Vertical bearing capacity of skirted circular foundations on Tresca soil

Capacité portante verticale des fondations-caissons circulaires sur sol de Tresca

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ABSTRACT: The vertical bearing capacity of shallow skirted foundations is investigated using the lower and upper bound theorems of plasticity. The soil is idealised as a rigid–plastic Tresca material having a linearly increasing profile of shear strength with depth. Axially symmetric geometry is considered, with embedment ratios (skirt length to foundation diameter) varying from 0 to 2. Results of a parametric study are presented graphically, in dimensionless form, for convenient use in design. The lower and upper bound bearing capacity factors are compared with previously published finite element results, and with data from physical modelling.

RÉSUMÉ: Une étude sur la capacité portante des fondations superficielles circulaires composé d'un caisson ouvert a été entreprise en utilisant le théorème des bornes supérieures et inférieures de la théorie de plasticité. Le sol a été modélisé en utilisant un matériau de Tresca, à comportement rigide–plastique, et ayant une résistance au cisaillement augmentant linéairement avec la profondeur. Une géométrie axisymétrique a été considérée, avec des rapports profondeur-diamètre variant de 0 à 2. Les résultats de l'analyse paramétrique sont représentés par abaques, dont les entrées sont des valeurs sans dimensions, ce qui rend leurs utilisations particulièrement commodes. Les bornes supérieures et inférieures de la capacité portante sont comparées à des résultats ayant été publiés auparavant, obtenus soit par des analyses par éléments finis, soit par des testes en laboratoire.

1 INTRODUCTION

Skirted foundations, or caissons, are commonly employed in both onshore and offshore geotechnical engineering. While acknowledging the importance of combined loading in many applications, and the added complexity of making realistic bearing capacity calculations for granular materials, this study focuses on the ultimate capacity of a skirted circular foundation subjected to purely vertical loading on undrained cohesive soil. Under these conditions, two main factors are responsible for the enhancement of bearing capacity (compared with that of a surface foundation). First, 'dead soil' becomes trapped within the caisson, effectively creating an embedded foundation at the level of the skirt tips, where the soil is usually stronger than that at the surface. Second, skin friction is mobilised on the exterior surface of the skirts, according to their relative roughness, and this may account for a significant fraction of the overall vertical capacity. Both effects can readily be incorporated into a classical bearing capacity analysis, utilising the bound theorems of plasticity.

2 THEORETICAL BACKGROUND

The lower and upper bound theorems are valid for any perfectly plastic material with an associated flow rule. As discussed by Chen (1975), the theorems have been implemented using a variety of analytical techniques, and have been applied to many geotechnical problems of practical significance. The majority of available plasticity solutions are for plane strain problems, and many of these solutions have been obtained in closed form. By contrast the derivation of lower and upper bounds for non-trivial problems in axial symmetry (or in three dimensions) invariably requires numerical computation of one sort or another.

Several authors have described the development of lower bound solutions for axially symmetric collapse problems using the method of characteristics. Martin & Randolph (2001) list some of the key references. It is first assumed that the stresses σ_{rr}, σ_{zz} and τ_{rz} satisfy the Tresca criterion ($\sigma_{max} - \sigma_{min} = 2s_u$). This means that the meridional stress state can be specified completely in terms of two auxiliary variables, often denoted σ and θ or p and θ. The hoop stress σ_{hh} is then equated with either the minor principal stress or the major principal stress in the meridional plane; this is the well-known 'full plasticity' hypothesis of Haar & von Karman. Taking compression to be positive, the assumption $\sigma_{hh} = \sigma_{min}$ would be appropriate when considering collapse of a circular foundation under downward load, while the assumption $\sigma_{hh} = \sigma_{max}$ would be appropriate for uplift loading (Shield, 1955). It can be shown that the collapse loads obtained in each case are of identical magnitude. Substitution of the various stress components into the equilibrium equations

$$\begin{aligned} &\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \tau_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{hh}}{r} = 0 \\ &\frac{\partial \tau_{rz}}{\partial r} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\tau_{rz}}{r} = \gamma \end{aligned} \qquad (1)$$

leads to a pair of hyperbolic partial differential equations (in the two auxiliary variables). These equations can be integrated numerically using a finite difference version of the method of characteristics. The mesh obtained during the solution indicates the orientation of the 'slip planes' on which $|\tau| = s_u$. In equation (1) the unit weight γ has been taken as zero, since its value has no effect on the bearing capacity factors N_c computed here.

Stress field construction typically commences from a free surface and terminates on a foundation boundary of specified roughness, ensuring that symmetry constraints (notably $\tau_{rz} = 0$ when $r = \theta$) are not violated. Stresses acting on the foundation are integrated to give the collapse load, which at this stage is only a putative lower bound. To establish the solution as a strict lower bound, it must be demonstrated that the 'partial' stress field in the vicinity of the foundation can be extended throughout the soil mass (generally taken to be semi-infinite) without violating the yield criterion. Computation of an acceptable extension field is often a difficult and tedious task, and this part of the solution procedure is frequently taken on trust, if it is acknowledged at all. The lower bounds presented in this paper have all been confirmed as rigorous solutions, although there is insufficient space to describe the extended stress fields in any detail.

An upper bound bearing capacity factor can be obtained from any collapse mechanism that satisfies both the flow rule of the plastically deforming soil and the velocity boundary conditions imposed by the foundation. For the Tresca soil and axially symmetric geometry considered here, the flow rule simply states that deformation must occur at constant volume:

$$\dot{\varepsilon}_{11} + \dot{\varepsilon}_{22} + \dot{\varepsilon}_{hh} = 0 \tag{2}$$

where the subscripts 1 and 2 denote any two orthogonal directions in the meridional plane (not necessarily the principal strain rate directions). For a Tresca material, the power dissipated in the velocity field is computed by integrating the absolutely largest principal strain rate over the volume of the mechanism,

$$\dot{W} = 2 \int_{vol} s_u \left|\dot{\varepsilon}\right|_{max} dvol \tag{3}$$

and by taking due account of any velocity discontinuities (Drucker et al. 1952). Equating with the work rate of the external applied loading gives a rigorous upper bound solution.

If the method of characteristics is used to construct a lower bound stress field, it is found (in axial symmetry as in plane strain) that the same mesh of characteristics can be used to construct a 'consistent' upper bound velocity field, where the principal strain rate directions are aligned with the principal stress directions. Unfortunately the upper bound obtained in this way does not always coincide with the lower bound. The reason is that the major principal strain rate can, in certain regions of the mechanism, become paired with the minor principal stress, and vice versa. A conflict can also arise between the hoop stress assumption ($\sigma_{hh} = \sigma_{min}$ or $\sigma_{hh} = \sigma_{max}$) and the sign of the hoop strain rate. It is therefore necessary to check for proper association between the consistent velocity field and the stress field before declaring that the bounds are coincident and the solution is exact. In fact the work involved in performing these checks (Shield, 1955) is only slightly less arduous than making a direct computation of the upper bound using equation (3). The latter approach has the advantage of always returning a valid upper bound solution, even if there are regions of conflict between the strain rates and stresses (cf. Mróz 1967, Murff et al. 1989).

3 CALCULATIONS AND RESULTS

Referring to Figure 1, all combinations of the following variables are now examined in a parametric study of bearing capacity: d/D = 0, 0.1, 0.25, 0.5, 1, 2; kD/s_{um} = 0, 1, 2, 5, 10, 20, ∞. Both smooth and rough skirt conditions are considered, but in all cases the 'effective base' of the foundation at skirt tip level is taken to be fully rough (see Martin & Randolph 2001).

For each combination of parameters, two lower bound stress fields and two upper bound velocity fields are formulated before selecting the optimum solutions. This is illustrated in Figure 2 for d/D = 0.25 and kD/s_{um} = 5. Stress fields of the type shown in Figure 2a have previously been considered by a number of authors (e.g. Martin 1994, Tani & Craig 1995). An acceptable extension field can always be constructed for this type of lower bound solution. Figure 2b shows the consistent velocity fields derived from Figure 2a. Note that there are significant discrepancies between the upper and lower bound bearing capacities. The alternative stress field shown in Figure 2c is of the type proposed by Martin & Randolph (2001). Here extensibility is crucial in determining the maximum allowable inclination of the artificial free surface, which provides the starting boundary condition for the partial stress field (denoted by solid lines). The simple two-variable collapse mechanisms shown in Figure 2d are described in more detail by Martin & Randolph (2001). The only non-zero velocities occur along a series of parallel streamlines, allowing straightforward computation of the plastic strain rates using a curvilinear axis system.

The results of the complete parametric study are shown in Figure 3. Only when d/D = 0 (corresponding to a skirtless rough circular footing at the soil surface) does the method of characteristics give a consistent upper bound that coincides with the lower bound. These exact N_c factors are 6.05, 6.95, 7.63, 9.23, 11.37 and 14.89 for non-homogeneity ratios kD/s_{um} of 0, 1, 2, 5, 10 and 20. When s_{um} = 0 it is obviously impossible to define an N_c factor, but in this case it can be shown analytically that the exact bearing capacity is $q_u = kD/6$. As the embedment ratio is increased, the consistent velocity field quickly becomes less efficient than the independent two-variable mechanism. Even when d/D = 0.1 the consistent upper bound is outperformed for every value of kD/s_{um} and both extremes of skirt roughness; this is a surprising result given its exactness when d/D = 0.

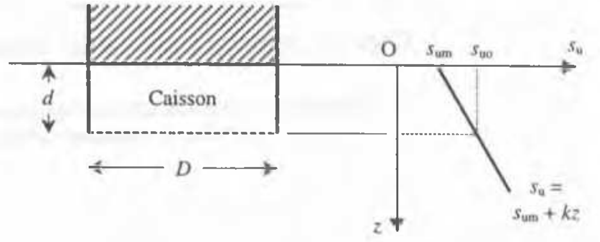

Figure 1. Circular caisson on non-homogeneous soil.

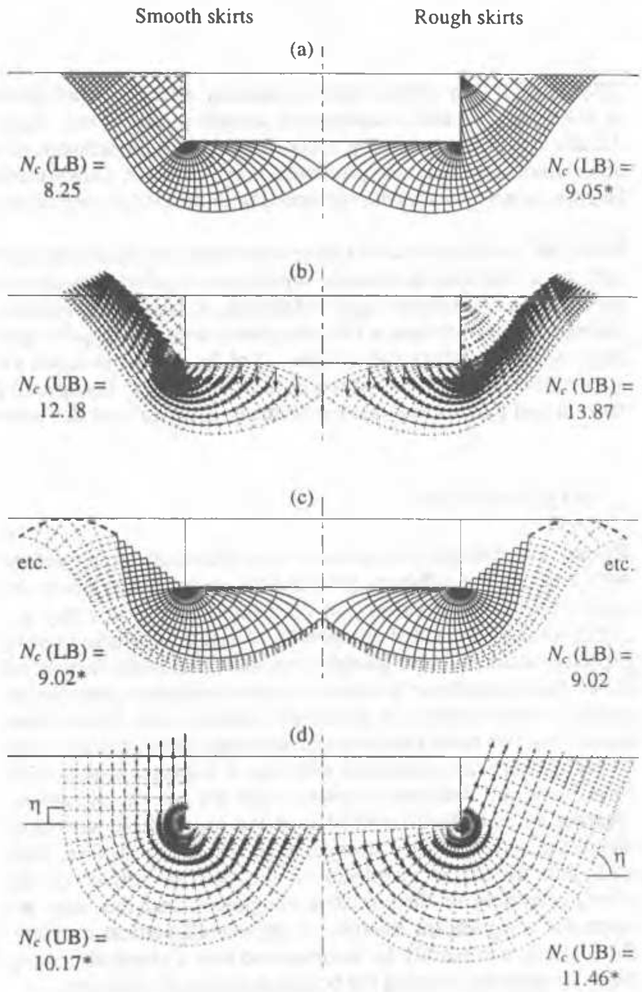

Figure 2. Typical stress and velocity fields (* denotes optimum choice).

For smooth-sided caissons, the optimum upper bound results (Fig. 3a) all involve an exit angle $\eta = 90°$ when $d/D \geq 0.1$. For rough-sided caissons, the optimum upper bounds (Fig. 3c) are obtained with an exit angle that becomes steeper with increasing embedment: over the full range of strength gradients, $\eta \approx 60°$ for d/D = 0.1 and $\eta \approx 80°$ for $d/D \geq 1$. Most of the upper bounds shown in Figures 3a and 3c involve a Prandtl-type wedge attached to the underside of the caisson; a Hill-type mechanism with lateral slip along the fully rough interface only becomes optimal when the 'local non-homogeneity ratio' kD/s_{uo} is greater than about 1 or 2. The lower bound solutions presented for smooth-sided caissons (Fig. 3b) are all obtained from stress fields with an inclined artificial free surface. By contrast the best lower bounds for rough-sided caissons (Fig. 3d) are generally obtained from the standard stress field, which fully exploits the available skirt friction; the alternative stress field is only optimal for a few combinations where d/D is small and kD/s_{um} is large.

The divergence of the bounds from their exact agreement at $d/D = 0$ is, regrettably, fairly rapid. For smooth-sided caissons, Figures 3a and 3b bracket the exact collapse load to better than ±10% over the full range of parameters, with the exception of a few combinations involving deep embedments. For rough-sided caissons the divergence is rather more severe. Figures 3c and 3d indicate that bracketing to within ±10% is only achieved for $d/D = 0.1$, gradually deteriorating to around ±19% at $d/D = 2$, regardless of the value of kD/s_{um}. A noteworthy feature of Figure 3b is that the lower bound curves all reach a peak between embedment ratios of 1 and 1.5 (when the artificial free surface inclination first reaches 90°). For homogeneous soil this peak bearing capacity is $N_c = 9.20$, attained at $d/D = 1.27$ and remaining constant thereafter. For non-homogeneous soil the peak bearing capacities are all in the range $N_c = 9.20$ to 9.60, subsequently reducing towards 9.20 as d/D increases.

4 DISCUSSION

In Figure 4 the plasticity solutions for homogeneous soil ($k = 0$) are compared with the results of some corresponding finite element analyses (Hu et al. 1999). The FE analyses were performed on a rigid, pre-embedded caisson with a fully rough base, using elastoplastic Tresca soil with $E/s_u = 500$. At shallow embedments ($0.1 \leq d/D \leq 0.5$) the FE results fall neatly between the lower and upper bounds, for both smooth and rough skirt conditions. At larger embedments the FE analyses show a transition from general bearing capacity failure to confined cavity expansion (Hu et al. 1999). As this type of mechanism takes over it becomes increasingly inappropriate to make comparisons with bound solutions for rigid–plastic soil, though it is interesting (and probably fortuitous) that when $d/D = 2$ the FE predictions still lie above the lower bound curves. For design purposes it is recommended that the lower bound N_c factors be used, but with no extrapolation to embedment ratios in excess of 2.

The semi-empirical recommendations of Skempton (1951) and Brinch Hansen (1975) for an embedded circular foundation are also indicated in Figure 4. Skempton's equation is the more conservative of the two, and is valid up to $d/D = 2.5$. Brinch Hansen treats the circular shape as an equivalent square of side $B = 0.5D\sqrt{\pi}$ and limits the applicability of his formula to $d/B = 1$, hence the truncated lines. In Figure 4a, the Skempton equation seems rather too conservative with respect to the lower bound and finite element results, at least for intermediate embedments (it compares more satisfactorily with the (inferior) lower bound N_c factors obtained using the standard stress field; see Martin 1994 or Martin & Randolph 2001). At shallow to intermediate embedments the Brinch Hansen formula shows better agreement with the lower bound curve. Brinch Hansen did propose a tentative extension to his formula to allow calculation of N_c at greater embedments, but strangely the transition between the two equations is discontinuous (Martin 1994). In Figure 4b the Skempton and Brinch Hansen predictions have been obtained by simply adding the effects of full skirt friction to the base capacities of Figure 4a. While this procedure is not explicitly ratified by either author for rough-sided foundations, it is commonly used in practice. Figure 4b suggests that the approach is justified, although the level of conservatism is not as great as that in Figure 4a.

To illustrate the application of the lower and upper bound results in practice, they have been used to predict the bearing ca-

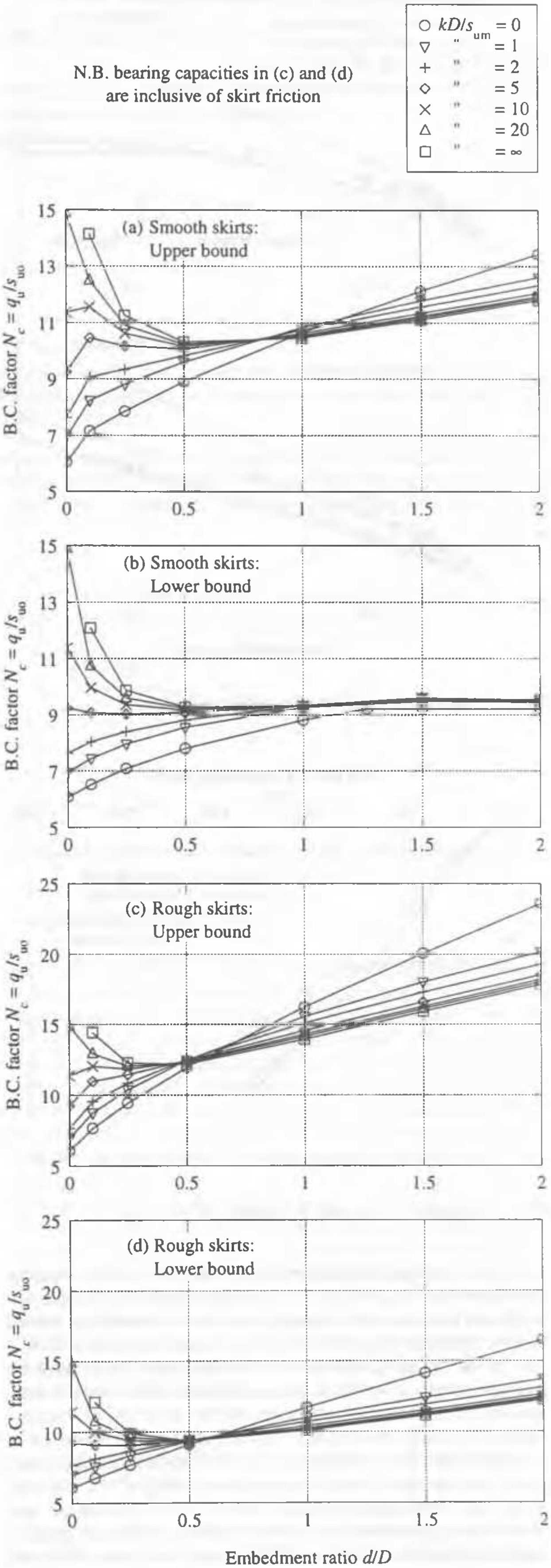

Figure 3. Results of parametric study of bearing capacity.

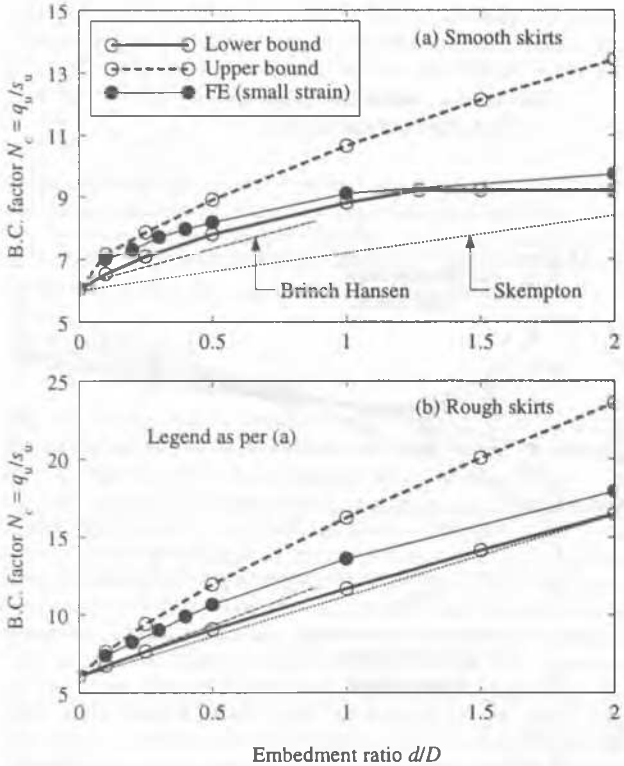

Figure 4. Bearing capacity factors for homogeneous soil.

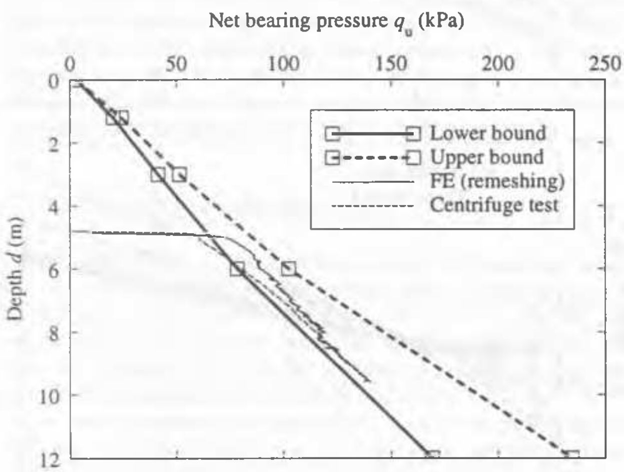

Figure 5. Penetration of circular caisson into calcareous silt.

pacity of a rough-sided circular caisson (D = 12 m) in normally consolidated soil (s_{um} = 0, k = 1.4 kPa/m). This set of properties was chosen to allow direct comparison with the centrifuge model test, and corresponding finite element simulation, reported by Hu et al. (1999). In the centrifuge test a model caisson with skirts of prototype length d = 4.8 m was installed to the 'touchdown' condition, then penetrated into the soil after a period of consolidation at constant vertical load. The top plate of the model caisson was flush with the skirts and of considerable thickness (much like the schematic design shown in Fig. 1) so that full contact was maintained between the side of the foundation and the soil during penetration beyond d = 4.8 m. Normally consolidated calcareous silt was used for the test, with the undrained strength profile assessed using miniature cone and T-bar penetrometers (the above value of k being derived from a linear fit to the upper 20 m of soil). The FE analysis was undertaken as a continuous penetration, with frequent remeshing and interpolation of the soil stresses and material properties. A rigidity index of E/s_u = 500 and the Tresca yield criterion were adopted. Full details of both the centrifuge test and the FE simulation, including justification of the assumption of fully rough skirts, are given by Hu et al. (1999).

As shown in Figure 5, the lower bound curve gives an excellent prediction of the initial bearing capacity at d = 4.8 m, and of the increase in observed capacity with depth. Note that for the centrifuge test, q_u represents the total measured bearing pressure minus the overburden pressure, $\gamma'd$. The finite element simulation also shows very good agreement with the bound solutions. The tendency for drift towards the lower bound with increasing embedment is consistent with Figure 4, although the maximum embedment ratio in this case is only d/D = 0.8. Hu et al. (1999) give a more detailed commentary on the centrifuge and FE results, particularly the rather slower mobilisation of ultimate bearing capacity exhibited by the former.

5 CONCLUSIONS

This paper contains a comprehensive parametric study of the bearing capacity of a skirted circular foundation (caisson) on undrained cohesive soil, under purely vertical loading. The soil is modelled as weightless and rigid–plastic with a Tresca yield criterion. The undrained strength is assumed to be isotropic, but non-homogeneous (increasing linearly with depth). Rigorous lower and upper bound plasticity solutions have been presented in the form of design charts. The closeness of bracketing of the exact collapse load deteriorates quite rapidly as the ratio of skirt length to foundation diameter increases. Nevertheless, the bound solutions compare favourably with both finite element simulations and semi-empirical bearing capacity formulae. They also correlate well with the observed load–penetration response of a model caisson on normally consolidated silt.

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