Interpretation of pressuremeter tests using non-linear elasto-plastic analysis

Interprétation de l'essai pressiométrique par analyse non linéaire élasto-plastique

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ABSTRACT: This paper presents an interpretation of self-boring pressuremeter test based the theoretical analyses of undrained cavity expansion in elasto-plastic soil with non-linear elasticity. The non-linear elasticity is represented by a power law function and a hyperbolic equation. Yielding of the soil is described by the modified Cam clay model. The theory was used to analyze a pressuremeter test in Singapore marine clay. Both the power function and hyperbolic equation are found to be suitable for deriving the non-linear elastic stiffness parameters form the reloading curves. Although the difference in the estimated in-situ effective horizontal stress based on linear and non-linear analyses is not very significant, conventional linear elastic analysis significantly overestimates the shear modulus at the onset of yielding and underestimates the corresponding shear strain. The overconsolidation ratio estimated from the pressuremeter test results is comparable to laboratory oedometer test results.

RÉSUMÉ: Cet article présente une interprétation de l'essai au pressiomètre auto-foreur sur la base d'analyses théoriques d'expansion de cavité en conditions non-drainées dans un sol élasto-plastique présentant une élasticité non-linéaire. L'élasticité non-linéaire est représentée par une loi puissance et une loi hyperbolique. L'écoulement plastique du sol est décrit par le modèle Cam-clay modifié. La théorie a été utilisée pour analyser un essai pressiométrique dans l'argile marine de Singapour. Les deux equations se sont avérées satisfaisantes pour dériver les paramètres de rigidité des lois d'élasticité non-linéaire à partir des courbes de rechargement. Bien que la différence entre les contraintes horizontales effective in situ estimées à partir des analyses linéaire et non-linéaire ne soit pas très importante, l'analyse conventionnelle linéaire élastique surestime le module de cisaillement de manière significative à l'amorce de l'é coulement plastique et sous-estime la déformation de cisaillement correspondante. Le rapport de surconsolidation estimé à partir de l'essai pressiométrique se compare bien à celui qui peut être déduit des essais oedométriques de laboratoire.

1 INTRODUCTION

Pressuremeter testing has advanced considerably since its introduction by Menard in 1956. Reports on this development were given by Mair & Wood (1987) and Clarke (1995) among many others. Existing pressuremeter testing can be divided into three main groups: prebored, self-boring and full-displacement types. The variation is mainly on the way the pressuremeter cylinder or probe is installed. The classic prebored pressuremeter test is performed in a predrilled hole. The self-boring pressuremeter test (SBPT) adopts a self-boring technique in the installation of the probe in an attempt to minimize soil disturbance. In the full-displacement test, the probe is pushed into the ground, resulting in large but consistent soil disturbance. Different interpretation methods have been developed to interpret these test results. The results of prebored pressuremeter test are often correlated empirically with specific design parameters in practice, although cavity expansion theory has been considered. The self-boring pressuremeter test and the full-displacement pressuremeter test are generally analyzed theoretically using the cavity expansion theory.

Solutions to the expansion of spherical/cylindrical cavity in elasto-plastic materials have been presented by numerous researchers. In most of these analyses, the elastic response was assumed linear. Wood (1990) discussed the importance of non-linear elasticity in the interpretation of pressuremeter test data. Recently, Bolton & Whittle (1999) provided a solution to the undrained expansion of a cylindrical cavity in a non-linear elasto-perfectly plastic material using a power law relation to describe the gradual reduction of soil stiffness with shear strain. Cao et al. (2001) presented a solution of undrained cavity expansion in an elasto-plastic soil with non-linear elasticity. The gradual reduction of soil stiffness with strain in the elastic stage was represented by a power law equation and also a hyperbolic equation. Yielding behaviour of the soil was described by the modified Cam clay (MCC) model. This paper describes the application of the theoretical solution to the interpretation of SBPT in the Singapore marine clay.

2 PROPOSED METHOD

In the theoretical analysis, soil disturbance due to installation is ignored. The length/diameter ratio of the probe is assumed to be sufficiently large so that the measurements taken at the midsection can be reasonably interpreted by the cylindrical cavity expansion theory. Derivations of the solutions of the undrained cylindrical cavity expansion in linear and non-linear elasto-plastic materials were reported by Cao (1997) and Cao (2001), respectively. The results are summarized briefly here.

Adopting the generally accepted convention, the shear stress τ and shear strain γ are defined as:

$$\tau = \frac{\sigma'_r - \sigma'_\theta}{2} = \frac{\sigma_r - \sigma_\theta}{2} \tag{1}$$

$$\gamma = \varepsilon_r - \varepsilon_\theta = 2\ln\left(\frac{a}{a_o}\right) \tag{2}$$

where σ_r and σ_θ are the total radial and circumferential stresses; the primes denote the effective stress; ε_r is the radial strain and ε_θ is the circumferential strain; a and a_o are the current and initial cavity radii, respectively.

The variation of shear stress with the shear strain is expressed using a power law function as follows:

$$(\tau - \tau_i) = G_l(\gamma - \gamma_i)^\beta \tag{3}$$

where G_l and β are the non-linear elastic parameters with $0 < \beta \leq 1$. The initial shear strain and shear stress are represented by γ_i and τ_i, respectively. For expanded cavity in an initially isotropic soil, γ_i and τ_i are 0. For cavity contraction or re-expansion from finite radius, γ_i and τ_i are those corresponding to the appropriate starting conditions.

If the hyperbolic equation is used, the relationship between τ and γ may be expressed as:

$$(\tau - \tau_i) = \frac{(\gamma - \gamma_i)}{1/G_i + (\gamma - \gamma_i)/\tau_r} \tag{4}$$

where G_i is the initial shear modulus and τ_r is a reference shear stress at infinite strain.

2.1 *Elastic analysis*

Based on the power law function, the solution for the cylindrical cavity pressure σ_a can be expressed as:

$$\sigma_a = \sigma_{ho} + \frac{1}{\beta} G_l \gamma^{\beta} \tag{5}$$

where σ_{ho} is the in-situ horizontal stress and γ is related to the cavity radius through equation (2). For cavity contraction or re-expansion, equation (5) can be expressed as:

$$\ln(\sigma_a - \sigma_{ai}) = \ln\left(\frac{G_l}{\beta}\right) + \beta \ln(\gamma_a - \gamma_{ai}) \tag{6}$$

where σ_{ai} and γ_{ai} are the cavity pressure and the cavity shear strain prior to contraction or re-expansion, respectively. The pore pressure at the cylindrical cavity wall is

$$u_a = u_o + G_l \gamma^{\beta}\left(\frac{1}{\beta} - 1\right) \tag{7}$$

where u_o is the in-situ pore water pressure.

Based on the hyperbolic equation, σ_a is

$$\sigma_a = \sigma_{ho} + \tau_r \ln\left(1 + \frac{G_i \gamma}{\tau_r}\right) \tag{8}$$

For cavity contraction or re-expansion, equation (8) can be written as:

$$\frac{d(\gamma_a - \gamma_{ai})}{d(\sigma_a - \sigma_{ai})} = \frac{1}{G_i} + \frac{(\gamma_a - \gamma_{ai})}{\tau_r} \tag{9}$$

The pore pressure at the cylindrical cavity wall is

$$u_a = u_o + \frac{\tau_r}{1 - 0.5\tau_r / G_i} \ln\left(\frac{1 + G_i \gamma / \tau_r}{1 + 0.5\gamma}\right) - \frac{\gamma}{1/G_i + \gamma / \tau_r} \tag{10}$$

Equations (5) to (10) are derived on small strain assumptions. Cao et al. (2001) have shown that the difference between small and large strain solutions is small.

2.2 *Plastic analysis*

After the initial yielding at the cavity wall, the cavity pressure can be approximately expressed as:

$$\sigma_a = \sigma_{ap} + s_u \ln\left[I_r\left(\frac{a^{m+1} - a_o^{m+1}}{a^{m+1}}\right)\right] \tag{11}$$

where s_u is the undrained shear strength at the critical state; I_r is the rigidity index. If the power law function is used,

$$\sigma_{ap} = \sigma_{ho} + \frac{s_u}{\beta} \tag{12}$$

If the hyperbolic equation is adopted,

$$\sigma_{ap} = \sigma_{ho} - \tau_r \ln\left(1 - \frac{s_u}{\tau_r}\right) \tag{13}$$

Equation (11) is the approximate closed-form solution for a non-linear elasto-plastic material. When β = 1 or $\tau_r \to \infty$, equation (11) reduced to is the closed-form solution for a linear elasto-perfectly plastic material.

At large strain, the effective cavity pressure (σ_a - u_a) can be expressed as

$$\sigma'_a = s_u + \sigma'_{vo}\left(\frac{OCR}{2}\right)^{\Lambda} \tag{14}$$

where σ'_{vo} is the in-situ effective vertical stress; *OCR* is the overconsolidation ratio; *Λ* is the plastic volumetric strain ratio.

3 INTERPRETATION OF TESTS IN SINGAPORE MARINE CLAY

Results from SBPT using a Cambridge self-boring pressuremeter with 6 strain arms were interpreted using the theoretical derivation. The test was conducted from an offshore jack-up platform in shallow sea at a reclamation site in the eastern part of Singapore. The mean sea level was 1.6 mCD (Chart Datum) and the elevation of seabed at the test location was at about -3 mCD. Site investigation revealed that the seabed was underlain by the Kallang Formation, a recent deposit of Singapore comprising predominately soft to firm Singapore marine clay. The marine clay deposit consists of two distinct layers separated by an intermediate layer of sand and/or reddish stiff silty clays. The upper layer is known locally as the upper marine clay (UMC) and the lower layer as the lower marine clay (LMC). The total thickness of the marine clay can vary from several metres to 50 metres across the site. The marine clay is underlain by the Old Alluvium of Singapore comprising dense, weakly cemented silty to clayey sand. The properties of the Singapore marine clay are summarized in Table 1.

Figure 1 shows a typical SBPT conducted in the lower marine clay at a depth of 19.3 m below the original seabed. Excess pore water pressure Δu_i was recorded during installation of the pressuremeter to the test depth. Although a pause of between 10 to 30 minutes was allowed for Δu_i to stabilize, a waiting period of 30 minutes was found not enough for Δu_i to dissipate fully in the Singapore marine clay. In this test, the total water pressure u_i prior to pressuremeter expansion was 415 kPa, The hydrostatic water pressure u_o calculated from the static water table was 239 kPa, giving a Δu_i of 176 kPa.

Figure 1 also shows that σ'_a which is a function of s_u and *OCR* became constant when $a/a_o > 1.03$.

From equation (6), the non-linear elastic parameters G_l and β can be determined from the unloading or reloading curve,

Table 1. Properties of Singapore marine clay at the site

Clay Layer	Bulk density (Mg/m³)	Water content (%)	Plastic limit	Liquid limit	OCR	Friction angle (°)
UMC	1.5 - 1.7	60 - 80	20 - 30	70 - 85	1 - 10	19 -25
LMC	1.6 - 1.7	45 - 65	20 - 30	70 - 85	1 - 2	19 - 25

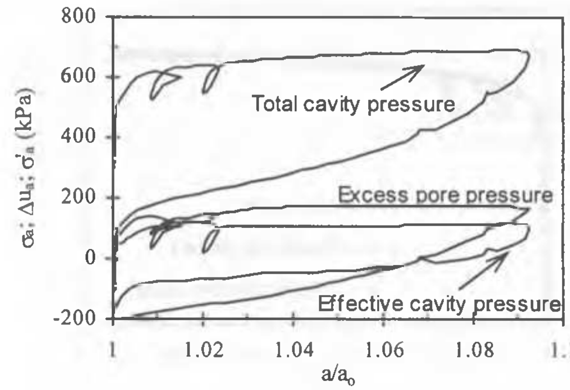

Figure 1. A typical self-boring pressure test results in the Singapore marine clay

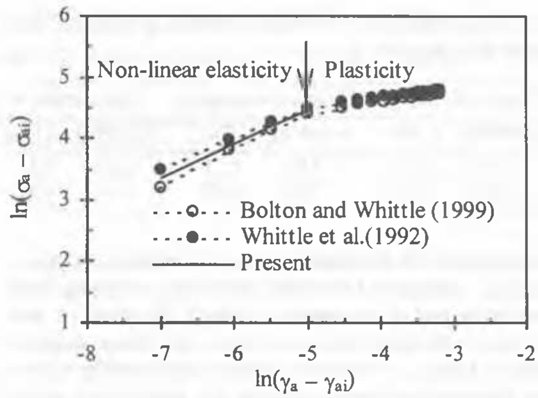

Figure 2. Deriving the power law relationship from a reloading loop

whereas G_i and τ_r may be estimated from small increments of $(\sigma_a - \sigma_{ai})$ and $(\gamma_a - \gamma_{ai})$ using equation (9). It is well known that the unloading curve from SBPT is affected more significantly by creep and consolidation in a clayey layer when compared to the reloading curve. However, some uncertainties remained in the selection of the origin for the reloading curve from SBPT data. Whittle et al. (1992) suggested using the bottom left corner of the unloading/reloading loop as the strain origin and the point where the pressure is the minimum in the loop as the pressure origin. Bolton and Whittle (1999) suggested taking the reversal point in the unloading/reloading loop as the origin. Adopting the power law function with the method suggested by Whittle et al. (1992) may overestimate the stiffness. Combining power law non-linearity with Bolton & Whittle's (1999) method, on the other hand, may underestimate the stiffness. Figure 2 shows the derived stress-displacement curves for the first reloading curve using the two proposed methods. Better results are obtained by locating the origin at an intermediate location between the point suggested by Whittle et al (1992) and the one by Bolton & Whittle (1999). As shown in Figure 2, the shape of the $\ln(\gamma_a - \gamma_{ai})$:$\ln(\sigma_a - \sigma_{ai})$ curve changes significantly when the soil at the cavity wall turns plastic. Similar results can also be found in the use of the hyperbolic equation. If the small increments of $(\sigma_a - \sigma_{ai})$ and $(\gamma_a - \gamma_{ai})$ of the hyperbolic equation is used, the choice of the origin does not significantly affect the stiffness parameters.

Figure 3 shows the determination of non-linear stiffness parameters by using the power law function and the hyperbolic equation. Both models work well. The measured points in the two reloading curves from SBPT practically fall onto the same straight lines for both cases. The non-linearly elastic stiffness parameters obtained are: G_l = 1.0 MPa, β = 0.60, G_i = 29 MPa and τ_r = 52 kPa. The two reloading curves in terms of total stresses give essentially the same stiffness parameters, implying that the soil stiffness parameters do not change during an undrained pressuremeter test. Thus the non-linearly elastic stiffness parameters derived from the pressuremeter test in clay can be considered as representative of those in-situ.

From the pressuremeter expansion curve at large strain, an undrained shear strength s_u of 39.3 kPa is obtained for the Singapore lower marine clay, as illustrated in Figure 4. The undrained shear strength can be considered as the yielding stress for the lightly overconsolidated soil with an *OCR* of between 1.2 to 2.5. It should be noted that s_u is independent of the elastic parameters. Both the linearly and non-linearly elastic analyses give the same value of s_u at the critical state.

Once s_u is obtained, *OCR* can be estimated from equation (14). The effective cavity pressure is estimated to be 112 kPa from Figure 1. Laboratory test indicated that Λ = 0.89 and σ'_{vo} = 117 kPa. The computed *OCR* is 1.17, compared with a value of 1.2 from the laboratory oedometer test.

Using the derived s_u and the non-linearly elastic stiffness parameters, the modulus and the shear strain at the onset of yield-

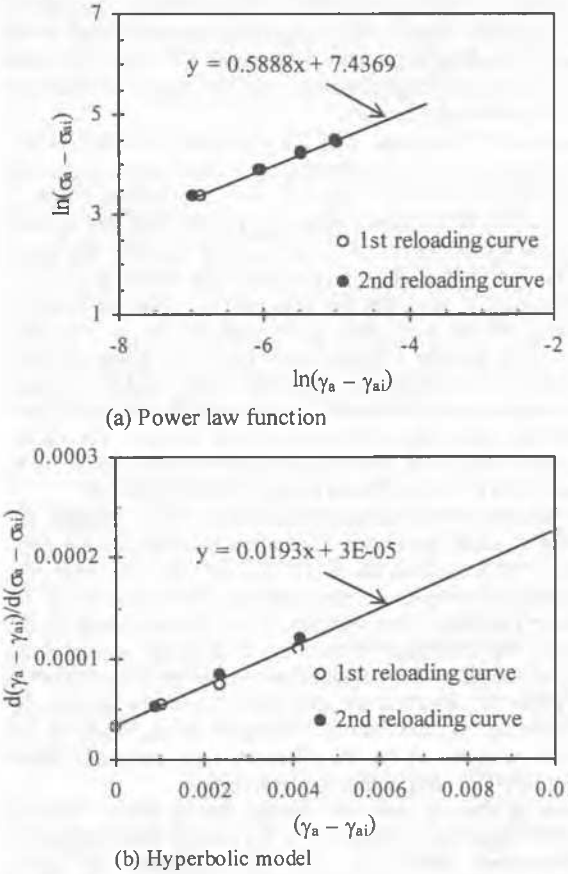

Figure 3. Determination of non-linearly elastic parameters from the reloading loops

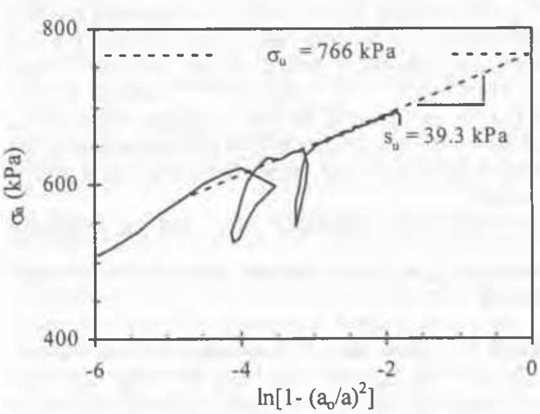

Figure 4. Determination of undrained shear strength and ultimate pressure from the pressuremeter expansion curve

Table 2. Stiffness parameters at the onset of yielding as estimated from non-linear and linear analyses

Loop no.	Power law function		Hyperbolic equation		Linear analysis	
	G_p (MPa)	γ_p (%)	G_p (MPa)	γ_p (%)	G_p (MPa)	γ_p (%)
1	9.58	0.41	7.10	0.55	12.5	0.31
2	9.58	0.41	7.10	0.55	12.2	0.32

ing can be calculated. In the traditional linearly elastic analysis, the modulus G_{ur} determined from the unloading-reloading loop is considered to be that at the onset of yielding. The modulus and the shear strain calculated from non-linear and linear analyses are provided in Table 2. The linear analysis significantly overestimates the modulus and underestimates the shear strain at the onset of yielding. The modulus interpreted using the hyperbolic equation is smaller than one obtained using the power law function. The error in estimating small increment of (σ_a - σ_{ai}) and (γ_a - γ_{ai}), especially when there are only few measured points in the unloading/reloading loops, may be a cause. With more measured points in the unloading/reloading loop, the degree of accuracy can be improved significantly.

Equation (11) indicates that the ultimate pressuremeter expansion pressure σ_u can be obtained by extending the expansion curve to $a/a_o \rightarrow \infty$, or $\ln[1 - (a_o/a)^2] \rightarrow 0$, as shown in Figure 4. Finally, the in-situ horizontal stress σ_{ho} can be estimated by substituting the known values of s_u, σ_u, G_p, β, or G_i and τ into equation (11). The various derived parameters are shown in Table 3. It is interesting to note that the values of σ_{ho} obtained from the non-linear and the traditional linear analyses fall in the same range. This is because a bigger radial stress and a smaller shear modulus at the elastic-plastic boundary were used in the non-linear elastic analysis, whereas a smaller radial stress and a bigger modulus were used in the linear elastic analysis. The values of σ_{ho} estimated by the two proposed non-linear analysis methods were close to that estimated using the lift-off method.

The deduced lift-off pressure is affected by any residual excess pore pressure generated during the insertion of the pressuremeter that remains at the start of the test, Δu_i. The value of σ_{ho} obtained by non-linear or linear analysis is also affected by Δu_i because σ_u estimated from total stress includes the effect of Δu_i. In this test, the difference between the in-situ total vertical stress and σ_{ho} estimated by the proposed method or the lift-off method is larger than $2s_u$, which is not admissible. Therefore, without the information on Δu_i, the correct estimation of σ_{ho} based on the total stress analysis or the lift-off method is impossible, especially for SBPT in soils with low permeability.

Assuming that Δu_i does not change during SBPT, which is reasonable where the permeability is very small, the effective in-situ horizontal stress σ'_{ho} can be estimated by using $\sigma'_{ho} = \sigma_{ho} - \Delta u_i - u_o$. The value of σ'_{ho} and the corresponding earth pressure at rest K_o are shown in Table. 3. The estimated value of K_o ranges from 0.56 to 0.74 from various methods. Results of K_o-consolidated triaxial tests on undisturbed samples recovered at the test site showed that K_o was between 0.6 and 0.7. By substituting laboratory determined the effective friction angle ϕ' and OCR into $K_o = (1 - \sin\phi')OCR^{\sin\phi'}$ (Mayne & Kulhawy 1982), K_o is estimated to be 0.65. It appears that the K_o values obtained by the non-linear analysis are comparable to the most probable in-situ value. The linear assumption could lead to a larger K_o value.

Since the in-situ total horizontal stress and the undrained shear strength estimated from non-linear and linear analyses are similar, the pressuremeter expansion curves predicted by the two analytical methods will also be similar. Figure 5 shows the pressuremeter expansion curves predicted by both the linear analysis and the non-linear analysis based on equation (11) as compared with the field expansion curve. There is a good agreement between the predicted and measured expansion curves. The difference between the expansion curves derived from the non-linear and linear analyses, is not significant for the Singapore marine clay, even in the initial expansion phased.

Table 3. Estimating σ_{ho}, σ'_{ho} and K_o from the pressuremeter test in Singapore marine clay

Loop	Lift-off			Power law			Hyperbolic			Liner analysis		
no.	σ_{ho}	σ'_{ho}	K_o	σ_{ho}	σ'_{ho}	K_o	σ_{ho}	σ'_{ho}	K_o	σ_{ho}	σ'_{ho}	K_o
1	480	65	0.56	483	68	0.58	488	73	0.63	500	85	0.73
2	480	65	0.56	483	68	0.58	488	73	0.63	501	85	0.74

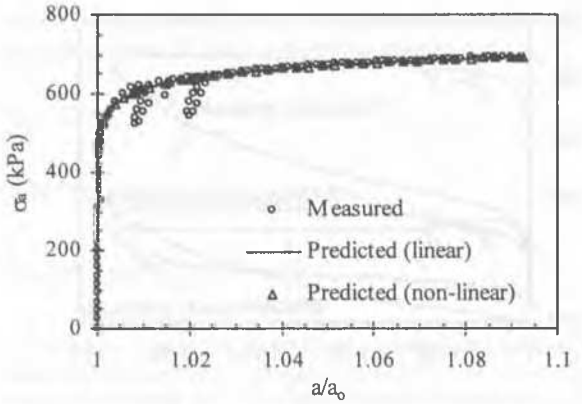

Figure 5. Comparison of predicted and observed expansion curves

4 CONCLUSIONS

An analysis method based on the non-linear elasto-plastic theory has been applied to the interpretation of results from a self-boring pressuremeter test in the Singapore marine clay. It has been shown that non-linear elastic stiffness parameters can be derived from the reloading curves using the power law function or the hyperbolic equation. The shear modulus at the onset of yielding is found to be significantly overestimated using the conventional linear analysis. The effective pressuremeter expansion pressure at large strain does not change during the pressuremeter expansion in an undrained test and can be used to estimate the overconsolidation ratio of soil. With the information of the excess pore pressure generated during the installation of the pressuremeter, the non-linear analysis can provide a reasonable estimate of in-situ effective horizontal stress that is comparable to the most probable field value.

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