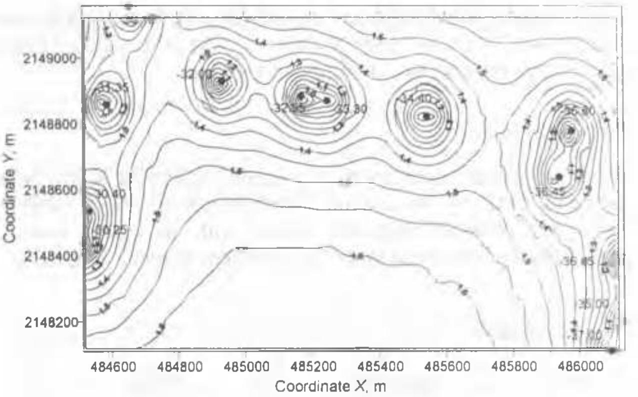

Figure 5. Contour map of the standard deviation of the estimation

The results of the estimation can also be represented by constructing a three-dimensional surface, assigning the value of the depth to the vertical coordinate (elevation) of each point. (Fig. 6).

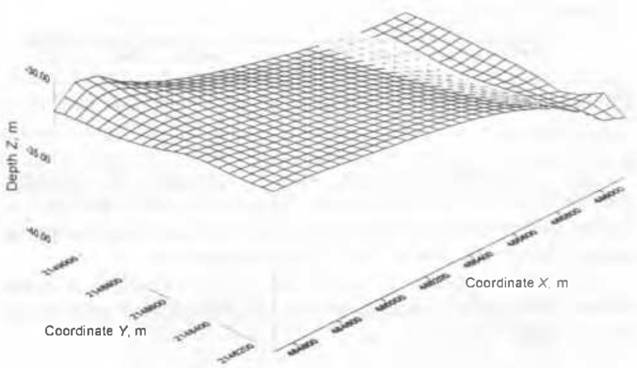

Figure 6. Representative surface of the expected hard layer depth

It can be seen that the kriging technique provides a softened interpolation of the data. The uncertainty associated to the estimation can also be represented by means of a three-dimensional surface (Fig. 7).

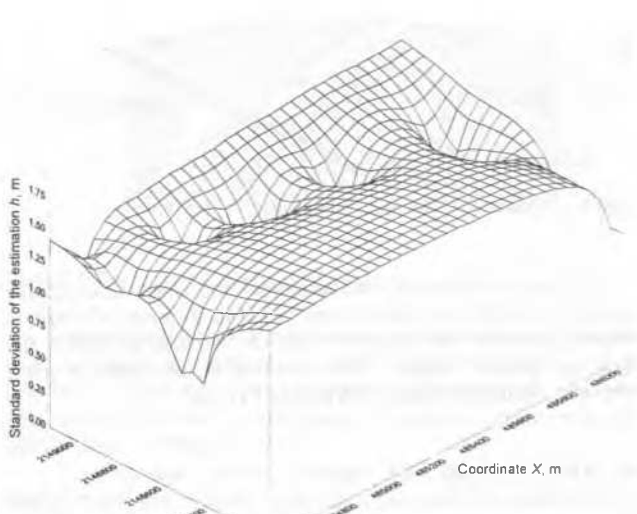

Figure 7. Representative surface of the standard deviation of the estimation.

In Figure 7, the depressions around the data show that, as was already mentioned, the uncertainty on the depth of the hard layer decreases in the area surrounding the borings

3.5 *Simulation*

Using conditional simulation, a possible realization of the field can be built taking into account the experimental data of Table 1. The image obtained coincides exactly with the known values. The results are presented in the contour map shown in Figure 8.

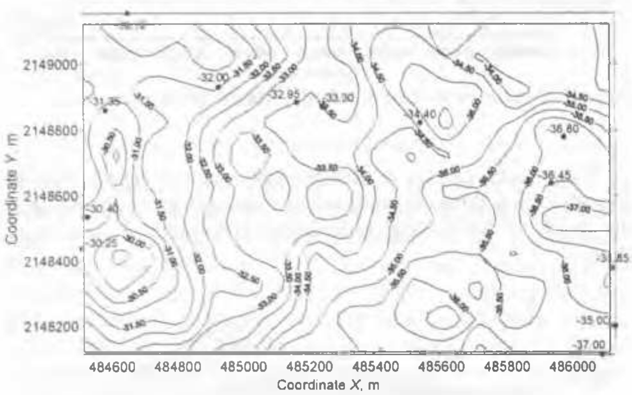

Figure 8. Contour map of the simulation of the hard layer depth

The generated contour map represents a possible configuration of the depth of the hard layer inside the field. It can be observed that the contours present a much more irregular pattern that in the case of the kriging interpolation.

In order to appreciate more clearly these variations, a three-dimensional surface such as the one presented on Figure 9 can also be built.

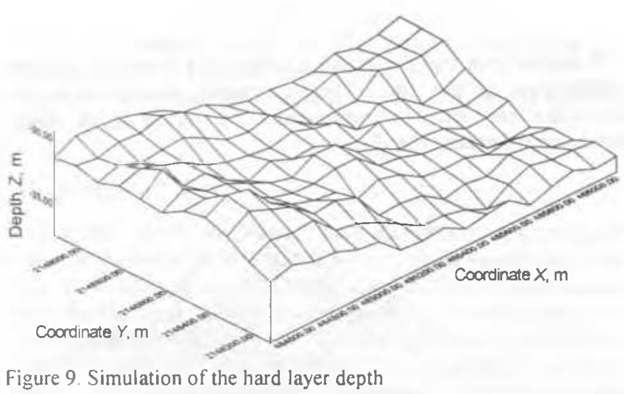

Figure 9. Simulation of the hard layer depth

Carrying out several simulations of the same field it also becomes possible to identify the possible location of extreme values, maximum or minimum, that can be reached within the field in specific points. This information can have a great relevance from the engineering point of view.

4 CONCLUSIONS

Geostatistical methods provide a rational tool for interpreting the available geotechnical information and to evaluate the space variability of the subsoil. These techniques can be used to evaluate systematically the results of soil exploration. They can be useful to eliminate a large part of the subjectivity introduced in traditional stratigraphical interpretations and will certainly be used much more frequently in the future.

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