Identification of dynamic properties of Mexico City clay in the frequency domain using downhole records

Identification des propriétés dynamiques de l'argile de la ville de Mexico dans le domaine de la fréquence en utilisant des registres d'accélération en puits

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ABSTRACT: Laboratory-determined soil dynamic properties are always (to different degrees) affected by sample disturbance, scale effects, deficient modeling of in situ conditions, and so on. The installation of vertical arrays of strong motion instruments and the ensuing records obtained during various seismic events, have opened the opportunity to explore other alternatives to evaluate soil dynamic properties by solving the inverse problem. In this paper, an analytical procedure that allows the solution of this problem in a simple way is presented and applied to a case history in Mexico City. The model assumes 1-D propagation of shear waves throughout homogeneous viscoelastic soil deposits. The results obtained here are compared with the velocities measured by means of field studies at Central de Abasto Oficinas (CAO) site with a P-S logging system. These comparisons show the potential of this procedure.

RÉSUMÉ: L´évaluation dans le laboratoire des propriétés dynamiques du sol sont toujours (a différents degrés) affectées par des troubles dans l'échantillon, effets de l'échelle, une modélisation incorrecte des conditions in situ, entre autres. L'installation d´arrangements verticaux d´accélérographes qui registrent le comportement du sol pendant plusieurs événements sismiques, a montré la possibilité d'explorer d'autres alternatives pour évaluer les propriétés dynamiques du sol en résolvant le problème inverse. Dans cette article, il est proposé une procédure simple pour résoudre ce problème et il est appliqué a un cas dans la ville de Mexico. Le modèle assume la propagation unidimensionnelle d´onde de cisaillement a travers un dépôt de sol homogène et viscoélastique. Les résultats obtenus sont comparés avec les vitesses d´onde mesurées dans le cite Central de Abasto Oficinas (CAO) avec un système P-S logging. Ces comparaisons montrent l´effectivité de cette procédure.

1 INTRODUCTION

The dynamic seismic response of a soil deposit depends on the material dynamic properties, the deposit geometry and the earthquake excitation characteristics. Herein, the response of the ground is computed using a 1-D wave propagation model and assuming a homogeneous, viscoelastic soil deposit. These approximations have been shown to yield adequate results when dealing with the deep clay deposits in Mexico City (e.g. Romo and Seed, 1986; Romo, 1995).

Regarding the evaluation of soil dynamic properties, it is common practice to resort either to laboratory or field test. As discussed in Romo et al (2000), these procedures involve uncertainties that may lead to erroneous evaluations of shear modulus and damping ratio values.

A procedure that has been gaining popularity and the approval by the profession in recent years is based on the observation of the response of sites properly instrumented. Once the ground motions are known at various points within the soil deposit for different earthquakes, the equivalent properties of the materials may be estimated by solving the inverse problem (system parameter identification). There exists a number of procedures to solve the inverse problem (e.g. Ljung, 1987).

In this paper, a simple procedure to achieve this purpose is proposed.

It basically consists in comparing the experimental and analytical responses. Once the amplitude Fourier spectrum of the response is computed, it is compared with the corresponding spectrum of the measured response. Then, an optimization procedure is activated that is based on the minimization of the overall error between both spectra. In doing this, sets of shear modulus (G) and soil damping (ξ) are varied until the minimum resulting error is obtained. The resulting values are the in situ equivalent properties of G and ξ.

Herein, this approach is applied to a case history in Mexico City (Central de Abasto Oficinas - CAO) and the results are discussed.

2 CAO SITE

This site is located according to the Mexico City geotechnical zonation within the lake zone, as indicated in Figure 1.

The vertical array at CAO site consists of a superficial accelerometer and three more located at depths of 12, 30 and 60 m, as indicated in Figure 2, where general soil-site characteristic are also given.

The horizontal distance between the surface accelerometer and the downhole accelerometers is 3 m and the horizontal distance between the downhole accelerometers (12, 30 and 60 m) is 1 m. The identification codes of these instruments according to the Mexican Database of Strong Earthquakes (Alcántara, 1997) are: CDAO, C166, C266 and C366, respectively. The downhole accelerometers are installed inside aluminium tubes of flexible walls without vertical confinement (e.g. borings were not backfilled). This arrangement has been shown to be advantageous because it allows the retrieval of the instrument for maintenance purposes. Furthermore, the aluminium casing is flexible enough to follow ground motions and preclude casing-soil interaction effects on recorded accelerations.

3 DYNAMIC RESPONSE OF CONTINUUM

The analytical model that represents the undimentional propagation of transversal waves in a viscoelastic continuos medium is:

$$G\frac{\delta^2 x}{\delta z^2} + \eta\frac{\delta^3 x}{\delta z^2 \delta t} = \rho\frac{\delta^2 x}{\delta t^2} \tag{1}$$

where G is the shear modulus [MPa], η is the viscosity coefficient [MPa*s], ρ is the medium density [kg/m^3], x is the transversal displacement [m] and z is the coordinate along the wave train propagates [m]. The solution is:

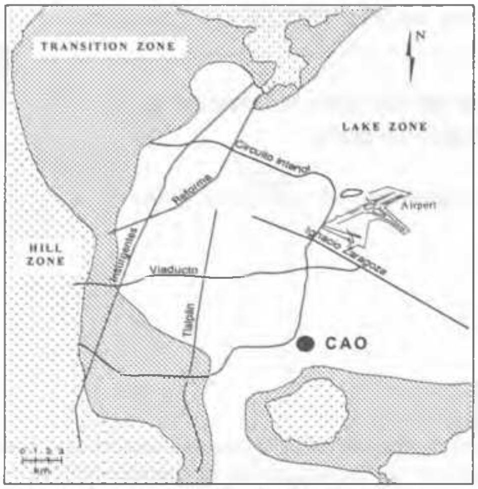

Figure 1. Mexico City geotechnical zoning.

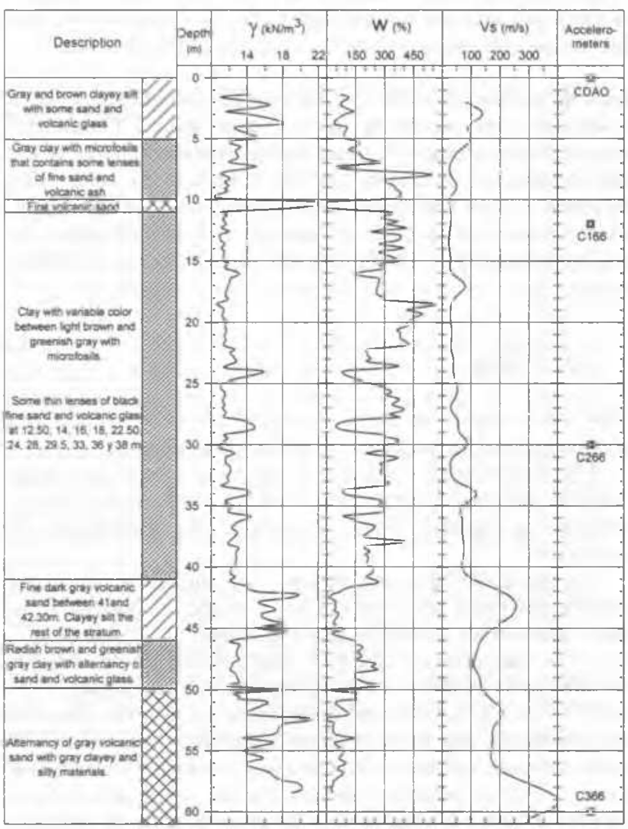

Figure 2. Profiles of stratigraphy, unit weight, γ, water content, w, shear wave velocity, Vs, and location of accelerometers at CAO site.

$$x_{(z,t)} = Ae^{i(\omega t + K^* z)} + Be^{i(\omega t - K^* z)} \qquad (2)$$

where A and B are the amplitudes of the transversal waves that propagate in the z negative and positive directions, respectively and ω is the circular frequency of the harmonic excitation. K* is the complex wave number and is defined as follows:

$$K^* = r_z^{1/2}\left[\cos(\phi_z/2) + \sin(\phi_z/2)i\right] \qquad (3)$$

where

$$r_z = \omega^2 \rho \Big/ G\sqrt{1+4\xi^2} \quad \text{and} \quad \phi_z = \tan^{-1}(-2\xi) \qquad (4)$$

The polar parameters of K * (Equation 4) have a representative variable of energy dissipation called damping ratio (ξ), which can be expressed mathematically in function of G, η and ω starting from the adopted definition of ξ in soil dynamics (Kramer, 1996), and presented in Equation 5.

$$\xi = \frac{1}{4\pi}\frac{\Delta W}{W} = \frac{\omega\eta}{2G} \qquad (5)$$

Assuming that the soil deposit is homogeneous, that the upper boundary is a free surface and the lower boundary is rigid, the solution of Equation 2 is given by:

$$x_{(z,t)} = 2A\cos(K^* z)e^{i\omega t} \qquad (6)$$

which represents a stationary transversal wave of amplitude equal to 2Acos(K*z). Equation 6 implies that points of the medium along z are not out of phase in time with respect to the excitation. This solution also establishes that the excitation must be harmonic with amplitude A, so the response will be a harmonic with amplitude 2Acos(K*z). Taking into account this, the response at a point of the medium at z=h, can be easily obtained using the transfer function, $TF_{H\rightarrow h}$:

$$x_{(z=h,t)} = (TF_{H\rightarrow h})\, Ae^{i(\omega t+\alpha)} \qquad (7)$$

$$TF_{H\rightarrow h} = \cos(K^* h) \big/ \cos(K^* H) \qquad (8)$$

where $Ae^{i(\omega t+\alpha)}$ is the excitation at a point in the medium at z=H.

In view that all data is been collected in terms of acceleration time histories, it is convenient to obtain the response in terms of accelerations. To achieve this, Equation 7 is derived twice with respect to time. Thus, the expression used to compute de accelerations at any depth and time in given by

$$\frac{\delta^2 x_{(z=h,t)}}{\delta t^2} = (TF_{H\rightarrow h})\, Be^{i(\omega t+\alpha)} \qquad (9)$$

The amplitude of the excitation in terms of the acceleration is B, which is equivalent to $-A\omega^2$. Note that $TF_{H\rightarrow h}$ is time invariant.

The process followed to evaluate the analytic response of a soil deposit is schematically indicated in Figure 3.

The first step is to assign the input motion in the time domain. Then, in step 2, this signal is mapped to the frequency domain applying the Discrete Fourier Transform, DFT. In step 3 the transfer function is computed using Equation 8, then, this is multiplied by the input motion to obtain the response in the frequency domain. Finally, step 4 performs the transformation of the response computed in step 3 to the time domain applying the Inverse Discrete Fourier Transform, IDFT.

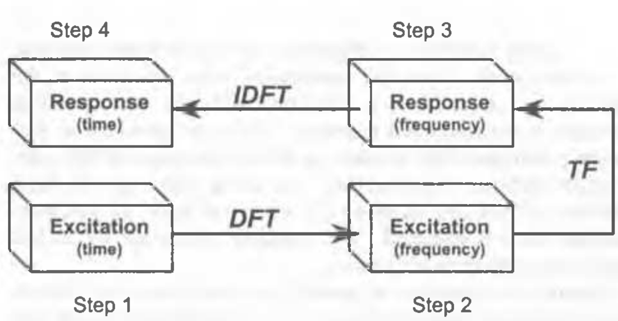

Figure 3. Process to evaluate the analytic response.

It is important to stress that the control point of the input motion not necessarily has to be assigned at the rigid boundary. It may be considered at any depth within the soil deposit.

Other transfer functions that allow to obtain the response of the soil in function of other movement can be readily computed (Carvajal, 2000). The TF that allows to obtain the response in function of the displacement at z=h based on the acceleration at z=h is:

$$TF_{Accel\mapsto Disp} = -1/\omega^2 \quad (10)$$

The TF that allows to obtain the response in function of the shear strain at z=h based on the displacement at z=h is:

$$TF_{Disp\mapsto ShearStrain} = -\tan(K^* h)\,K^* \quad (11)$$

The TF that allows to obtain the response in function of the shear stress at z=h based on the shear strain at z=h is:

$$TF_{ShearStrain\mapsto ShearStress} = G\,(1+2\xi\, i) \quad (12)$$

4 PARAMETER IDENTIFICATION SYSTEM PROPOSED

To determine the dynamic properties of the soil deposit (solving the inverse problem), a Dynamic Parameter Identification System, DPIS, was implemented.

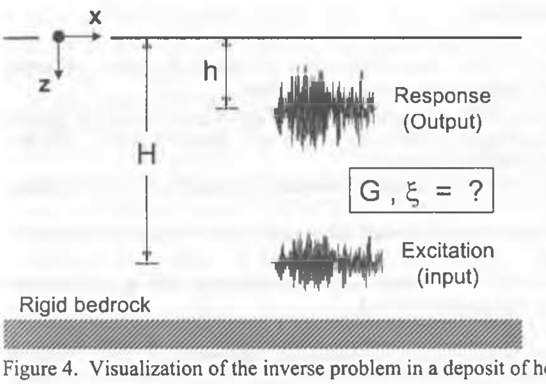

Figure 4. Visualization of the inverse problem in a deposit of homogeneous soil.

The DPIS takes the input record that corresponds to the excitation and calculates the theoretical response as indicated in Figure 3, considering a pair of values of G y ξ. Then compares the theoretical response (dashed line in the Figure 5) with the recorded response (continuous line) in the frequency domain and estimates an error based on the differences in areas (for each frequency increment, δω) between the amplitude Fourier spectra of the recorded and computed motions (shaded area), as indicated in Figure 5. The absolute value of this difference is always considered in the minimization process.

Applying this procedure, theoretical responses are calculated for G values between 0.1 and 20 MPa and for ξ varying from 0 to 30 %. The optimum dynamic parameters are those that induce the minimum error between the theoretical response and the one measured in the field. Although the soil density was considered constant in this study, it can easily be included in the optimization procedure.

The DPIS calculates the transfer function, considering G and ξ frequency-independent. It is worth mentioning that this parameter identification system needs not to determine angular strains, because they are the product of the dynamic parameters of the medium and the excitation, therefore, the system identifies the parameters based on the variables measured in the field (e.g. accelerations).

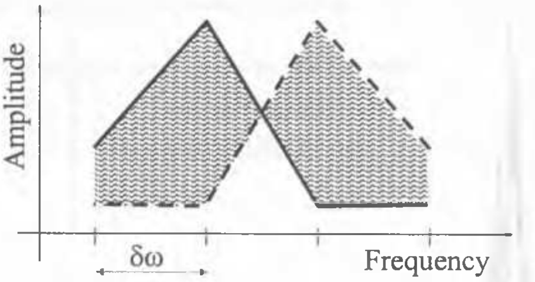

Figure 5. Error approach between the analytical response and the experimental one.

5 CAO SITE CASE HISTORY

To evaluate the dynamic properties of CAO site clays, the recorded motions by instruments C166 and C266 were processed. The acceleration histories recorded with the instrument C266 were considered as the input motion and the records of the accelerometer C166 were taken as the experimental response. Both horizontal components were considered. Seismic events included here are shown in Table 1 and the average value of the dynamic parameters identified in each component is included in Table 2. The soil unit weight considered in the analyses was the average (γ=12 kN/m^3) value obtained from the unit weight profile included in Figure 2.

Table1. Dynamic properties of the clay identified with the DPIS.

Event	Component	G (MPa)	ξ (%)
03/31/93	N00E	2.57	9.6
	N90W	3.07	8.3
09/10/93	N00E	2.60	5.4
	N90W	2.07	18.3
10/24/93	N00E	2.26	11.8
	N90W	2.57	13.5
05/23/94	N00E	3.25	5.5
	N90W	3.14	9.5
12/10/94	N00E	3.16	8.4
	N90W	2.24	11.0

Table 2. Average properties in each component.

Component	G (MPa)	ξ (%)
N00E	2.77	8.1
N90W	2.62	12.1

In Figure 6 it is illustrated by way of example the pseudo spectrum of amplitudes of the experimental response and the analytical response (calculated with the optimum set of G and ξ determined with DPIS) of the earthquake of the 10/24/93 - N00E. In this figure, it is observed that the two responses are practically the same in the whole interval of frequencies except for the high frequencies (1.7 to 2.2 Hz). This behavior could be due to the influence of the stiffer stratum lying above the C166 instrument making the homogeneous-deposit hypothesis on which the procedure in based on a bit debatable. However, due to the great contrast of stiffness between the soft clay and the stiffer strata of the deposit, it is still possible to identify the dynamic properties of the clay carrying out the analysis for the low frequency range.

In the Figure 7 the experimental $TF_{H\to h}$ is illustrated (calculated from the records at h=12 and H=30 m) and analytic $TF_{H\to h}$ (calculated with Equation 8) for the earthquake of the N00E component of the 10/24/93 event. In this figure it is appreciated that the analytic $TF_{H\to h}$ follows closely the tendency of the experimental $TF_{H\to h}$, except for the interval between 1.7 and 2.2 Hz, possibly due to the above given explanation.

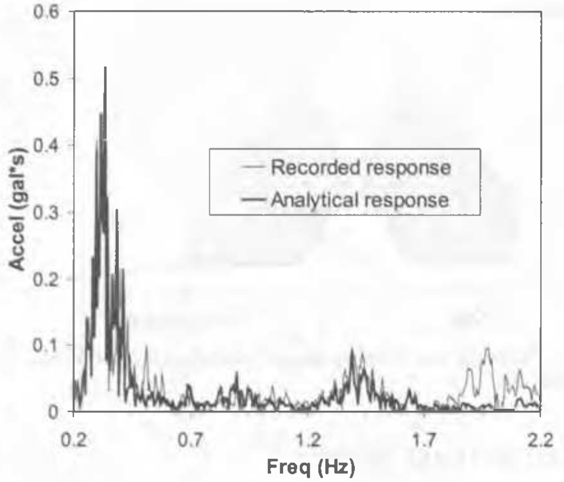

Figure 6. Comparison between experimental and analytical acceleration spectra for the 10/24/93 (N00E) earthquake.

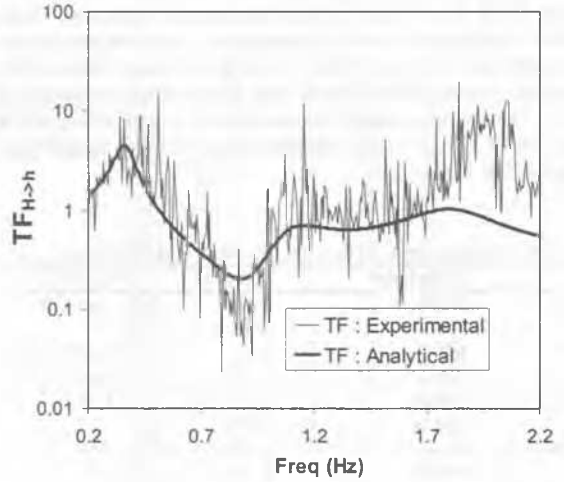

Figure 7. Experimental and analytic $TF_{H\rightarrow h}$ for the earthquake of the 10/24/93 (N00E).

The shear stress-strain curve computed with Equations 10, 11 and 12, at mid-section (h=21 m), is shown in Figure 8. The average shear strain was 1.5×10^{-2} %, far from the threshold for linear behavior ($\gamma \approx 1.0 \times 10^{-1}$ %), of Mexico City clay.

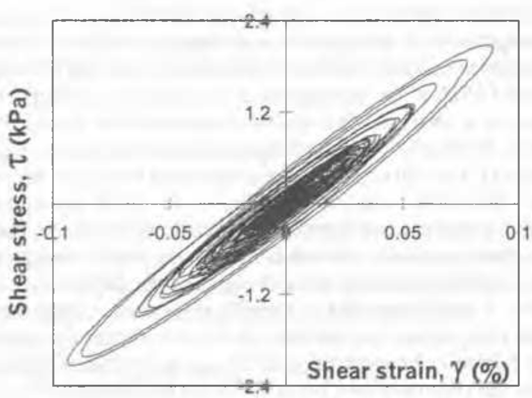

Figure 8. Shear stress strain loops at h=21 m for the earthquake of the 10/24/93 (N00E).

All earthquakes analyzed in this article were of low intensity (maximum acceleration at h=12 m ≈ 20 gals) and therefore they induced shear strains that remained within the elastic interval of the curve G-γ of the clay of Mexico city.

6 CONCLUSIONS

The average G value obtained from P-S logging testing at the site (≈2.5 MPa, using the shear wave velocity profile depicted in Figure 2) is similar to the average G value obtained by means of the DPIS (≈2.7 MPa).

The ξ values computed with DPIS are much higher than those obtained from laboratory testing. This difference may be due to a) geometric damping included in DPIS computations and b) soil-structure-interaction effects induced by nearby, relatively massif, one-story building.

Further analyses, not included here, carried out with the PDIS considering sub-intervals of frequency (Carvajal, 2000), showed that the frequency content of the earthquake may affect the equivalent dynamic properties of the clay at the CAO site. This effect shows as an increase of the value of G and a reduction in ξ, with the frequency. The value of G may be increased an 100 % and that of ξ can be decreased an 80%. In fact, the accuracy of the method is improved significantly particularly at the higher frequency range. According with these results, it seems fair to acknowledge that the procedure proposed in this paper represents a viable alternative for the estimation of soil dynamic properties.

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