Predicting the stability of a reinforced soil retaining wall by the generalized limit equilibrium method

Estimation de la stabilité d'un mur de soutènement en sol renforcé à l'aide de la méthode généralisée de l'équilibre limite

D.A.Chrysikos, C.I.Papantonopoulos & D.K.Atmatzidis – *Department of Civil Engineering, University of Patras, GR-265 00 Rio, Greece*

ABSTRACT: The generalized limit equilibrium method recently extended for the analysis of reinforced soil structures offers the possibility to perform local and global stability analysis and estimate the corresponding factors of safety for elements of the soil-wall-reinforcement system. The method is applied in order to analyze the stability of a well documented case of a reinforced soil retaining wall. Computed values for the failure load present a remarkable stability for various reasonable assumptions concerning grid shape and friction angles and fall within a ±10% range from the actual failure load, thus indicating the reliability of the method for stability analysis.

RÉSUMÉ: La méthode généralisée de l'équilibre limite récemment étendue pour inclure l'analyse des structures d'un sol renforcé offre la possibilité d'effectuer une analyse de stabilité locale ou globale et d'estimer les facteurs de sécurité correspondants aux éléments du système sol-mur-renforcement. Cette méthode est appliquée afin d'analyser la stabilité dans un cas bien documenté d'un mur de soutènement d'un sol renforcé. Les valeurs calculées de la charge à la ruine présentent une stabilité remarquable pour une gamme d'hypothèses raisonnables concernant le réseau des éléments et l'angle de frottement et sont tous dans la marge ±10% de la charge à la ruine actuelle, indiquant ainsi la fiabilité de la méthode en ce qui concerne l'analyse de stabilité.

1 INTRODUCTION

Reinforced soil retaining walls, initially introduced by Vidal (1966), are, by now, used extensively throughout the world. The numerous design procedures available to date can be classified in three broad groups as limit equilibrium methods, limit analysis methods and complete stress-strain analysis methods, each with its own advantages and limitations. Limit equilibrium methods are simpler to use, need less material parameters but cannot predict displacements and have to treat separately internal and external equilibrium. It is a known fact that the results vary rather widely according to the particular method used. Examples of this type of analysis are those presented by Steward et al. (1977), Broms (1978), Collin (1986), Bonaparte et al. (1987), Leshchinsky & Perry (1987) and Schmertmann et al. (1987).

Limit analysis methods using finite element techniques have been introduced by Lysmer (1970), Anderheggen & Knoepfel (1971) and have been applied by Pastor (1976), Bottero et al. (1980), Tamura et al. (1984) and Sloan (1988) to slope stability and foundation problems. More recently the method has been extended in order to treat reinforced earth problems by Asaoka et al. (1994) and Kodaka et al. (1995). Finite element formulation of these methods leads to rather complicated and large non linear programming problems. It appears that, up to now, this approach is not used in practice.

Complete stress-strain analysis methods are based on some form of elastoplastic formulation and solve the problem numerically, usually by the finite element method. They seem to depend considerably on the particular constitutive equations adopted and especially those concerning soil-reinforcement interactions (Leshchinsky 1992). A relatively recent comparative study using a variety of finite element codes resulted in significant over and under predictions with respect to the actual results of full scale experiments (Wu 1992b).

A practical alternative to the above mentioned procedures is the generalized limit equilibrium method, initially introduced by Papantonopoulos (1972, 1979) and Papantonopoulos & Ladanyi (1973). The method integrates limit equilibrium and limit analysis concepts and has a static and kinematic approach, the first being a direct extension of the classical limit equilibrium methods. The static approach of this method has been recently extended by Chrysikos (1998) and Papantonopoulos & Chrysikos (2001) in order to analyze reinforced soil retaining wall problems. As a direct extension of the classical limit equilibrium methods, this approach addresses directly the stability problem using finite element discretization and yields a simple linear programming problem (Danzig 1951). This method offers the possibility to perform local (internal) and global (external) equilibrium stability analysis in one step. In addition, it is possible to evaluate local factors of safety, maximum load, minimum required reinforcement strength and to quantify the effect of various strength parameters on the stability of the wall. The advantage is that the analytical procedure remains simple and requires a minimum amount of information on material parameters.

In this paper, after a brief description of the generalized limit equilibrium method and a summary of the relevant experimental data from a full scale retaining wall with cohesionless backfill, a first attempt is made in order to validate this method by comparing the predictions with the actual measurements as well as with the results of other theoretical predictions.

2 THE GENERALIZED LIMIT EQUILIBRIUM METHOD

In order to formulate the problem according to this method one has to proceed as follows:

1. Separate the area of concern in a number of elements
2. Formulate the appropriate equations of equilibrium for all elements
3. Formulate the inequalities corresponding to the failure criteria
4. Choose a variable or a function of the involved variables, called objective function, that has a physical or engineering meaning and whose the maximum or minimum value is of interest, and
5. Solve the resulting mathematical problem by maximizing or minimizing the objective function subject to the constraints (equations and inequalities) mentioned above.

A reinforced soil retaining wall system can be discretized using three typical elements, namely triangular soil elements (Fig.1a), reinforcement elements (Fig. 1b) and wall facing elements (Fig.

1c). Such systems are shown in Figures 2 and 3.

Equilibrium equations are derived from the free body diagram for each of the typical elements. The unknowns involved are the normal and shear stresses σ_{ij}, σ_{ji} and τ_{ij} at the interfaces ij, the tensile forces N_i of the reinforcement at nodes i and the shear forces T_{fi} and moments M_i at the nodes i of the facing elements.

The resulting mathematical problem (linear program) can be summarized as follows:
Maximize or minimize:

$$Z = \sum_{j=1}^{n} d_j x_j \tag{1}$$

subject to:

$$\sum_{j=1}^{n} a_{ij} x_j = b_i\ ,\quad i=1,\ldots,m_1 \tag{2}$$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i\ ,\quad i=1,\ldots,m_2 \tag{3}$$

Relations (1), (2), (3) are all linear and, consequently, the problem can be solved using linear programming techniques (Danzig 1963). These relations correspond to: a) the objective function (external loads, local or global margins of safety), b) the m_1 equilibrium equations for all elements and c) the m_2 inequalities for the failure criteria, namely, the shear strength along the interfaces (soil-soil and soil-reinforcement), the tensile strength of the reinforcement and the shear and bending strength of the facing. The unknowns x_j are the internal stresses and the unknown external loads. Parameters a_{ij}, b_i and d_j are either constants or variables, depending on geometry, strength parameters and unit weight according to the case. The detailed mathematical formulation and the corresponding equations can be found in Chrysikos (1998) and Papantonopoulos & Chrysikos (2001).

3 THE DENVER WALLS EXPERIMENTS

The Denver walls experiments are well known and were the subject of an International Symposium (Wu, 1992a). The experiments include cases of reinforced soil walls with cohesionless and cohesive backfills. Details on the experimental results and procedures are presented by Wu (1992c, d).

The work presented herein concentrates on the case of the cohesionless backfill. The dimensions and the experimental setup are shown schematically in Figures 2 and 3. The width of the wall was approximately 1.2m and the whole structure was rigid to insure plain strain conditions.

The cohesionless material was an Ottawa sand with unit weight γ_t=16.8 kN/m^3. The angle of internal friction φ as measured with triaxial tests was equal to 39°. Based on the work of Conforth (1964) and Meyerhof (1963) the estimated angle for plane-stain conditions is approximately equal to 43°. This value is in agreement with that used by several contributors in Wu (1992a). The reinforcement consists of a non woven thermally bonded polypropylene geotextile 0.28mm thick and with a mass per unit area equal to 98.5gr/m^2. Tensile strength N_{max} was equal to 5.8 kN/m as measured with tensile tests similar to the wide width tensile test. The angle of friction, δ, at the soil-geotextile interface was determined experimentally and was equal to 27°.

The facing of the walls consisted of timber logs interconnected by ½" thick plywood boards at their back face. Each reinforcement sheet was fastened between the plywood boards and the timber logs with screws and placed flat toward the back of the wall as shown in Figure 3. The maximum bending moment of the facing element M_{max} was measured experimentally and

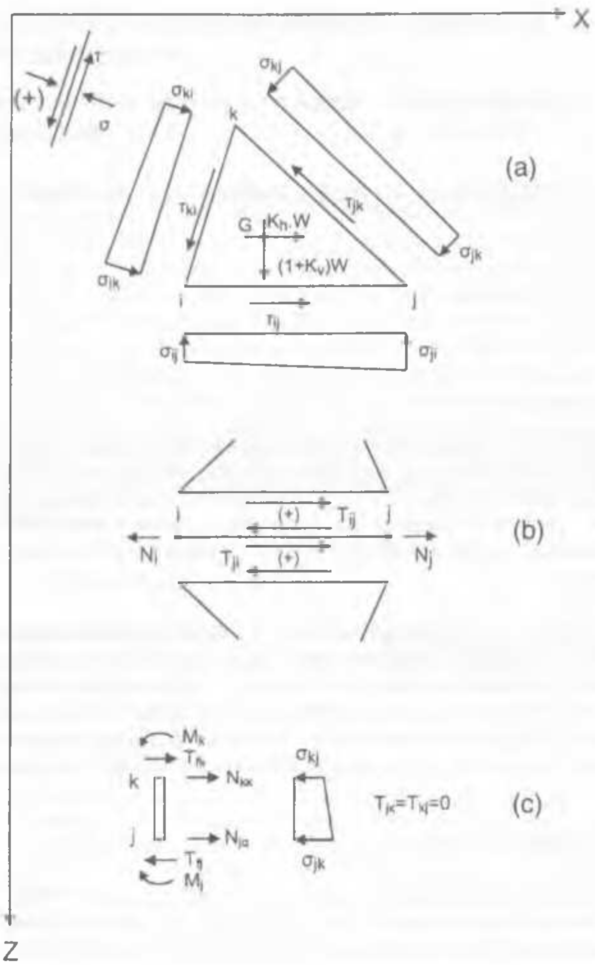

Figure 1. Free body diagrams for three typical elements.

was found equal to 0.8 kN.m/m at the yield point and 1.35 kN.m/m at the ultimate state. The maximum shear force T_{max} was estimated to be greater than 10kN/m in any case.

The wall was loaded with air-bags and failed at a pressure q_{max}=200 kPa. Predictions of the maximum load and corresponding deformations were made by 15 authors and the results are summarized by Wu (1992b). Some predictions did not give an answer for the question of the maximum load.

Nine research groups used finite element codes with various plastic models, five of them being of the Kondner-Dunkan-Chang type. The maximum pressure predicted was:

q_{max}=83, 90-110, 124, 131, >138, 200, 296, >365, and >517 kPa.

Three predictors using conventional methods gave:

q_{max} = 10~21, 41, and 165 kPa

It turns out that, out of nine finite element predictions, 5 underestimated and 3 overestimated considerably the failure load, with only one being successful. All conventional methods underestimated the failure load, two of them being totally out of range.

4 STABILITY ANALYSIS

Stability analysis has been performed by defining as objective function to be maximized the pressure q on the top surface of the backfill.

The finite element mesh depends mainly on the size and shape of the wall and the distance between reinforcements. Two meshes were used, as shown on Figures 2 (mesh A) and 3 (mesh B), in order to estimate the effect of the mesh geometry on the results. Due to the type of the loading, the inclined sides of the

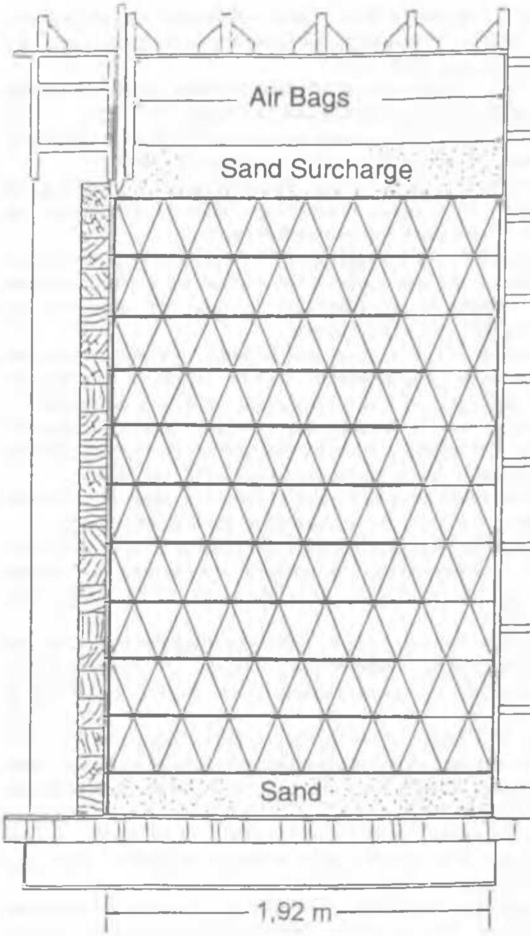

Figure 2. Denver wall (mesh A).

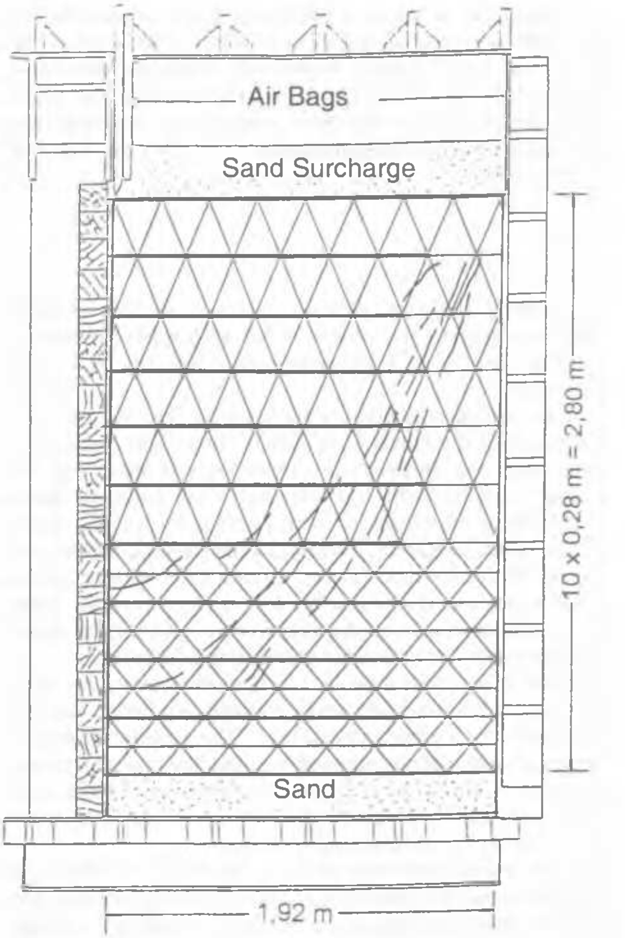

Figure 3. Denver wall (mesh B) and observed slip lines.

triangular elements were chosen to form an angle of approximately 45° + φ/2 with the horizontal for mesh A and part of mesh B. In mesh B, the four lower soil layers are formed by two rows of triangular elements.

The choice of mesh B was inspired by the fact that slip surfaces are generally curved. For this particular problem the actual slip surfaces are shown in Figure 3. So the mesh was constructed in such a way that the left inclined sides of the triangular elements follow approximately the orientation of the slip lines.

Nine cases were analyzed, seven using mesh A and two using mesh B with characteristic strength parameters. Friction between wall facing and soil has been neglected. All results are summarized in Table 1 and are discussed in the following.

First the analysis was performed for the typical estimated parameters mentioned in section 3, using meshes A and B (cases 5A and 9B in Table 1). The maximum load, q_{max}, for each mesh was equal to 180.3 kPa and 219.8 kPa, respectively. These values are within the ±10% range with respect to the measured. However, comparing these two cases it turns out that the effect of the mesh geometry is important since for the same parameters, in the second case, the load is increased by ~22% with respect to the first one.

In order to estimate the sensitivity of the results to reasonable variations of the parameters φ, M_{max}, and T_{max}, five more cases were analyzed (1A to 4A and 8B) taking M_{max}=0.8 kNm/m (yield value), T_{max}=0kN/m, φ=39° and N_{max}=5.8 kN/m.

Based on the results presented in Table 1 it is observed that:

1. Cases 1A and 4A differing only in φ (39° and 43°) show that φ is an important parameter since a variation of 10% increases the maximum load by 24% and the same is true for cases 8B and 9B.
2. Cases 3A and 4A differ only in T_{max} (0 and 10 kN/m). It appears that the results are less sensitive to variations of this parameter.

Case	Soil friction angle φ	Facing bending strength M_{max}	Facing shear strength T_{max}	Reinf. tensile strength N	Failure load q
No.	°	kN.m/m	kN/m	kN/m	kPa
1A	39	0.8	10.0	5.8	145.0
2A	43	0.0	0.0	5.8	161.4
3A	43	0.8	0.0	5.8	161.4
4A	43	0.8	10.0	5.8	179.3
5A	43	1.35	10.0	5.8	180.3
6A	43	1.35	10.0	(0.5)	0
7A	43	1.35	10.0	(5.8)	180.3
8B	39	1.35	10.0	5.8	179.5
9B	43	1.35	10.0	5.8	219.8

Table 1. Results of the Denver wall analyses with cohesionless backfill

3. Cases 2A and 3A as well as cases 4A and 5A differing only in M_{max} show that the effect of this parameter is negligible.

Obviously the above observations are valid for the range of variation of the parameters examined and concern this particular problem.

Concluding the sensitivity analysis, it is encouraging to observe that with reasonable assumptions (cases 4A, 5A, 8B, 9B) and with meshes with different geometries, the theoretical predictions of the maximum load are within the ±10% range of the actual load. It can be also observed that the range of values for all cases analyzed (145.0 ~219.8 kPa) is better than the range of the results predicted by the different finite element codes and reported in the previous paragraph.

Furthermore, in order to illustrate the possibilities of the method, its flexibility and its capacity to evaluate internal equilibrium, the tensile strength N of the geotextile was considered as the objective function. Then the minimum value of N necessary to keep the system in equilibrium was requested for two cases with q=0 and $q=q_{max}=180.3$ kPa. The answer was $N_{min}=0.5$ and $N_{min}=5.8$ kN/m respectively (values in parentheses, cases 6A and 7A, Table 1). It is observed that the tensile strength N=0.5 kN/m is sufficient if there is no surcharge load while, as expected, it becomes equal to N_{max} for the limit load $q_{max}=180.3$ kPa.

5 CONCLUSIONS

The generalized limit equilibrium method, as recently extended to analyze reinforced soil structures, has been applied in order to analyze the stability of a well documented retaining wall with a cohesionless backfill.

The method has successfully predicted the failure load. Computed values of the failure load present remarkable stability for various reasonable assumptions concerning grid shape and friction angles and fall within a ±10% range from the actual failure load, indicating the reliability of the method for stability analysis. The observed stability contrasts with the wide spread scattering of the results reported for various finite element codes. Parametric analysis shows that for the particular problem examined the friction angle and the shape of the grid are the factors that affect mostly the calculated values of the failure load.

The flexibility of the method is further demonstrated by direct calculation of the minimum tensile strength required to keep the system stable for a given external load. The capacity to perform directly such calculations coupled with the fact that a byproduct of the analysis are the local factors of safety along interfaces offers the possibility to evaluate internal equilibrium and define factors of safety for various modes of failure.

The successful prediction of the failure load is an additional confirmation that the generalized limit equilibrium method provides to the practicing engineer a valuable alternative to evaluate stability while preserving the simplicity and extending the possibilities of the classical limit equilibrium approach.

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