Wave equation analyses for seismic grouped piles

Analyses de l'equation des ondes pour des piles sismiques groupées

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ABSTRACT: Numerical procedures of the wave equation analysis for seismic grouped piles are presented in this paper. The piles connecting to a cap are analyzed with the free-field excitations of the soils. Rational soil stiffness and damping accounting for the pile-to-pile interactions are suggested. The kinematic and inertial interactions of the superstructure and foundation are attainable from the proposed solution. Validation of the analysis is shown with those from the finite element analysis.

RESUME : Les processus numériques des analyses de l'équation des ondes pour des piles sismiques groupées sont presentés dans cette communication. Les piles connectées à un bouchon sont analysées avec les excitations des sols en plein air. On suggère de prendre en considération la résistance du sol rational et les comptes des amortisseurs dans les interactions d'une pile à une autre. Dans la solution proposée ci-dessous, les interactions kinématiques et inertes de la superstructure et de la fondation seront accessibles. La validation de cet analyse sera prouvée à l'aide du résultat tiré de l'analyse des éléments finis.

1 INTRODUCTION

The wave equation analysis has been suggested in modeling the pile response since 1960s. It can be solved with the finite difference schemes at both the frequency and time domains. These solutions are often conducted to monitor the pile deformations due to the mechanical loads. Such analysis is also applicable to the static ones if the governing equation became time independent. In addition, one could estimate the pile's capacity and integrity from such analysis with the field test records. Summary of the research works regarding to wave equation analysis of the pile foundations can be found in Prakash & Sharma (1990). To include the raft-soil-pile interactions for such analysis, the author (Chang & Yeh, 1999, Chang & Lin, 1999, Chang et al., 2000a, Chang et al., 2000b, Chang & Wen, 2001) has suggested the numerical procedures to model the behavior for a group pile foundation. These formulations are adequate in dealing with the mechanical pile vibrations. For the pile response due to the earthquake shaking, the existing model needs modifications. The procedures required to apply the wave equation analysis in solving the seismic problems of the pile foundation are suggested next.

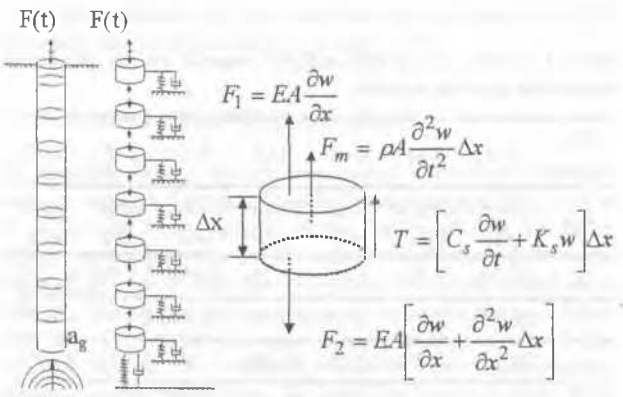

Figure 1. Discrete system of the pile

2 NUMERICAL PROCEDURES

To monitor the earthquake response of the discrete structural system as shown in Figure 1, the free-field excitation of the soil stratum needs to be obtained prior to the analysis. Such analysis provides a one-dimensional soil amplification solution of the site. The EQ motions are decomposed to vertical and horizontal ones. With the computed time histories of the soil deformations, the seismic loads, resistance, damping and the inertia forces from the soils can be applied to the pile segments as a function of time, and then to solve for the corresponding pile displacements. The foundations can be either newly installed or mounted with the time dependant sustained loads from the

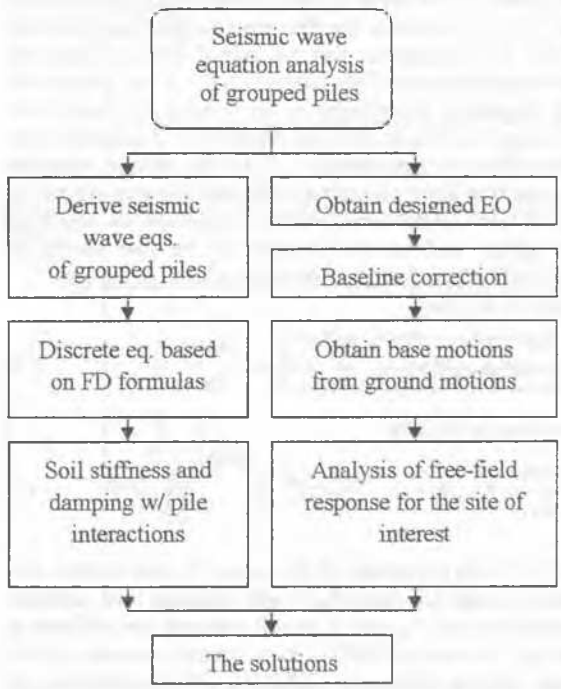

Figure 2. Numerical procedures of the analysis

superstructure. With proper boundary conditions at the pile cap, kinematic and inertial interactions of the structural system can be modeled. Figure 2 illustrates the numerical procedures required for such analysis.

3 ANALYTICAL FORMULATIONS

3.1 *Pile shaft*

In this paper, discussions are made only for the vertical response of the pile. The governing differential equations of the pile segment exciting vertically can be written as:

$$\frac{\partial^2 W}{\partial x^2} = \frac{1}{V_{c,p}^{\ 2}}\frac{\partial^2 W}{\partial t^2} + \frac{C_{s,p}}{E_p A_p}\frac{\partial W}{\partial t} + \frac{K_{s,p}}{E_p A_p}W \tag{1}$$

where W = the relative pile displacements, $V_{c,p}$ = compression wave velocity of the pile; E_p =Young's modulus of the pile; A_p= cross-section area of the pile; $C_{s,p}$ and $K_{s,p}$= damping coefficient and stiffness of the soils along the pile shaft. Eq. (1) can be written in the difference form such that:

$$W(i,j+1) = \frac{1}{A+B}\begin{bmatrix} W(i+1,j)+(2A-C-2)W(i,j) \\ +W(i-1,j)+(B-A)W(i,j-1) \end{bmatrix} \tag{2}$$

where $A = \dfrac{\Delta x^2}{V_{c,p}^{\ 2}\Delta t^2}$, $B = \dfrac{C_{s,p}\Delta x^2}{2\Delta t E_p A_p}$, $C = \dfrac{K_{s,p}\Delta x^2}{E_p A_p}$

Considering the earthquake loads from the soils, the relative deformations between the pile and the soils can be expressed by subtractions of the absolute deformations. Therefore following algebraic equation is obtained.

$$W_p(i,j+1) = \frac{1}{A+B}\begin{bmatrix} [W_p(i+1,j)-W_s(i+1,j)] \\ +(2A-C-2)[W_p(i,j)-W_s(i,j)] \\ +[W_p(i-1,j)-W_s(i-1,j)] \\ +(B-A)[W_p(i,j-1)-W_s(i,j-1)] \\ +(A+B)[W_s(i,j+1)] \end{bmatrix} \tag{3}$$

where W_p and W_s are the absolute displacement of the pile and the soil. Eq. (3) indicates that the absolute pile displacements under the EQ excitations can be solved directly from the absolute displacements of the adjacent soils. If the presence of the pile foundation is negligible in the seismic soils, one could simply use a free-field analysis to obtain the soil displacements, and then substitute them into Eq. (3) for the desired solutions. This is similar to those suggested in the multiple-step analysis of the soil-structure interaction problems. In addition, the equations of the highest and lowest elements of the pile should be modified with proper boundary conditions listed as follows.
At the top of the pile:

$$E_c A_c' \frac{\partial W}{\partial x} = K_{b,c}W + C_{b,c}\frac{\partial W}{\partial t} + E_p A_p \frac{\partial W}{\partial x} \tag{4}$$

At the bottom of the pile:

$$E_p A_p \frac{\partial W}{\partial x} = K_{t,p}W + C_{t,p}\frac{\partial W}{\partial t} \tag{5}$$

where E_c= Young's modulus of the cap; A_c'= cross-section area of effective cap; $C_{b,c}$ and $K_{b,c}$= soil damping and stiffness underneath the cap; $C_{t,p}$ and $K_{t,p}$= soil damping and stiffness at the pile tip. The discrete forms of these equations can be derived with the central difference schemes. Their solutions are dependent of the soil displacements too.

3.2 *Pile cap*

The equation of the pile cap is similarly derived. Eq. (6) is expressed for the effective pile cap connecting to the interior piles where the side soil resistance can be ignored.

$$\frac{\partial^2 W}{\partial x^2} = \frac{1}{V_{c,c}^{\ 2}}\frac{\partial^2 W}{\partial t^2} \tag{6}$$

where $V_{c,c}$= the compression wave velocity of the cap. With the boundary conditions at the top and bottom of the pile cap, one can establish the discrete formulations for the analysis.
At the top of the cap (assuming free surface):

$$E_c A_c' \frac{\partial W}{\partial x} = 0 \tag{7}$$

At the top of the cap (assuming prescribed loads):

$$E_c A_c' \frac{\partial W}{\partial x} = -P(t) \tag{8}$$

In which, P(t) is the loading time history to account for the inertia effects of the superstructure. It can be obtained by a separate analysis, for example, from a dynamic structural analysis for a bridge system. At the bottom of the cap, the equation is the same as Eq. (4).

4 EFFECTS OF PILE-TO-PILE INTERACTIONS

To include the pile-to-pile interactions, approximate dynamic interaction factor (Dobry & Gazetas, 1988) is adopted herein. The interaction factor α between two piles is suggested as:

$$\alpha = (S/r)^{-1/2}\exp(-\beta\omega S/V_s)\exp(i\,\omega S/V_s) \tag{9}$$

where S= pile spacing, r= pile radius, ω= circular frequency, β= material damping ratio of the soil, and V_s= soil shear-wave velocity. By assuming that the settlements under the raft are equal, dynamic impedance functions of the soils along arbitrary piles in the pile group are computed. For example, the dynamic impedance of the soils for a 2×2 square-shape group-pile foundation, K_s^* can be written as:

$$K_s^* = K_S / [1 + 2\alpha(S) + \alpha(\sqrt{2}S)] \tag{10}$$

where K_s = the original impedance of the soil along a single pile. The author has suggested the soil impedance functions for the piles of 3×3 and 4×4 orientations (Chang et al., 2000b, Chang & Wen, 2001). To compute the load distributions of different piles, one needs to conduct a matrix analysis.

Table 1. Ratios of the soil stiffness reduced by the pile-to-pile interactions and their originals

Fdt. type	4×4	4×4	4×4	3×3	3×3	3×3	2×2
S/d	Corner pile	Center pile	Edge pile	Corner pile	Center pile	Edge pile	Each pile
2	24.3%	5.3%	16.8%	32.2%	6.8%	21.9%	41.3%
3	26.3%	9.0%	19.5%	34.3%	13.4%	26.7%	46.3%
4	28.1%	11.8%	21.7%	36.4%	17.8%	30.1%	49.9%
5	29.6%	14.1%	23.5%	38.2%	21.2%	32.8%	52.7%

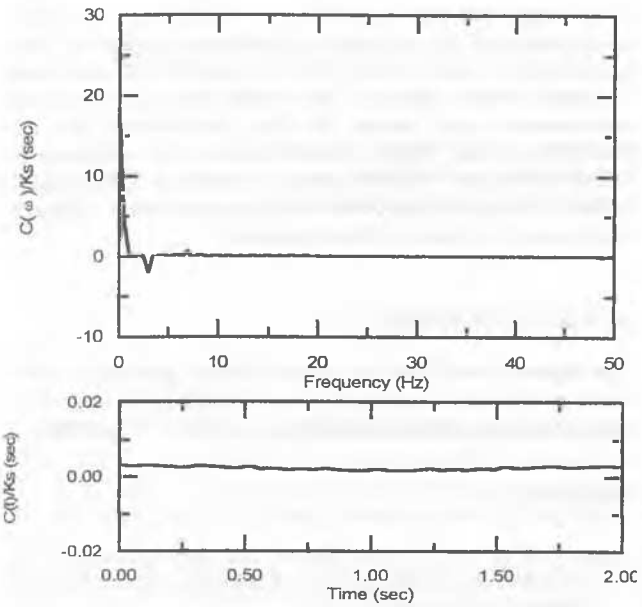

Figure 3. Frequency and time dependent ratios of soil damping and stiffness

4.1 *Effective pile cap*

The size of an effective cap can be computed according to the loads distributed at the piles (Chang et al., 2000b). By assuming that the load distributions are similar to the transmitting forces, the cross-section area of the effective cap is obtained. The cap is taken as an embedded shallow foundation, where the underneath soils are modeled by the Novak's simplified impedance.

4.2 *Soil stiffness and damping*

The soil stiffness derived from the theoretical *t-z* and *Q-z* equations were suggested for the analysis. A time-dependent viscous damping coefficient, based on frequency-dependent nature of the structure, has been suggested to implement with the stiffness in solving the time-dependent wave equations. According the author (Chang et al., 2000b), the loads on different piles will be distributed equally. The pile-to-pile interactions will reduce the soil stiffness around the pile. Table 1 lists the reduced soil stiffness at different piles in the pile groups. The corresponding damping ratios can be solved following the model suggested by the author (Chang & Yeh, 1999, Chang et al., 2000c). Discrete data of the time-dependent damping are obtained from a FFT routine. Figure 3 illustrates the ratios between the damping coefficients and the stiffness of the soils for the 2×2 pile foundations where *S/d*=3. The soils around the effective raft are modeled as those suggested for embedded shallow foundations (Novak, 1974).

5 OBTAINING FREE-FIELD MOTIONS

The one-dimensional seismic excitations of the soils onto the piles are computed from a free-field response analysis for the site of interest. Such analysis can be conducted with the finite element technique, or to be solved with the 1-D wave propagation model and the simplified lump-mass structural analysis. To analyze the equations of motion of the soil layer under the EQ excitations, the relative deformations of the structural system are obtained with the base accelerations induced by the earthquake. In order to obtain proper base motions of the site, the seismic accelerogram recorded at the ground surface should be modified to obtain the base motions of that site. This can be simply done by obtaining first the frequency-spectrum of the accelerogram, and then multiplying it with the analytical 'transfer function' represented for the ratios of the accelerations occurring at the base (bedrock) and those at the ground surface of that site. This procedure can be found in Roesset (1977). This computation would complete a frequency-domain convolution and prepare a base-acceleration spectrum to solve for the corresponding accelerogram. Notice that the discrete wave equations would be expressed in terms of the displacements only. To analyze the soil deformations more carefully, a baseline correction procedure (Kramer, 1996) is suggested to eliminate the offsets of the velocities and displacements appearing after the quake.

6 NUMERICAL EXAMPLE AND COMPARISON

Assuming that a square-shape group-pile foundation with 2×2 piles is constructed in a soil stratum where the averaged shear wave velocity of the soils is 150 m/sec, and the Poisson's ratio is 0.4. 30-meter long round concrete piles with 1m diameter are installed vertically, and connected to an embedded cap with the dimensions (L×W×H) of 5m×5m×2m. The free-field motions of the stratum induced by the vertical accelerations of the Kobe earthquake were obtained from the FEM analysis using computer program ABAQUS. Figure 4 illustrates the relative and absolute ground motions obtained at the free-field ground surface under the earthquake following the discussed concerns. Notice that the absolute soil displacement time histories at various depths of the site were then applied to the discrete wave equations and solving for the foundation response under the same earthquake. Figure 5 shows the relative and absolute displacement time history obtained at the free surface of the cap.

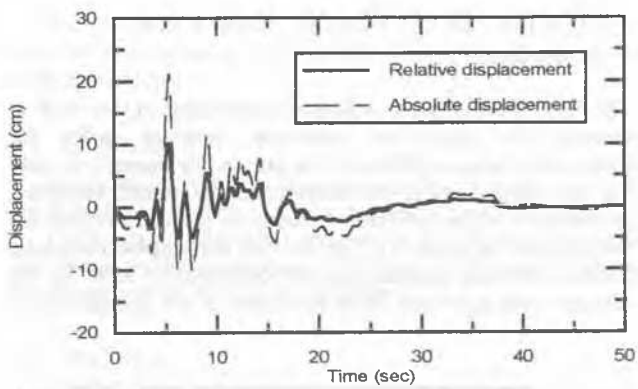

Figure 4. Relative and absolute free-field ground displacement time history

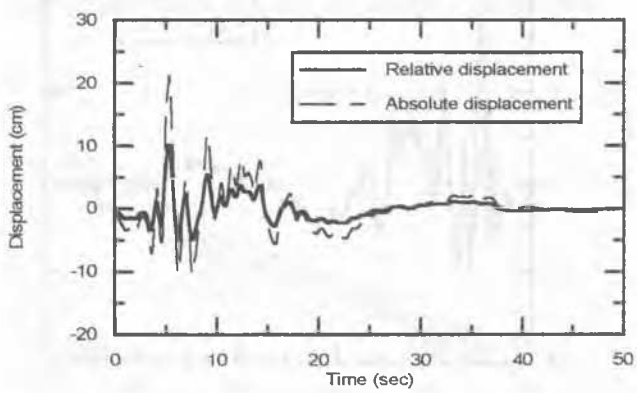

Figure 5. Relative and absolute displacement time history of foundation

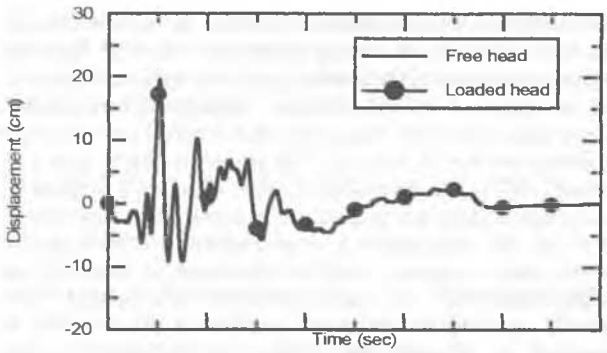

Figure 6. Effects of the sustained loads on fdt. displacement time history

Comparing Figures 4 and 5, one can find that the foundation will follow closely to the ground response under the earthquake shaking. It is rational to see that the foundation will mainly move as the stratum does. This observation implies that the seismic design of the structure on deep foundations may be improved by designing an effective isolation and barrier between the foundation and the structure. The rigid connection of the pier column for the bridge structures and the underneath foundation could be modified. On the other hand, Figure 6 compares the absolute displacement time history at a free cap and that under a sustained load. It seems that the foundation displacements are only governed by the EQ shaking in these cases. Further study to understand the dynamics influence of the sustained loads including the load pattern, magnitude and the duration is now carried. Validation of the proposed modeling is made comparing the solutions of the embedded pile foundation to those obtained from the 2-D FEM analysis from ABAQUS program. Figure 7 plots these results where good agreements between the solutions can be found.

7 SUMMARY

The direct wave equation analysis is reassessed in this work to monitor the group-pile foundation behavior under the earthquake loadings. Effects of the pile-to-pile interactions onto the cap-connected piles were considered. The solution requires a pre-analysis of the free-field motions of the construction site subjected to the same EQ. The resolved soil displacement-time histories are then applied onto the foundation to solve for the discrete wave equations. From the results of the example study, it is found that the embedded pile foundation will move consistently with the site under the earthquake excitations. This phenomenon is also revealed by the compatible solutions from the finite element analysis. This observation implies that the conventional rigid design for the pier-column and the foundation of the bridge structures needs to be improved to reduce the impacts from the ground movements. The sustained loads of the superstructure were found to have a trivial influence on the seismic response of the foundation.

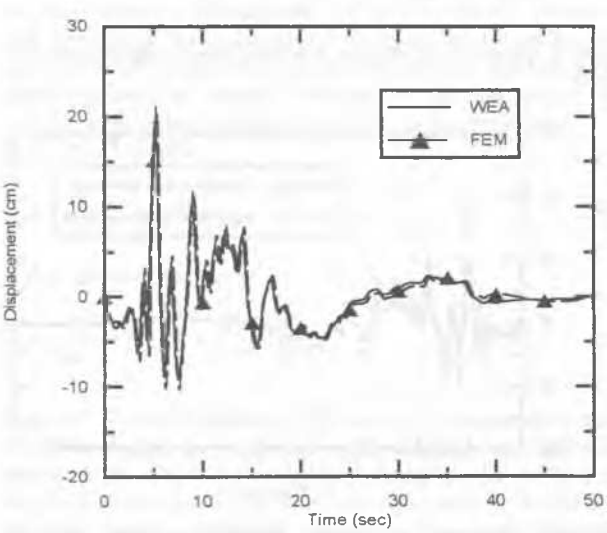

Figure 7. Comparisons of the solutions with those from FEM analysis

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