Use of serviceability limit state calculations in geotechnical design

L'utilisation des calculs a l'état limite de service pour le dimensionnement géotechnique

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ABSTRACT: In recent years a number of new codes of practice, based on the limit state design philosophy, have been developed for geotechnical design. Generally these limit state design codes devote more attention to ultimate limit states than to serviceability limit states. The different reliabilities adopted for ultimate and serviceability limit state designs are examined as are the situations when the design of a foundation is governed by the serviceability limit state requirements. A method for selecting the characteristic values of stiffness parameters is proposed and the suitability of common methods for analysing serviceability limit states is discussed.

RÉSUMÉ: Ces demières années un bon nombres de documents d'application, basés sur le concept de calcul à l'état limite de service, ont été developpés pour le dimensionnement géotechnique. Générallement, ces codes de calculs à l'état limite prêtent plus attention aux états limites ultimes qu'aux états limites de service. Les auteurs examinent les différentes propositions adoptée pour les calculs à l'état limite de service, et étudient les situations où le dimensionnement d'une fondation est dicté par les besoins de l'état limite de service. Une méthode de selection des valeurs caractéristiques des paramètres de raideur est proposée, et le bien-fondé des méthodes classiques pour l'analyse aux états limites des service est discuté.

1 INTRODUCTION

The limit state philosophy has been adopted as the basis for a number of new geotechnical design codes, including the prestandard version of Eurocode 7, ENV 1997-1 (1994). When using limit state design, a distinction is made between ultimate limit states (ULSs), which involve the safety of people and/or the safety of the structure, and serviceability limit states (SLSs), which, as defined in the draft standard version of the head Eurocode, Draft prEN 1990: Basis of Design (2000), concern:

- The functioning of the structure or structural members under normal use;
- The comfort of people;
- The appearance of the construction work.

In limit state design, it is necessary to check that the occurrence of both ultimate and serviceability limit states is sufficiently unlikely. However the main focus in limit state codes is on designing against the occurrence of ULSs; the codes generally provide fewer design rules and guidance with regard to designing against the occurrence of SLSs. In the published literature on limit state design, a similar focus on ULS design exists, with few papers on SLS design. For example, of the total of 68 papers published at the International Symposium on the Limit State Design in Geotechnical Engineering held in Copenhagen in 1993, only 7 dealt with SLS design, while the remaining papers were primarily concerned with ULS design. Similarly, of the 13 papers presented at the International Workshop on Limit State Design in Geotechnical Engineering held in Melbourne in 2000, no paper was principally concerned with SLS design, although one paper examined the forms of p-y curves for laterally loaded piles.

2 LIMIT STATE DESIGN

2.1 *Design Principles*

The principles and requirements for the safety, serviceability and durability of structures are established in Draft prEN 1990. This code describes the basis for structural design upon which the various structural Eurocodes for different materials, such as Eurocode 7 for designs involving ground materials, are based. It also provides guidelines regarding structural reliability. According to Draft prEN 1990, a structure shall be designed and executed in such a way that it will, during its intended life, with appropriate degrees of reliability and in an economical way:

- Sustain all actions and influences likely to occur during execution and use, and
- Remain fit for the use for which it is intended.

2.2 *Degrees of Reliability*

Different degrees of reliability are adopted for ultimate and serviceability limit states and for different reference periods. The recommended minimum reliability index β values in Draft prEN 1990 for ultimate and serviceability limit states for structural members for a reference period of 50 years are shown in Table 1, together with the corresponding probabilities of failure assuming a normal distribution. According to Draft prEN 1990, the β values have been evaluated assuming lognormal or Weibull distributions for material and structural resistance parameters and model uncertainties and normal distributions for self-weight and variable actions.

The recommended minimum β value in Table 1 for ultimate limit state designs is 3.8, corresponding to a probability of failure of 7.4×10^{-5} or about 1 in 13,500. According to prEN 1990, ultimate limit state designs using the partial factors given in that code and in the other Eurocodes generally lead to structures with a β value greater than the recommended minimum value of 3.8.

In the case of serviceability limit state designs, the recommended minimum β value for a reference period of 50 years is 1.5, corresponding to a probability of a serviceability limit state occurring during this period of 6.7×10^{-2} or 1 in 15. This recommendation is obtained from deformation calculations using unfactored representative values of the actions (loads) and unfactored characteristic values of the stiffness parameters.

Draft prEN 1990 stresses that the calculated reliability index and the corresponding probability of failure are only notional values and do not necessarily provide an indication of the actual frequency of failure. They are used as operational values for code calibration purposes and for comparing the reliability levels

Table 1. Recommended minimum β values in prEN 1990 for structural members for a reference period of 50 years

	Minimum values for β	Probability of failure
Ultimate limit state	3.8	7.4×10^{-5}
Serviceability limit state	1.5	6.7×10^{-2}

of structures. Furthermore, it is stated that the actual probability of failure of structures is significantly dependent on human error, which is not considered in partial factor design.

Geotechnical design differs from structural design in that soil is a natural material, unlike materials such as steel and concrete that are manufactured under controlled conditions and whose properties are specified. As a natural material, the properties of soil are well known to be much more variable. In addition, uncertainties in geotechnical designs occur due to limitations in the available models. Consequently a wider range of β values is normally obtained in geotechnical than in structural analyses.

Taking account of the coefficients of variation of the actions and soil properties in geotechnical designs, Meyerhof (1993) gave the order of magnitude of lifetime probabilities of stability failures of about 10^{-4} for foundations on land and 10^{-3} for earth retaining structures and retaining walls. For serviceability limit states, Meyerhof reported that using partial factors of unity on all characteristic loads and characteristic deformation and compressibility properties of soils, gives a reliability of 95%. This reliability is equal to a probability of failure is 0.05. A probability of failure of 0.05 is close to the value of 0.06 corresponding to the β value of 1.5 given in Table 1.

2.3 *Governing Limit State*

In many geotechnical designs, particularly in those for spread foundations, the size of the structure is determined by SLS rather than by ULS requirements. The following square pad foundation examples, one on cohesionless soil and the other on cohesive soil, demonstrate this. SLS requirements are also likely to govern the design of retaining walls in urban environments where there are structures or services behind the retaining wall.

2.3.1 *Pad Foundation on Cohesionless Soil*

The first example consists of a square pad foundation, supporting a permanent load only and resting on sand with $\phi'_k = 35°$ and a constant equivalent elastic modulus, $E'_k = 25$ MPa. The bearing pressure (characteristic load/foundation area) for this foundation is plotted against the ULS and SLS design widths in Figure 1, based on Orr & Farrell (1999). The graphs in Figure 1 were obtained using the equations in ENV 1997-1 for drained bearing resistance and foundation settlement for a maximum settlement of 25 mm. The graphs show that for small foundations, i.e. for small loads, the design width is governed by the ULS, while for larger foundations, i.e. larger loads, the design width is governed by the SLS. In this example the change from the foundation design being governed by the ULS to the SLS occurs at a foundation width of about 1.5 m. As the soil stiffness reduces or as the soil strength increases for the same soil stiffness, the SLS governs the foundation design at smaller foundation widths and loads.

2.3.2 *Pad Foundation on Cohesive Soil*

The second example is that of a square pad foundation supporting a permanent load only and resting on cohesive soil. An appreciation of the limit state governing the design of such a foundation may be obtained from the empirical relationships established between the SPT N value and the drained Young's modulus, E' and the undrained shear strength, c_u, such as those proposed by Stroud and Butler (1975). The following relationships are adopted for a low plasticity glacial soil:

Table 2. β values and probabilities of failure reported by Meyerhof (1993) for service life of 50 to 100 years

	β	Probability of failure
Ultimate limit state:		
Foundations	about 3.5	about 10^{-4}
Earthworks	about 3.0	about 10^{-3}
Retaining structures	about 3.0	about 10^{-3}
Serviceability limit state:		
Earth structures and foundations	about 1.5	about 5×10^{-2}

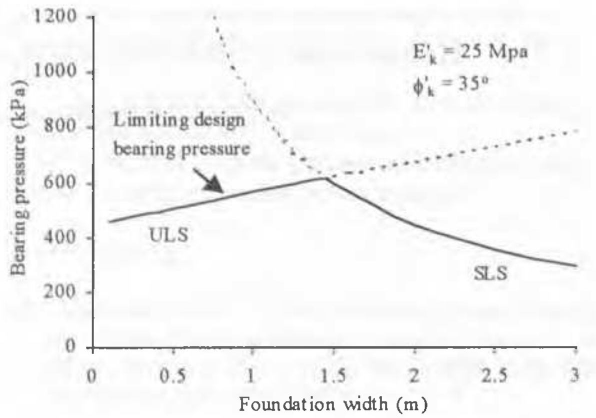

Figure 1. Bearing pressures versus ULS and SLS design foundation widths for pad foundation example on sand

$$E' = 1500N \text{ kPa} \quad (1)$$

$$c_u = 5N \text{ kPa} \quad (2)$$

from which the following relationship is obtained:

$$E' = 300c_u \quad (3)$$

In this example the foundation is at a depth of 1 m in soil with weight density $\gamma = 22$ kN/m^3. Considering first undrained conditions and using the undrained bearing resistance equation with bearing resistance factor, $N_c = 5.14$, shape factor = 1.2 and partial material factor of 1.4 on the characteristic undrained shear strength, c_{uk} given in ENV 1997-1, the ULS bearing pressure is:

$$p_{ULS} = 5.14 \times 1.2 \times c_{uk} / 1.4 + 22 \quad (4)$$

Using the equation for elastic settlement in ENV 1997-1 with the characteristic drained Young's modulus, E'_k and with the settlement coefficient, f = 0.8, and assuming the maximum acceptable settlement is 25 mm, and a width B, the SLS bearing pressure is:

$$p_{SLS} = 0.025 \times E'_k / (B \times 0.8) \quad (5)$$

Equating the bearing pressures obtained from Equations (4) and (5) and using the relationship between E'_k and c_{uk} in Equation (3) yields the following equation for B when the undrained ULS and the SLS conditions governing the design coincide:

$$B = \frac{9.375c_{uk}}{(4.41c_{uk} + 22)} \quad (6)$$

For drained conditions, the soil is assumed to have $\phi'_k = 35°$. Using the equation for the drained bearing resistance in ENV 1997-1 and the partial factor of 1.25 on tan ϕ'_k, the bearing pressure at the drained ULS is calculated and equated to SLS bearing pressure as above to obtain the graph of B against c_{uk} (assuming $E'_k = 300c_{uk}$) in Figure 2 when these governing conditions coincide.

The graphs in Figure 2 show that, in the design of a pad foundation for the chosen soil and loading conditions, the size of the foundation is governed by the ULS condition when the foundation is small and by the SLS condition when the foundation is larger. The undrained ULS condition (Equation 6) governs for c_{uk} values up to 172 kPa, with maximum widths ranging from 1.5 to 2.1 m for $10 < c_{uk} < 172$ kPa, while the drained ULS condition governs for larger c_{uk} values and for larger widths.

The critical characteristic permanent load, F_{ck} on the foundation when the ULS and SLS conditions coincide is obtained by multiplying the appropriate bearing pressure by the foundation area and is plotted in Figure 3. For undrained conditions, the following equation is obtained for F_{ck} by multiplying the pressure from Equation 4 by the width from Equation 6:

$$F_{ck} = q_{ULS}B^2 = \frac{87.89c_{uk}^2}{(4.41c_{uk} + 22)^2} \quad (7)$$

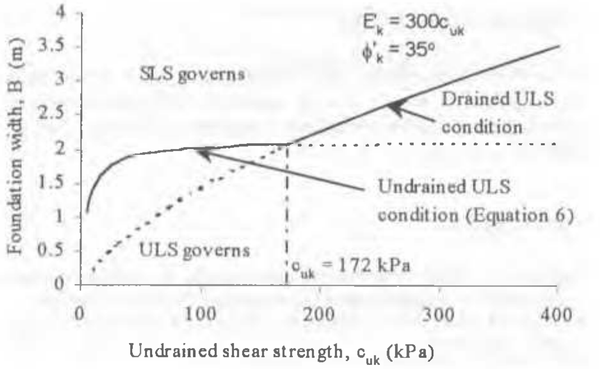

Figure 2: Graphs showing the maximum foundation widths for which the ULS is the governing condition

This relationship is plotted in Figure 3 and shows that, for undrained conditions, F_{ck} increases approximately linearly with c_{uk} for c_{uk} greater than 100 kPa and is given approximately by:

$$F_{ck} = 20c_{uk} \quad (8)$$

Similar relationships may be determined for soils with other combinations of strength and stiffness.

3 SERVICEABILITY CALCULATIONS

3.1 *Direct and Indirect Methods*

According to ENV 1997-1, pad foundations may be designed using direct or indirect methods. In the direct method, separate calculations are carried out to check ultimate and serviceability limit states. Ultimate limit states are checked using a calculation model for failure in the ground, with factored values of the characteristic material parameters or resistances and/or the loads. Serviceability limit states are checked using a deformation model for the soil with unfactored characteristic stiffness parameter values and unfactored actions and action effects.

Where no reliable model is available for a specific limit state, an indirect method may be adopted. This involves analysis of the other limit states using factors to ensure that the specific limit state is sufficiently improbable. For example, where there is no reliable model to calculate settlements or where the deformation parameters are unreliable, a ULS calculation involving the ground strength parameters may be used to design against the occurrence of an SLS. In this situation, characteristic ground strength parameter values are used together with a factor of safety chosen so as to avoid the occurrence of an SLS as well as a ULS. In practice, in some geotechnical design situations, the strength parameters of the ground, i.e. the $\tan\phi'$, c' and c_u values, are known with much greater confidence than the deforma -tion parameters and consequently the indirect approach outlined above may be used. This method is very similar to the traditional method adopted for the design of foundations in which a sufficiently large global factor of safety is chosen to prevent the occurrence of excessive settlements as well as to avoid failure.

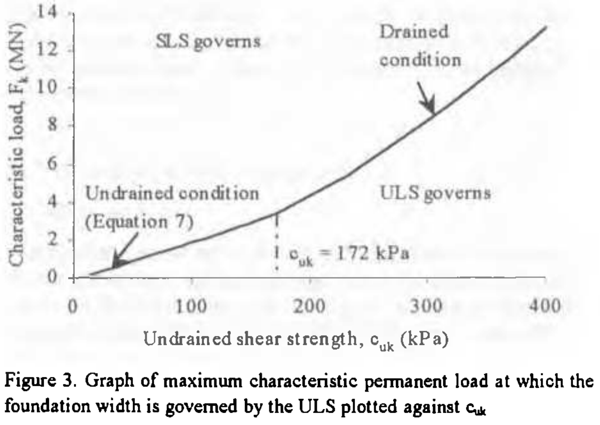

Figure 3. Graph of maximum characteristic permanent load at which the foundation width is governed by the ULS plotted against c_{uk}

3.2 *Characteristic Parameter Values*

When designing to EN 1997-1, characteristic values of the soil deformation parameters are required for SLS calculations. As there are rarely sufficient test results in geotechnical designs to use a purely statistical approach to determine the characteristic value, ENV 1997-1 states that the characteristic value of a soil parameter shall be selected as a cautious estimate of the value affecting the occurrence of the limit state. This definition applies equally to deformation parameters selected for SLS calculations as to strength properties selected for ULS calculations. If statistical methods are used, the characteristic value should be derived such that the probability of a worse value governing the limit state is not greater than 5%. Selection of the characteristic value of a soil property shall take account of variability in the property value, the volume of ground involved in the limit state, geological and background information, and uncertainty in the calculation model, unless this is allowed for directly in the calculation model. The fact that model uncertainty shall be taken into account when selecting the characteristic value is an important feature of designs to ENV 1997-1.

The problem in practice in selecting the characteristic value of a deformation parameter is deciding how cautious this value should be. Normally deformation parameter values for use in geotechnical designs are selected on the basis of a limited number of tests and comparable experience, as defined in ENV 1997-1, taking into account the particular deformation model to be used and the particular design situation.

Schneider (1997) has proposed a method for obtaining the characteristic value of strength parameters based on a simple statistical approach. On the basis of comparative calculations, he has shown that a good approximation is obtained when the characteristic value is chosen as one half a standard deviation below the mean value. Adopting the same approach in the case of deformation parameters and setting the standard deviation, σ equal to the mean value of the particular parameter, X_m multiplied by its coefficient of variation, V, the characteristic value, X_k is given by:

$$X_k = X_m - 0.5\sigma = X_m(1 - 0.5V) \quad (9)$$

Most methods for calculating ground settlements or deformations, whether traditional elasticity based methods or modern elastic-plastic models, require knowledge of the elastic parameters. However, due to the non-linearity of soil and the limited number of tests normally carried out at any particular site to investigate the soil's stress-strain behaviour, the information required to analyse the deformation parameters statistically is rarely obtained. Hence there are few published values of V for Young's modulus and Poisson's ratio or the other deformation parameters commonly used in SLS calculations. Some values of the coefficient of variation for soil parameters that have been reported in the literature are provided in Table 3. These values show that the coefficients of variation for most soil deformation

Table 3. Reported values for coefficient of variation, V for soil parameters

Parameter	Source	Coefficient of variation, V
Deformation Parameters		
C_c	Meyerhof (1993)	0.25 – 0.5
	Cherubini (2000)	0.087 – 0.6
	Cherubini's mean value:	0.314
m_v	Schneider (1997)	0.2 – 0.7
	Schneider's recommended value if limited test results	0.4
E'	Meyerhof (1993)	0.2 – 0.5
ν	Pula & Wyjadlowski (2000)	< 0.2
In situ parameters		
SPT N & CPT q_c	Meyerhof (1993)	0.3 – 0.5
Strength		
$\tan\phi'$	Meyerhof (1993)	0.05 – 0.15
c_u, c'	Meyerhof (1993)	0.2 – 0.6

It should be noted that sample disturbance and experimental errors can result in increased V values being obtained from laboratory tests.

parameters are generally in the range from 0.2 to 0.5 and are much greater than the coefficient of variation for tanϕ', which is generally in the range 0.05 to 0.15.

If the V value for the elastic modulus E' ranges from 0.2 to 0.5, then, using Equation 1, the characteristic E'_k value is in the range $0.9*E'_m$ to $0.75*E'_m$. Schneider (1997) has recommended that, in the case of limited test data, a value of V = 0.4 be used for m_v in design calculations. If this V value is also assumed for the elastic modulus in the case of limited test results, then the characteristic E'_k value for use in serviceability limit state calculations, when designing to ENV 1997-1, is given by:

$$E'_k = 0.8E'_m \quad (10)$$

If there is significant uncertainty in the model used to calculate the deformations, it may be necessary to select an even more cautious value for E'_k than that obtained using the approach outlined above. For example it may be appropriate to use $E'_k = 0.7E'_m$.rather than $0.8E'_m$. in the case of limited test results and an uncertain calculation model.

3.3 *Linear Elastic Analyses*

Although it is known that the stress-strain behaviour of soil is highly non-linear, it is common practice when calculating the settlement of foundations, to adopt a linear elastic analysis. This approach has proved to be reliable provided an appropriate elastic modulus value is selected which takes account of the strain and stress levels and the nature of the deformations. Farrell et al. (1997) have shown that this simple linear approach gives reliable results as non-linear factors involved compensate for each other.

3.4 *Finite Element Method in Limit State Designs*

At present not much research has been carried out into the use of the finite element method (FEM) for limit state designs in geotechnical engineering. Bauduin et al. (2000) have considered the use of the FEM in limit state design, but have concentrated on ULS design. The purpose of finite element calculations in the case of SLS design is to check that the displacements or other movements at service (unfactored) loads do not exceed the limiting values. When increasing the load from 0 to failure in finite element analyses using unfactored characteristic values for the actions and soil stiffness and strength parameters, the load-displacement curve obtained predicts the required displacements at the serviceability load. If, however, the load is increased from 0 to failure using factored soil strength and stiffness parameters, the calculated displacements, particularly for non-linear soil, are likely to exceed those obtained using unfactored parameters and hence do not provide reliable information about the displacements at the service load. Consequently, when using the finite element method for limit state designs, it is necessary to carry out separate analyses to check the serviceability and ultimate limit states if the ultimate limit states are analysed using factored strength parameters.

4 CONCLUSIONS

The importance of the serviceability limit state in the context of limit state design has been discussed. The authors have shown how, for a pad foundation, the SLS governs the design as the size of the foundation increases and as the load supported by the foundation increases. For a pad foundation on cohesive soil, the range of foundation widths over which the ultimate limit state governs the design is limited. An example is given showing how, taking into account undrained and drained conditions, relationships can be established to predict when the design of a pad foundation changes from being governed by the ULS to being governed by the SLS,. The characteristic value of a stiffness parameter for use in SLS calculations may be selected as 0.95 to 0.7 times the mean value, depending on the parameter and its variability.

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