Method of a variable level of shear strength mobilization for calculation of the strength of soil masses

La méthode du degré variable de la mobilisation du la résistance au cisaillement pour de la stabilite des sols

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ABSTRACT: An approximate method based on evaluation of the degree of shear strength mobilization along the vertical faces of partitioned slices of a shearing mass is described for strength calculations. Comparison is made with other solutions, including exact solutions, for problems involving determination of the bearing capacity of earth beds and the soil pressure on a retaining wall.

RÉSUMÉ: Une méthode approximative basée sur l'évaluation du degré variable de la mobilisation de la résistance au cisaillement au long des faces verticales des sections d'un massif mouvant est décrite pour le calcul de la stabilite. La solution proposée est comparée à des autres variantes, y compris la solution exacte des problemes de la determination de la force portante des fondations et de la pression des sols à l'énclos.

Calculations of the bearing capacity of beds, slope stability, and the limit pressure on retaining structures are some of the critical elements of geotechnical analysis. Methods based on partitioning of a shearing soil mass into slices with vertical walls such as method of circular-cylindrical slip surfaces are used successfully in all of these calculations. Our Modified Method of Force Equilibrium (MMFE) (Fedorovsky & Kurillo, 1997) belongs to the above-described category. The development of MMFE, which makes it possible to improve its accuracy in principle, while retaining advantages inherent to the MMFE (allowance for the inhomogeneity of the geological and hydrogeological structure of the mass, fulfillment of equilibrium and strength conditions, no restriction on the shape of the desired failure surface), is described in this study.

As in the majority of familiar methods used to calculate slope stability (Huang 1983), the calculations are performed in the MMFE for plane strain. The inclination of the forces of interaction between slices (sliding pressure) is assumed constant (as a rule, equal to zero; this goes into the safety margin). The safety factor f is introduced for stability along the slip surface, which indicates that if the strength characteristics (c and $\tan\varphi$) along the entire surface under consideration is reduced by a factor f, the mass on the whole is in a state of limit equilibrium. For a given trial slip surface (the lower boundary of a shearing mass), the safety factor for stability is, as in Spencer's method [3], found from the condition whereby the sliding pressure on the lower edge of the slide is equated to zero. The safety factor for slope (embankment) stability is the minimum of the factors f with respect to all possible slip surfaces. The strength at which the desired minimum is attained is potentially the most dangerous failure surface (Fig. 1). This approach can also be used to calculate the bearing capacity, replacing f by the safety factor f_p with respect to load.

In the MMFE, the geometry of the slope and its geo-logy and hydrogeology, as well as trial slip surfaces are approximated using continuous piecewise-broken lines, and the search for the failure surface reduces to a search for the positions of the nodal points of its approximation for a given number n of slices. The minimum of the function of a finite number of variables with constraints is sought by the method of coordinate-to-coordinate descent on the basis of the algorithm adopted in Chernousco's method of local variations (Chernousco & Banichuk 1973). Both

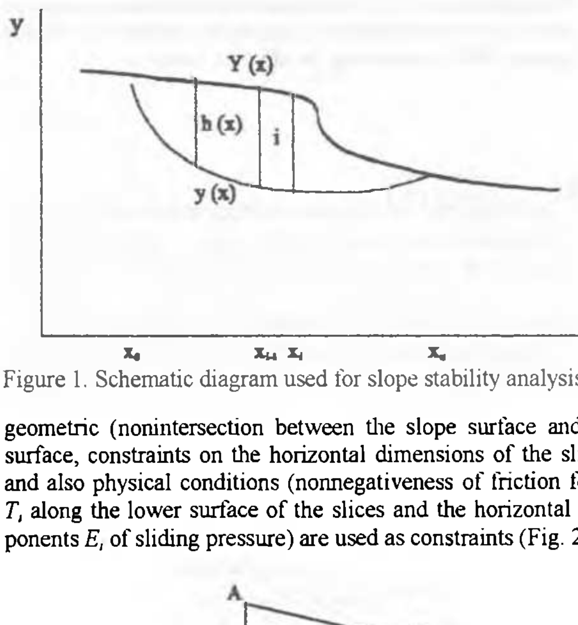

Figure 1. Schematic diagram used for slope stability analysis

geometric (nonintersection between the slope surface and slip surface, constraints on the horizontal dimensions of the slices), and also physical conditions (nonnegativeness of friction forces T_i along the lower surface of the slices and the horizontal components E_i of sliding pressure) are used as constraints (Fig. 2).

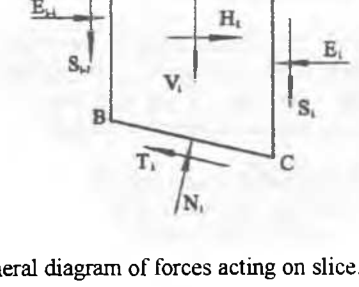

Figure 2. General diagram of forces acting on slice.

For a given finite-section break, which approximates the desired line of failure, the safety factor f is determined in the following manner. At the initial point x_0 (see Figure 1), the forces $E_0 = S_0 = 0$, since the corresponding vertical face of the first slice is either absent, or stress-free (crack). T_i and N_i as well as S_i and E_i can be found as functions of f from the equations of force equilibrium of this slice with allowance for the relationships between these four quantities. This operation can then be repeated successively for all slices (see Figure 2). As a result, we obtain the nonlinear equation

$$E_n(f)=0 \qquad (1)$$

by whose solution we can also determine the desired value of f for a given trial line of failure. In the case where there is a safety factor with respect to load (i.e., the factor that must be multiplied by the assigned surface or volume load in order that the mass proceed to limit equilibrium), the solving equation is found to be linear in terms of f_p; this appreciably simplifies the calculation. (See Fedorovsky & Kurillo (1997) for a detailed description of the numerical procedure.)

Calculations for actual slopes under different hydrogeological conditions, which were performed using the SLIDE program that implements the MMFE, indicated the latter's sufficiently high effectiveness (Fedorovsky & Kurillo, 1997). Due to the lack of "supporting" exact solutions, however, it is difficult to assess the accuracy of the results obtained. Required exact solutions are available for bearing-capacity problems, for example, Prandtl (1920) solution. The results presented in Table 1 for solution of the problem of determination of the reduced bearing capacity p_{ult}/c of a weightless cohesive bed beneath a strip foundation for φ = 30° (Fig. 3a) suggest that the MMFE yields a very inaccurate, although conservative (as distinguished from the calculation performed in accordance with a logarithmic-spiral slip surface (Fedorovsky 1985) estimate of the ultimate load p_{ult}.

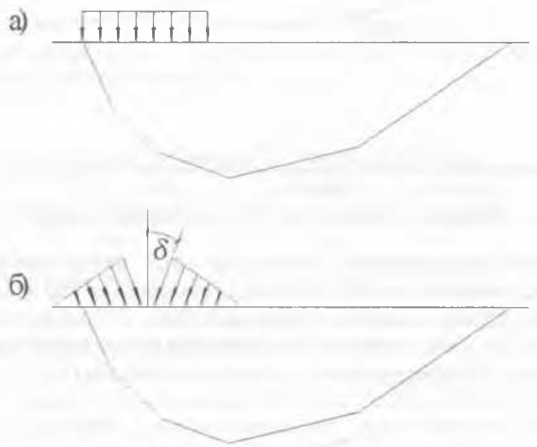

Figure 3. Diagram used to calculate bearing capacity or cohesive (a) and cohesionless (b) beds.

It is curious that the solution given by the MMFE for the case in question agrees with Gersevanov (1948) solution. The reason for this, like the basic cause of the inaccuracy of this approximate solution, consists in disregard of tangential interaction between slices.

This point is revealed in examining Prandtl's solution. Let us construct a vertical that intersects the upthrust (shear) zone, and calculate the integral S of the shear stresses and level of shear-strength mobilization within the limits if this zone

$$\zeta = S\Big/\int_0^h(\sigma_x \tan\varphi + c)dz \qquad (2)$$

where h is the depth of the upthrust zone at the point in question, σ_x is the horizontal effective stress, and z is the distance from the surface.

It is found that in this case, the level of mobilization depends on the position of the vertical. ζ is equal to zero from the back edge to the midpoint of the plate, and it then increases, attaining a maximum value of 1 near the front edge; it then decreases again, returning to zero, where the vertical intersects the rectilinear boundary of the Rankine zone. Hence follow the three assumptions;

1) the angle of incline of the forces of interslice interaction is rejected in view of the inaccuracy of this quantity for cohesive soils and is replaced by the level of shear-strength mobilization ζ, which is suitable for any soils;
2) the quantity ζ is rendered variable over the length of the upthrust (slide); and,
3) as a first approximation, an attempt is made to relate ζ to the geometry of the body of the overflow (slide).

Table 1. Comparison of the bearing capacities of a weightless cohesive bed beneath a strip footing for φ = 30° obtained by different methods

Solution	1	2	3	4	5	6
p_{ult}/c	30,14	37,61	13,86	28,58	28,27	28,05
l/b	4,290	4,764	3,000	4,893	4,778	4,690
d/b	1,585	1,693	1,732	1,366	1,494	1,496

Notation: b - width of foundation; l - width of upthrust zone on free surface; d - maximum depth of overflow zone;

Alternate schemes of solutions: 1) Prandtl (1920); 2) logarithmic spiral (Fedorovsky 1985); 3) Gersevanov (1948) and by MMFE; 4-6) in accordance with MVLM for n = 3, 6, and 12, respectively.

If the first two assumptions are natural conclusions drawn from analysis of the situation, the last assumption, by no means an indisputable one (in the general case, the level of shear-strength mobilization more than likely will depend not on geometry alone), can be justified with comparative simplicity, and, in final account, by the results obtained.

It should be stated that the need to refine existing methods, for example, by accounting for the variability of the angle of force interaction between slices, has been realized for some time. In Morgenstem and Price (1965) familiar method, for example, the tangent of this angle was assumed variable and was assigned in advance with an accuracy to an unknown factor. The principles of the assignment are unclear, however, and in effect, a linear function is assigned as a rule; this does not correspond perfectly with reality.

Turning to analysis of Prandtl's solution, let us formulate assumptions concerning the relation between the level of mobilization and the geometry. As is apparent from this solution, the more curved the slip line (the level of mobilization is equal to zero where this line is rectilinear) and the thicker the shearing zone, the high the level of mobilization. Physically, this is completely explicable: deformations are intensified in areas where the curvature is large, and the greater the thickness of the deformed zone, the greater the portion of deformation that is realized in a form of shear rather than bending. The simplest dimensionless combination in which both these quantities are brought into play

$$\kappa = hd^2y/dx^2 = (Y-y)d^2y/dx^2 \qquad (3)$$

where $Y(x)$ is the equation of the surface of the bed (slope), and $y(x)$ is the equation of the slip line.

To convert from κ to the level of mobilization ς, let us use the generalized hyperbolic function (Levachev et al. 1986)

$$\zeta = k\kappa \Big/ \left(1 + |k\kappa|^m\right)^{1/m} \qquad (4)$$

which satisfies the following basic requirements: a monotonic increase, tends to 1 when $\kappa \to \infty$, is equal to zero when $\kappa = 0$, and, moreover, is sufficiently flexible due to the possibility of selecting the controlling dimensionless parameters k and m. It is natural to assume that the level of mobilization is calculated as the mobilized portion of the strength with allowance for the soil's safety factor, i.e.,

$$S = \zeta(\overline{E\tan\varphi} + \bar{c}h)/f \qquad (5)$$

where E is the integral of the horizontal effective normal stress in a given vertical section over the height of the upthrust or slide zone (in the latter case, this is the horizontal component of the sliding pressure), and $\overline{\tan\varphi}$ and $\bar{c}$ are strength characteristics averaged over the height.

The method of strength calculation, which has been obtained as a result of successive implementation of this assumption, possesses appreciable differences from the MMFE, and can be called the Method of a Variable Level of shear-strength Mobilization (MVLM). The MVLM has been tested in the same trial example. Results with respect to both approximation of the exact slip line, and the value of the ultimate load obtained (Table 1) are found to be considerably more accurate even for the simplest selection of the parameters $k = m = 1$.

A similar effect is obtained for the more complex problem of determining the bearing capacity of a weighted cohesionless bed with $\varphi = 30°$. It is evident from Table 2, where an exact Fedorovsky & Vorob'ev solution (corresponding to Lundgren and Mortensen (1953) solution for $\delta = \varphi$) is compared with the MVLM solution. It is natural that here the contact stress diagram is assumed to be triangular and symmetrically inclined in conformity with the adopted angle of contact friction (Fig. 3b), and p_{ult} is recognized as the average pressure against the plate base in the limit state. The approximate load diagram adopted in this calculation (Fig. 3b) results in a certain limit load on the high side, especially for large angles of contact friction; this contributes to a more precise definition of the solution for the case in question.

Table 2. Comparison of the bearing capacities $2\,p_{ult}/\gamma b$ of a cohesionless bed with $\varphi = 30°$ beneath a strip footing obtained by different methods

Solution	Angle of contact friction δ, deg						
	0	5	10	15	20	25	30
Exact	7,15	9,87	11,75	13,14	14,03	14,51	14,75
MVLM	6,20	8,07	9,67	11,43	12,99	13,93	14,80

An another essential distinction among two problems which are represented schematically in Figures 3a and 3b lies in the fact that in the first case a slip line begins from a rear edge of the plate both in either exact and approximate solutions, whereas in the second case the line begins from a point inside the plate base. It corresponds to the fact, that a hard core beneath the plate occupies the whole base in the first case and only the central part of the base in the second case. In the second case the hard core is even degenerated for an ideally smooth plate base ($\delta = 0$; Hill's scheme).

Hence MVLM gives considerably more exact solutions of the bearing capacity problem than other known methods of the limit equilibrium theory (see, for example, Brinch Hansen 1966). MVLM holds usual advantages of numerical methods as well, so as to allow for the soil inhomogeneity, a loading path and so on. The last property is particularly important for calculation of bearing capacities of offshore gravity base foundations. The following model example (Fig. 4) serves to illustrate it.

a)

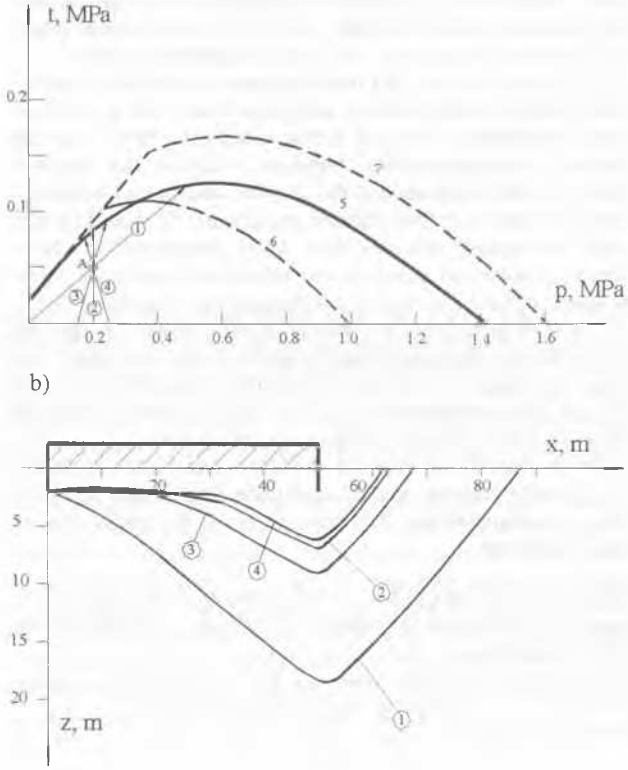

Figure 4. Foundation stability envelope (a) and failure lines (b): 1-4 – loading paths and respective failure lines; envelopes: 5 – MVLM; 6 – Brinch Hansen (1970); 7 – Caquot & Kerisel (1953)

Let us consider gravity platform of width $b = 50$ m with skirt of depth $d = 2$ m. The soil has an effective unit weight $\gamma' = 10$ kN/m^3; a friction angle $\varphi = 20°$ and a cohesion $c = 10$ kPa. Analysis is performed for drained and plane strain conditions. The mean design loads at platform base level follow: vertical $p_1 = 0.2$ MPa; horizontal $t_1 = 0.05$ MPa; moment load is absent (the design load .is shown as point A in Figure 4a). Results of MVLM calculations are presented in Table 3

Table 3. Safety factors for different loading paths

Loading path	1	2	3	4
Safety factor	2.57	1.73	1.92	1.57

The greatest safety factor is obtained for the loading path 1 (t/p = const; Figure 4a). It corresponds to the deepest upthrust (shear) zone (Fig. 4b). Lesser shear zone and safety factor take place for the path 2, where the vertical load is constant. If the value of p increases (3) or decreases (4) (both variants are quite possible in the case of ice or wave loading) the respective safety factor increases or decreases as well. All limit points for different paths belong to the stability envelope 5 (Fig. 4a)

Homogeneity of soil in the above problem makes possible to assess an exactness of a general formula for bearing capacity (Brinch Hansen 1970), that in our case can be written as

$$p_{ult} = 0.5\gamma' b N_\gamma i_\gamma + (q + a) N_q d_q i_q - q - a \qquad (6)$$

where $q = \gamma' d$ – a surcharge at the level of skirt tips; $a = c \cot\varphi$.

In DNV (1992) bearing capacity factors are adopted as follows: N_q – according to Prandtl's solution; and

$$N_\gamma = 1.5(N_q - 1)\tan\varphi \quad \text{Brinch Hansen (1970)}$$
$$\text{or} \quad 2.0(N_q + 1)\tan\varphi \quad \text{Caquot \& Kerisel (1953)} \qquad (7)$$

The first version is very close to the exact value of bearing capacity of sandy soil (Lundgren & Mortensen 1953), and the second version is well above that value. Stability envelopes which are plotted according to eqs. (6, 7) are presented in Figure 4a.

It is known that eq. (6) underestimates the bearing capacity when bearing capacity factors are exact. Curve 6 (Fig. 4a) confirms this opinion. Caquot & Kerisel's version (curve 7), on the contrary, overestimates the foundation stability. An envelope shape (i.e. the dependence of the ultimate load on its inclination) compares well with that obtained numerically. It should be also noted that a plane (tip-to-tip) shear which corresponds to a rectilinear envelope part occurs in the numerical analysis for lesser values of vertical load than in calculations according to eq. (6).

It must be added that the bearing capacity formula makes use of an effective foundation width when the load is eccentric. The usual Gersevanov – Meyerhof's formula for the effective width leads to the underestimation of the bearing capacity (see Fedorovsky 1989). The MVLM has not such drawback.

Results from the MVLM for the active and passive pressures of an ideally granular soil on a retaining wall (Tables 4 and 5) are no less satisfactory. It is apparent from the results cited in these tables that:

Table 4. Coefficients of active earth pressure on the vertical retaining wall for the cohesionless soil with φ = 30° obtained by different methods

Solution	Angle of contact friction δ, deg						
	-30	-20	-10	0	10	20	30
1	0,2661	0,2827	0,3046	0,3333	0,3748	0,4477	0,75
2 .	0,2574	0,2794	0,3038	0,3333	0,3737	0,4411	0,75
3	0,263d	0,2830	0,3045	0,3333	0,3746	0,4443	0,75
4	0,2500	0,2784	0,3037	0,3333	0,3738	0,4419	0,75
5	0,2617	0,2808	0,3043	0,3333	0,3741	0,4435	0,75

Table 5. Coefficients of passive earth pressure on the vertical retaining wall for the cohesionless soil with φ = 30° obtained by different methods

Solution	Angle of contact friction δ, deg						
	-30	-20	-10	0	10	20	30
1	0,75	1,450	2,160	3,000	3,958	4,939	5,670
2	0,75	1,548	2,204	3,000	4,080	5,737	8,743
3	0,75	1,552	2,215	3,000	3,971	5,018	6,000
4	0,75	1,573	2,214	3,000	4,041	5,297	6,384
5	0,75	1,461	2,165	3,000	3,968	4,961	5,765

Alternate schemes of solutions: 1) exact obtained by integration of Boussinesq equations (Fedorovsky & Vorob'ev 2001); 2-4) when the slip line is selected, respectively, in the form of a straight line (Coulomb's solution), a logarithmic spiral, and straight line - logarithmic spiral - straight line as in Prandtl solution (Fedorovsky & Vorob'ev 2001); 5) MVLM when n = 12

- the MVLM yields results, which are close in terms of accuracy to those obtained by the method of a logarithmic spiral and which are markedly superior to those obtained by other approximate methods in terms of accuracy, including the Coulomb method, which is normally used in Standards.
- use of the MVLM for calculation of the passive pressure yields better results than for the active pressure, since in the latter case, the shearing mass apparently extends more along the vertical than the horizontal, and its partitioning into vertical slices is less effective; and,
- use of the MMFE for calculation of the earth pressure on a rough retaining wall is extremely difficult.

Thus, the proposed method is probably a rather reliable and accurate means of solving a variety of problems of soil mechanics and geotechnical engineering, which do not submit (or submit with difficulty) to exact solution. As an example, let us state that it has been used to solve problems involving the bearing capacity of offshore gravity structures in shear (with allowance for the existence of a skirt along the base and variation in pore pressure in the soil under a wave loading), the stability of slopes where anti-slide pile structures exist, and the pressure against a retaining wall with the existence of a surcharge and a complex ground-surface contour, which until now have not been satisfactorily resolved.

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