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Correlation between penetration resistance and relative density of sandy soils

La corrélation entre la résistance de pénétration et la densité relative des sols sableux

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ABSTRACT: It is known that the penetration resistance depends on the grain size of soils and that fines-containing sands have smaller SPT N values than clean sands. When correlating the SPT blow count and relative density of sandy soils, it is essential to offer a proper interpretation for these effects. This paper presents an empirical correlation between N and D_r that allows for the important effects of grain-size and fines content of soils, and is therefore applicable to various kinds of sandy soils, including clean sands, sands with fines and gravelly sands. The proposed correlation is based on data of high-quality undisturbed samples secured by means of the ground freezing technique and measured SPT resistance of natural sandy soil deposits in Japan.

RÉSUMÉ: C'est bien connu que la résistance à la pénétration dépend de la granulométrie des sols et que les sables argileux ou silteux ont la valeur N des essais de Standard Pénétration plus moins que les sables propres. Quand on fait une corrélation entre la valeur N et la densité relative des sols sableux, il est essentiel de prendre en considération une interprétation propre de ces effets. Une corrélation empirique est présentée entre N et Dr qui compris les effets important de caractéristiques granulométriques et la présence de particules argileuses ou silteuses dans les sols. Par conséquent, cette corrélation est applicable aux sols sableux différents : les sables propres de même comme les sables argileux, silteux ou graveleux. La corrélation proposée est établie sur les résultats obtenus des échantillons non remaniés de haute qualité, congelés in-situ, et la résistance mesurée de Standard Pénétration des sols sables naturels de Japon.

1 INTRODUCTION

The standard penetration test (SPT) is routinely used in the geotechnical engineering practice for in situ investigation of soil deposits. Due to the longtime and worldwide use of this test, comprehensive data have been accumulated in the past resulting in a number of SPT-based empirical correlations for sandy soils. In these correlations, the SPT blow count is used as a parameter for in situ characterization of field deposits. On the other hand, extensive information on the behaviour of sandy soils has been obtained from laboratory tests on reconstituted and undisturbed samples. In these laboratory studies, the relative density D_r is commonly identified as a principal factor influencing the deformation and strength characteristics of sands. Thus, in order to establish a correspondence between the field and laboratory parameters for soil characterization, the first step would be to provide a link between the penetration resistance and relative density of sandy soils.

Based on chamber test data, Meyerhof (1957) proposed a correlation between the SPT N value and relative density D_r for clean sands. In essence, the Meyerhof's expression specifies that the ratio between the normalized SPT N value and the square of the relative density, N_1/D_r^2, is fixed at 41. It has been recognized, however, that the penetration resistance depends on the grain size of soils, and that fines-containing sands have lower SPT blow count than clean sands. Scrutiny of the correlation between penetration resistance and relative density has revealed consequently that the ratio N_1/D_r^2 depends on the grain size of sands and that this ratio gradually decreases with the decreasing mean grain size or increasing fines content of sand.

The objective of the present study is to establish an improved correlation between the SPT blow count and relative density that allows for the important effects of grain size and fines content of soils. By using SPT measurements of field deposits along with data of high-quality undisturbed samples, an empirical correlation between N and D_r is derived in the following. A key factor in the established N-D_r link is the introduction of an efficient measure for the grain-size characteristics of sandy soils, resulting in a correlation that is applicable to various kinds of sandy soils, including clean sands, sands with fines and gravelly sands.

2 BACKGROUND

According to the well known expression of Meyerhof (1957), the penetration resistance is assumed to increase with the square of the relative density and be in direct proportion to the effective overburden pressure of the sand

$$N = \left(17 + 24\frac{\sigma_v'}{98}\right)D_r^2 \qquad (1)$$

where N is the SPT blow count, σ_v' is the effective overburden pressure in kPa and D_r is the relative density expressed as a ratio, not a percentage. Skempton (1986) expressed the relation of Equation 1 in a general form as

$$N = \left(a + b\frac{\sigma_v'}{98}\right)D_r^2 \qquad (2)$$

Substituting $\sigma_v' = 98$ kPa in Equation 2, the expression is reduced to

$$\frac{N_1}{D_r^2} = a + b \qquad (3)$$

where N_1 is the normalized penetration resistance to an overburden pressure of 98 kPa, i.e. 1 kgf/cm². Note that in the original definition of Meyerhof the ratio N_1/D_r^2 or the parameter $(a + b)$ was assumed to take a value of 41.

Tatsuoka et al. (1978) examined the accuracy of the original Meyerhof's expression by using results of SPT on
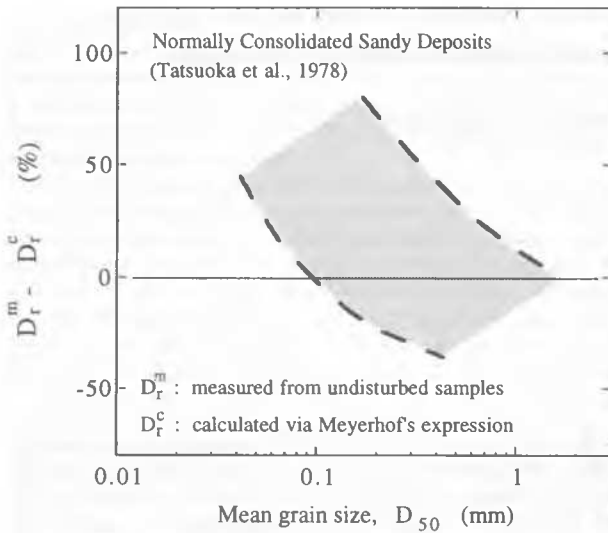
Figure 1. Deviation of measured relative densities from those calculated according to the Meyerhof's expression

Figure 2. Variation of void ratio range with mean grain size of soils

normally consolidated sandy deposits along with in situ relative densities of conventional undisturbed samples. It was found that the correlation of Equation 1 or using $(a + b) = 41$ tends to underestimate the relative density of fine sands and silty sands as indicated in Figure 1 where the difference in relative density between the one measured on undisturbed samples and that estimated via Equation 1 is plotted versus the mean grain size D_{50}. In essence, the relation displayed in Figure 1 shows that the parameter $(a + b)$ tends to decrease with decreasing mean grain size of sands, and that accordingly the ratio N_1 / D_r^2 of fines-containing sands is smaller than that of clean sands. Similar conclusions were reached in more recent studies by Skempton (1986) and Ishihara (1993).

3 VOID RATIO RANGE

The scatter of the data in Figure 1 is somewhat too large for a reasonably accurate evaluation of the relative density to be made based on D_{50}. Thus, more appropriate measure for the grading properties of sandy soils is needed so as to quantify the grain-size effects in the correlation between the relative density and penetration resistance. When evaluating comparatively grading properties of various sandy soils however, one encounters difficulties in identifying an appropriate measure that is indicative of the overall grading characteristics of a given soil. In this context, the conventional parameters readily available from gradation curves, such as the mean grain size D_{50}, uniformity coefficient U_C and fines content F_C, are deficient in a sense that each of them describes only a single grading feature. With this in mind, the void ratio range $(e_{max} - e_{min})$ may be suggested alternatively as one of the measures for identifying the grain-size characteristics of sandy soils, where $(e_{max} - e_{min})$ represents the difference in the void ratio between the loosest and densest packing states achieved in standard laboratory procedures. It is important to recall that the density of packing or volume of voids in the soil skeleton is related to the grain-size distribution and fines content of soils, and therefore, the grading properties of a given sandy soil are reflected in the values of the extreme void ratios (e_{max}, e_{min}) and void ratio range $(e_{max} - e_{min})$ of the soil. Even though it is an indirect measure of the gradation, it was found that the void ratio range is a very good indicator of the overall grain-size characteristics of sandy soils (Cubrinovski & Ishihara 1999, 2000). This is amply illustrated in Figure 2 where a correlation
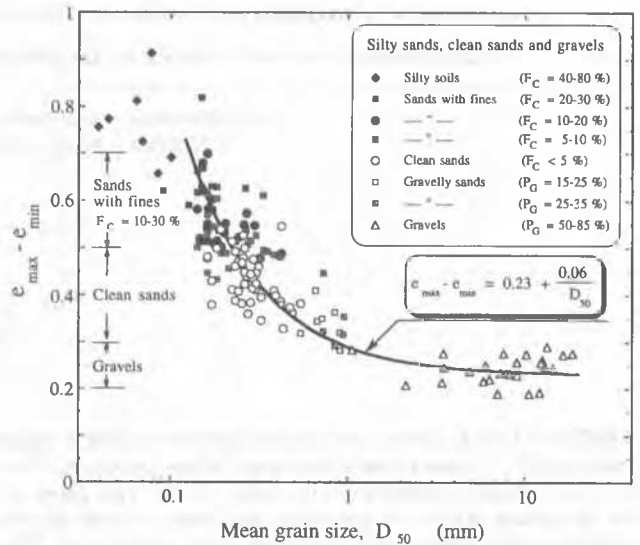
between the void ratio range and mean grain size of various cohesionless soils is shown. The main features of the void ratio range may be summarized as follows: (a) in general, $(e_{max} - e_{min})$ has values in the range between 0.25 and 0.65 for sands with less than 30 % fines; (b) the value of $(e_{max} - e_{min})$ gradually increases with decreasing mean grain size or increasing fines content, and (c) for a given mean grain size, $(e_{max} - e_{min})$ is larger for sands containing more fines.

4 PROPOSED CORRELATION BETWEEN N AND D_r

4.1 Generalized Expression

On the basis of the expression of Meyerhof and existing evidence for the effects of grain size on the penetration resistance of sandy soils, it can be assumed that the SPT blow count is controlled primarily by the relative density, effective overburden stress σ'_v and grain-size composition of soils, which may be expressed as

$$N = \frac{C_D}{C_N} D_r^2 \qquad (4)$$

where C_N is a function of σ'_v, whereas C_D is a parameter representing the grain-size effects and is therefore a function of $(e_{max} - e_{min})$. The C_N function proposed by Liao and Whitman (1986) was adopted:

$$C_N = \left(\frac{98}{\sigma'_v}\right)^{1/2} \qquad (5)$$

where σ'_v is given in kPa. This function is actually used to normalize a given N value to the blow count corresponding to an overburden pressure of 98 kPa $(N_1 = C_N N)$, and therefore, Equation 4 may be also written as

$$\frac{N_1}{D_r^2} = C_D \qquad (6)$$

Thus, it is in fact postulated that the ratio N_1/D_r^2 depends only on the grain-size properties or $(e_{max} - e_{min})$ value of soils.

4.2 Empirical Correlation

Data of undisturbed samples recovered from field deposits with known SPT N values were compiled and used to examine the
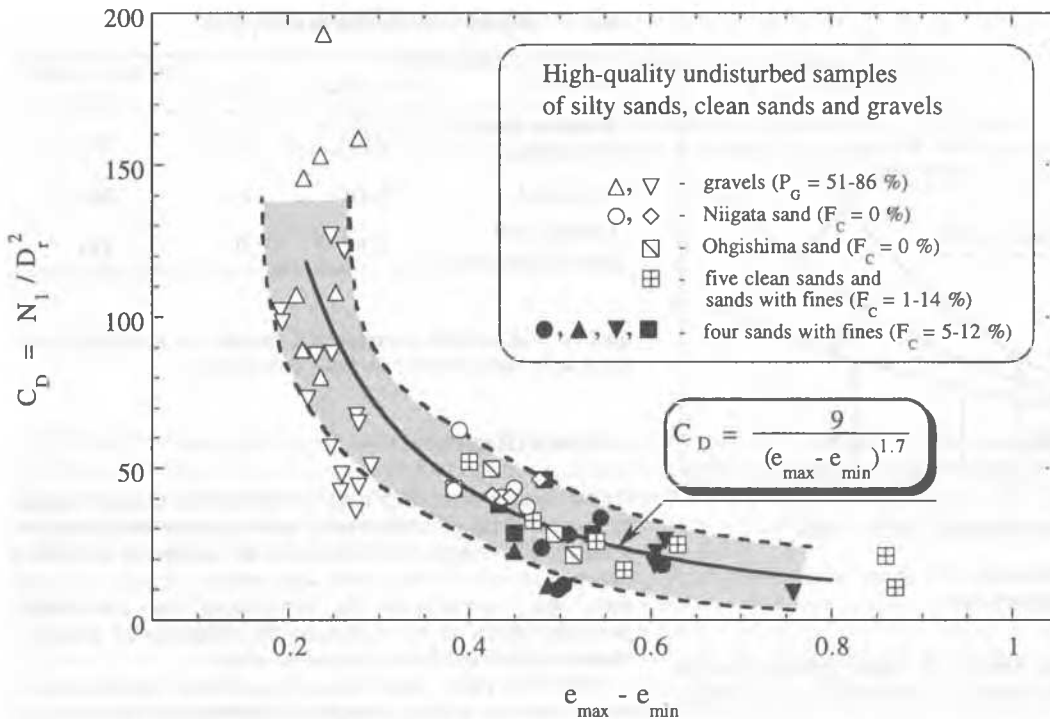
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Figure 3. Correlation between N_1 / D_r^2 and void ratio range

link between the ratio N_1 / D_r^2 and void ratio range. Due to a high sensitivity of the relative density on sample disturbances, only high-quality undisturbed samples mainly secured by means of the ground freezing technique were used in the development of the correlation. Various cohesionless soils ranging from silty sands to gravels have been considered in the investigation as indicated by the $(e_{max} - e_{min})$ values of the examined soils which were anywhere between 0.20 and 0.85. For each undisturbed sample, the limiting void ratios (e_{max}, e_{min}) and relative density were evaluated by standard laboratory test procedures while the N_1 value corresponding to the undisturbed sample was calculated from the known SPT blow count and sampling depth. Eventually, for each sample the ratio N_1 / D_r^2 was calculated and plotted against the $(e_{max} - e_{min})$ value of the soil, as shown in Figure 3. It is evident that the ratio $N_1 / D_r^2 = C_D$ decreases with increasing void ratio range from a value of about 100 for gravels to a value of about 10 for soils with $(e_{max} - e_{min})$ value of approximately 0.80. The correlation shown in Figure 3 may be approximated as

$$C_D = \frac{9}{(e_{max} - e_{min})^{1.7}} \qquad (7)$$

Now, introducing Equations 5 and 7 into Equation 4, the following empirical correlation between the SPT N value and D_r is derived

$$N = \frac{9 D_r^2}{(e_{max} - e_{min})^{1.7}} \left(\frac{\sigma_v'}{98}\right)^{1/2} \qquad (8)$$

where D_r is defined as a ratio and σ_v' is given in kPa. It is important to note that, in this expression, the SPT blow count corresponds to an energy rod ratio of about 78 % from the theoretical free-fall energy, and that in the case of gravels, the N value represents the blow count of large penetration test (LPT). Solving for D_r, Equation 8 may be also written as

$$D_r = \left\{ \frac{N(e_{max} - e_{min})^{1.7}}{9} \left(\frac{98}{\sigma_v'}\right)^{1/2} \right\}^{1/2} \qquad (9)$$

The proposed empirical correlation is considered to provide a rational link between the penetration resistance and relative density for a wide range of cohesionless soils, encompassing soils from silty sands to gravels. It may be seen in Figure 3 that the scatter of the data for gravels is larger than that of sands indicating that lower level of accuracy should be expected when the correlation is applied to gravels. A detailed evaluation of the accuracy of the correlation may be found in Cubrinovski & Ishihara (1999) where relevant comparisons with SPT-based liquefaction criteria derived from case histories are also given.

4.3 Illustrative Example

In order to illustrate how the effects of different grading properties are manifested in the proposed $N - D_r$ correlation, compared in the following are three sandy soils with a void ratio range of 0.30, 0.41 and 0.625 respectively. As shown in Figure 2, the $(e_{max} - e_{min})$ value of 0.30 is typical of coarse clean sands and gravelly sands, $(e_{max} - e_{min}) = 0.41$ is a representative value for medium fine clean sands while $(e_{max} - e_{min}) = 0.625$ is typical of sands with approximately 20 % fines. Assuming $\sigma_v' = 98$ kPa and entering Equation 8, for each soil or $(e_{max} - e_{min})$ value as above, a relationship between the normalized SPT blow count and relative density was defined, as shown in Figure 4. In this figure, the C_D value for each soil, as calculated by Equation 7, also is indicated. It is evident in Figure 4 that the soils have remarkably different $N_1 - D_r$ relationships. For example, if a normalized SPT blow count of $N_1 = 10$ has been measured in the field, then this will indicate a relative density of about 38 % for the gravelly sand deposit while the same blow count in the deposit of sand with 20 % fines will indicate a relative density as high as 71 %. In the case of the clean sand deposit, the corresponding relative density takes an intermediate value of around 49 %. The comparison of the soils as above is summarized in Table 1.

These comparisons clearly demonstrate the importance of the grain-size characteristics on the $N - D_r$ relationship of sandy soils. It is interesting to note that the original expression of Meyerhof, for which $C_D = 41$, provides a reasonable estimate for clean medium fine sands, whereas it
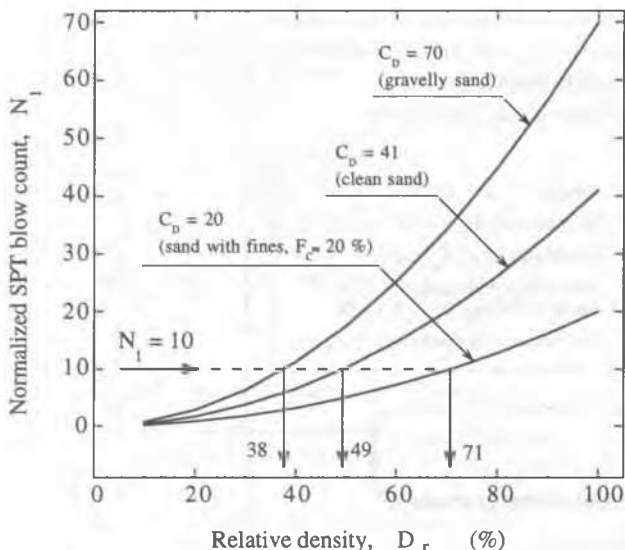
Figure 4. $N_I - D_r$ relationships for three sandy soils with different grain-size characteristics

Table 1. Computed relative densities for $N_I = 10$

Soil type	$(e_{max} - e_{min})$	C_D	D_r (for $N_I = 10$) (%)
Sand with fines ($F_C \approx 20\%$)	0.625	20	70.7
Clean sand	0.410	41	49.4
Gravelly sand (coarse clean sand)	0.300	70	37.8

quality undisturbed samples are needed to scrutinize more rigorously these effects in field conditions.

overestimates the relative density of coarse (gravelly) sands, and conversely, it underestimates the relative density of fines-containing sands.

4.4 Effects of Lateral Stress

The data used in developing the correlation of Figure 3 are from natural deposits, and hence, it may be considered that the above correlation is directly applicable to normally consolidated soils. To incorporate the effects of a general lateral stress state on the $N - D_r$ relationship, results of a recent study by Salgado et al. (1997) concerning the influence of lateral stress on the CPT (cone penetration test) resistance were used as described below. Here, the key assumption is that the effects of lateral stress on the SPT resistance are equivalent to those of the CPT resistance. Following the suggestion of Salgado et al. (1997), the penetration resistance for a general lateral stress state N_K can be defined as

$$N_K = \frac{N}{C_K} \qquad (10)$$

where N is the SPT blow count for a normally consolidated deposit corresponding to that of Equation 8, and C_K is a function of the lateral stress defined as

$$C_K = \left(\frac{K_0}{K}\right)^m \qquad (11)$$

Here, $K = \sigma'_h / \sigma'_v$, σ'_h is the effective lateral stress, K_0 represents the K value for a normally consolidated soil while m is an exponent which increases with D_r. Typical values of the exponent are $m = 0.5$ for $D_r = 50\%$ and $m = 0.3$ for $D_r = 90\%$ respectively. Introducing Equations 10 and 11 into Equation 8 results in the following expression for the SPT blow count which allows for the effects of lateral stress:

$$N_K = \frac{9 D_r^2}{(e_{max} - e_{min})^{1.7}} \left(\frac{\sigma'_v}{98}\right)^{1/2} \left(\frac{K}{K_0}\right)^m \qquad (12)$$

Whereas results of chamber test data support the modelling of lateral stress effects as above (Salgado et al. 1997, Skempton1986), direct field evidence, including data of high-

5 CONCLUSIONS

Based on data from high-quality undisturbed samples secured by means of the ground freezing technique and measured SPT resistance of natural soil deposits, an empirical correlation between the SPT blow count and relative density of sandy soils was established. In developing the correlation, particular attention was given to the influence of grain-size characteristics and fines content of soils.

The void ratio range $(e_{max} - e_{min})$ was employed as a measure for the grading properties of sandy soils and was used for quantifying the grain-size effects in the $N - D_r$ correlation. It was found that the ratio N_I / D_r^2 steadily decreases with the increasing void ratio range of soils, from a value of about 100 for gravels to a value of around 10 for fines-containing sands. This dependency was formulated into a simple $N - D_r$ relationship that is applicable to a wide range of cohesionless soils, encompassing soils from silty sands to gravels. It is important to note that in the proposed correlation, the SPT blow count corresponds to an energy rod ratio of 78 % from the theoretical free-fall energy and that LPT resistance is used for gravels.

Recent solutions suggested for the cone penetration resistance were used to tentatively introduce the effects of a general lateral stress state in the proposed $N - D_r$ relationship.

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