# An instability analysis of cut slope using an anisotropic strain-softening model for soft rock

## Analyse de l'instabilité des ruptures de pente utilisant un modèle anisotropique de déformations avec antiécrouissage pour une roche tendre

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ABSTRACT: In the paper, the Adachi-Oka elasto-plastic model with strain softening is extended to simulate the anisotropic behavior of soft sedimentary rock. Using the model, a finite element analysis of a soil-water coupled problem is conducted to investigate the progressive failure of a cut slope in soft rock. The mechanical behavior of the cut slope, including such factors as changes in the excessive pore-water pressure, the redistribution of stress in the ground due to strain softening, the propagation of a shear band, and progressive failure, are discussed in detail. Furthermore, the influence of the anisotropic properties of the geologic materials on the stability of the slope is also investigated.

RÉSUMÉ: Les auteurs de contributions pour compte-rendus de congrès doivent taper celles-ci de façon à permettre à l'éditeur de les reproduire directement par photographie. Afin d'obtenir un style uniforme tout au long du volume, toutes les contributions devront être préparées strictement selon les instructions données ci-dessous. Un imprimeur laser doit être utilisé pour imprimer le texte. Le texte prêt à reproduire sera imprimé uniquement en noir par l'éditeur. A l'usage des auteurs, des moules de fichiers pour MS Word 6.0 (et supérieurs) sont fournis. Dans cette article, le modèle élasto-plastique d'Adachi-Oka avec anti-écrouissage est étendu afin de simuler le comportement anisotropique d'une roche sédimentaire molle. Un code de calcul aux éléments finis avec couplage sol-eau est utilisé avec ce modèle pour analyser la rupture progressive d'une pente de roche molle excavée. Le comportement mécanique d'une pente excavée, incluant des facteurs comme les variations de la surpression interstitielle, la redistribution des contraintes dans le sol due à l'anti-écrouissage, la propagation des bandes de cisaillement, la rupture progressive, seront discutés en détail. En outre, l'influence des propriétés mécaniques des matériaux géologiques sur la stabilité de la pente sera également dégagée.

## 1 INTRODUCTION

It is commonly known that soft sedimentary rock can be linked to many geotechnical engineering problems, such as the instability of cut slopes and foundations. Generally speaking, the mechanical behavior of soft sedimentary rock is elasto-plastic, dilatant, strain hardening-strain softening, and time dependent. Compared to other geologic materials formed in the same epoch, the void ratio of the sedimentary rock is relatively large and a special structure forms during sedimentation. Its mechanical behavior during shearing is largely dependent on the confined stress and the pore-water pressure. The cementation, existing in the structure, deteriorates due to a breakdown of the structure. Various processes, such as large shearing deformation, cyclic drying-wetting, and stress release can cause such a breakdown. Therefore, the softening behavior of soft sedimentary rock becomes a very important factor in the long-term stability of cut slopes

It is known that the progressive failure of cut slopes is usually caused by the following two factors, namely, (a) the deterioration of the structure of the geologic materials due to swelling and weathering during and after the cutting of the slopes, and (b) a reduction in the apparent shear strength due to the dissipation of negative pore-water pressure caused by the rapid excavation of the slopes.

Meanwhile, soft sedimentary rock usually shows anisotropic behavior in its strength, deformation, and dilatancy due to the anisotropic microstructure formed in the sedimentary process.

Adachi et al. (1994) proposed an elasto-viscoplastic model that not only can describe the time dependency, but also the strain softening of the geologic materials. A finite element analysis was conducted for a cut slope under a drained condition to investigate the progression of the cut slope. Adachi et al. (1999) also conducted a soil-water coupled finite analysis, based on a strain-softening elasto-plastic model, to investigate the instability of a cut slope. In the present paper, an isotropic constitutive model with strain softening (Adachi and Oka, 1995) is extended to an anisotropic model which not only can describe the strain-softening behavior and dilatancy, but also the anisotropic behavior of soft rock. This is done by introducing Boehler's anisotropic theory and generalized Hooke's theory. The validity of the proposed model is verified by a series of laboratory tests. Based on the model, a soil-water coupled finite element analysis is conducted for a cut slope under different sedimentary directions. Differences in the long-term stability, due to the different sedimentary directions, are estimated quantitatively through the analysis proposed in this paper.

## 2 CONSTITUTIVE MODEL AND FEM PROCEDURE

In this section, a soil-water coupled consolidation analysis using the finite element method is conducted to analyze the progressive failure of a cut slope with different sedimentary directions. For a boundary-value problem related to the soil-water coupled analysis, a backward finite difference scheme is adopted for water flow in the finite element analysis based on Biot's theory. The equilibrium equation in the finite element scheme can be given as follows:

$$\begin{bmatrix} K & L \\ L^T & -\alpha \end{bmatrix} \begin{Bmatrix} d\delta \\ u_e|_{t+dt} \end{Bmatrix} + \sum_i^4 \alpha_i u_{ei}|_{t+dt} = \begin{bmatrix} [dF] + [dF_R] + [L](u_e|_t) \\ 0 \end{bmatrix} \quad (1)$$

where

$$[K] = \int_V [B]^T [D][B] dV \quad (2)$$

$$[L] = \int_V [B]^T [M] dV \,,\; [M]^T = [1 \quad 1 \quad 0] \quad (3)$$

$$[dF_R] = \int_V [B]^T \{d\sigma_R\} dV \,,\; \{d\sigma_R\} = [D]\{d\varepsilon^p\} \quad (4)$$

$C = [D]^{-1}$ is the flexibility matrix of anisotropic materials in the transverse symmetric plane and can be evaluated with the following equation, namely,

$$[C]=\begin{bmatrix} 1/E_x & -\nu_{yx}/E_y & -\nu_{zx}/E_x & 0 & 0 & 0 \\ -\nu_{yx}/E_y & 1/E_y & -\nu_{yx}/E_y & 0 & 0 & 0 \\ -\nu_{xz}/E_x & -\nu_{yx}/E_y & 1/E_x & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2G_{yx} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/2G_{yx} & 0 \\ 0 & 0 & 0 & 0 & 0 & (1+\nu_{xz})/E_x \end{bmatrix} \quad (5)$$

in which five independent parameters, $E_x$, $E_y$, $\nu_{xz}$, $\nu_{yx}$, and $G_{yx}$ generally need to be determined. The anisotropic model which can describe the strain-softening behavior and the anisotropic behavior of soft rock, extended by introducing Boehler's anisotropic theory and a generalized form of Hooke's theory into the original Adachi-Oka model (Kobayashi, 2000), can be expressed as

$$d\varepsilon_{ij}^{p} = \Lambda\left[\frac{\bar{\eta}_{ij}}{\bar{\eta}} + \left(\bar{M} - \bar{\eta}\right)\frac{\delta_{ij}}{3}\right]\left[\frac{\eta^{*}_{kl}}{\eta^{*}} - \eta^{*}\frac{\delta_{kl}}{3}\right]\frac{d\hat{\sigma}^{*}_{kl}}{\hat{\sigma}^{*}_{m}} \quad (6)$$

$$\Lambda = M_f^{*2} / G' / \left(M_f^{*} - \eta^{*}\right)^2, \bar{\eta}_{ij} = \hat{s}_{ij} / \left(\hat{\sigma}_m + b\right) \quad (7)$$

where the stress history tenser is given by

$$\sigma^{*}_{ij} = \sigma_{ij0} + \int_0^z \frac{1}{\tau}\exp\left(-\frac{z-z'}{\tau}\right)\left(\sigma_{ij}\left(z'\right) - \sigma_{ij0}\right)dz' \quad (8)$$

in which $\hat{\sigma}$ is Boehler's transform stress tensor based on a simple anisotropic theory and is expressed as

$$\hat{\sigma} = \left\{(\alpha + \gamma - 2\beta)tr(\hat{M}\sigma)\right\}\hat{M} + \gamma\sigma + (\beta - \gamma)(\hat{M}\sigma + \sigma\hat{M}) \quad (9)$$

where $\hat{M}$ is the structural tensor and can be given by the following equation:

$$[\hat{M}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (10)$$

Generally speaking, there are 5 parameters in a transverse symmetric model. It is obvious that Equation 9 gives a simplified form of Boehler's anisotropic model in which three plastic anisotropic parameters, $\alpha, \gamma$, and $\beta$ are used.

Table 1. Isotropic parameters of the ground.

| | | | |
|---|---|---|---|
| $E$ (MPa) | 100.0 | $M^{*}_{f}$ | 1.00 |
| $\nu$ | 0.30 | $b$ (MPa) | 0.87 |
| $\gamma'$ (g/cm$^3$) | 1.0 | $\sigma_{mb}$ (MPa) | 16.0 |
| $k$ (cm/sec) | $10^{-6}$ | $\bar{M}_m$ | 1.25 |
| $G'$ | 452 | $\tau$ | 0.025 |

In addition to Young's modulus and Poisson's ratio, 6 parameters are involved in the original model. For the anisotropic model proposed in this paper, three additional elastic moduli and three additional plastic parameters need to be determined through conventional triaxial compression tests.

Figure 1 shows the failure criteria of the Adachi-Oka model and the integration scheme used in the numerical analysis.

Figure 2 shows a comparison of the stress-strain-dilatancy relations of the isotropic ground whose parameters are listed in Table 1. It is found that the calculated relations agree well with the theoretical ones.

## 3 PROGRESSIVE FAILURE IN ISOTROPIC GROUND

Figure 3 shows the finite element mesh adopted in the analysis of the cut slope. The ground is 1000 m in length by 360 m in depth. The height and the slope gradient of the cut slope are 150 m and 5:1, respectively. The numbers of node and 4-node isoparametric element are 1120 and 1053, respectively. In the soil-water coupled analysis, the excessive pore-water pressure is taken as the unknown variable. The initial stress field of the model ground is a gravitational field with a value of $K_0 = 0.43$.

Figure 4 shows changes in the distribution of the stress-history ratio. In the residual state, the cohesion or the cementation of the geologic materials tends to be zero. Only the frictional strength, which depends on a confining stress, remains. In this case, the stress-history ratio will be the same as the stress ratio; it takes the value of $M^{*}_{f}$. The value of $\eta^{*}$ is kept constant. It increases abruptly at the toe of the slope, 4.57 years after the completion of the excavation, and then the phenomenon propagates to other regions. 6 months later, a failure band forms from the toe to the surface, in which the $\eta^{*}$ reaches the residual value. Finally, an unstable block appears in the slope, taking the band as the boundary which connects the stable area of the ground.

Figure 5 shows changes in the distribution of the plastic shear strain. Similar to the stress-history ratio, shear strain develops very quickly in a zone after 4.57 years. The propagation of the shear zone, in which a large shear strain occurs, takes the same form as the failure zone shown in Figure 4.

Figure 6 shows changes in the distribution of excessive pore-water pressure with time. At the time immediately following the completion of the excavation, a large excessive pore-water pressure develops in the ground, resulting in an apparent shear strength that keeps the slope stable. After 4.57 years, it dissipates gradually and the failure zone shown in Figure 7 begins to develop due to a loss in the apparent shear strength. At that moment, the excessive pore-water pressure reaches its minimum value. When the shear zone forms, strain softening occurs and a dilatancy develops in some areas, resulting in an increase in excessive pore-water pressure, as shown in Figure 6.

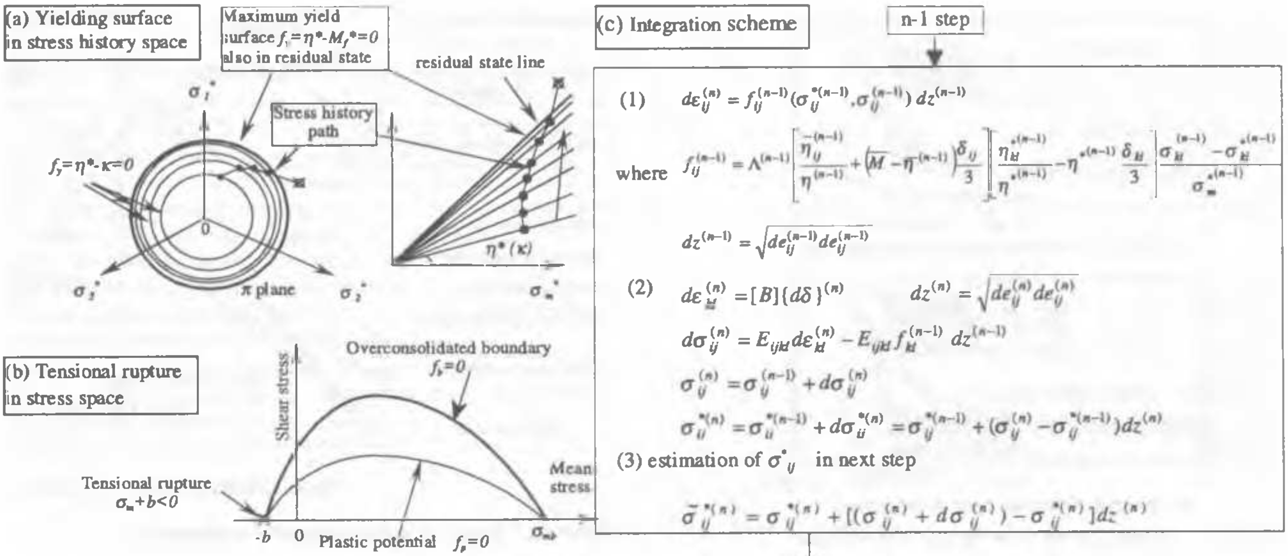
From Figures 4-6, it is clear that due to the dissipation of excessive pore-water pressure brought about by the excavation, the ground of the cut slope loses its apparent strength and a strain softening occurs in some areas. Then, a redistribution of stress leads to the start of the propagation of the softening zone, resulting in the formation of the failure band and the shear zone. The failure band develops gradually, and a global failure in the cut slope finally occurs.

In order to clarify the mechanism of the progressive failure, the time history of such field quantities as stress ratio, etc., in individual elements is studied in detail. Two groups of elements located in the shear zone, one grouped along a horizontal line and the other grouped along the slope surface, are considered.

Figure 7 shows changes in the stress-history ratio with time for the elements. It is clear that the stress-history ratio remains constant for a long time, and then it increases abruptly to the failure line.

Figure 8 shows changes in the stress ratio with time and the stress-strain relations. It is seen that the stress ratio increases very slowly, but it does not change for a long time. Meanwhile, the plastic strain is very small. When the ratio reaches its peak value, strain softening occurs and plastic strain develops very quickly, resulting in a sharp reduction in the stress ratio. It is found that the time at which the strain softening occurs is different for different elements, showing a clear propagation of the softening zone. In Group A, the softening propagates from inner to outer parts, while for Group B, it propagates from lower to upper parts. In both cases, the softening starts from the shear band.

Figure 9 shows the relation of strain rate and time. It is found that although strain softening occurs at different times for



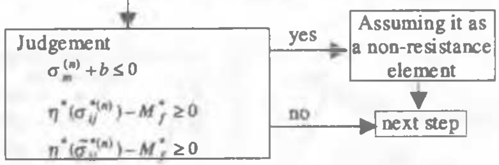




Figure 1. Failure patterns and the integration scheme adopted in the numerical analysis.

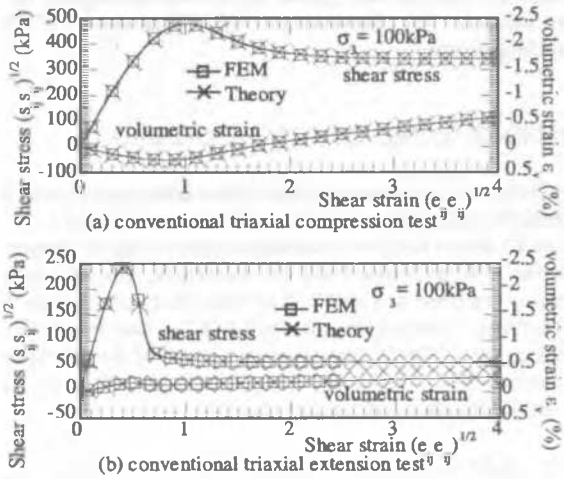



Figure 2. Comparison of stress-strain-dilatancy relations obtained from theory and FEM.

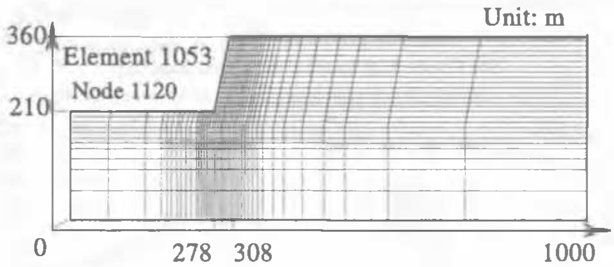



Figure 3. Finite element mesh.

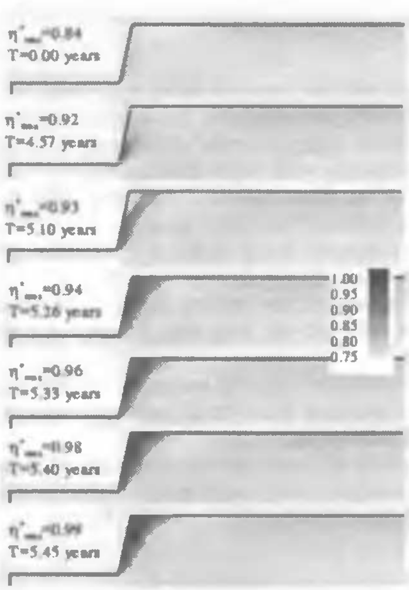


Figure 4. Distribution of stress-history ratio with time.

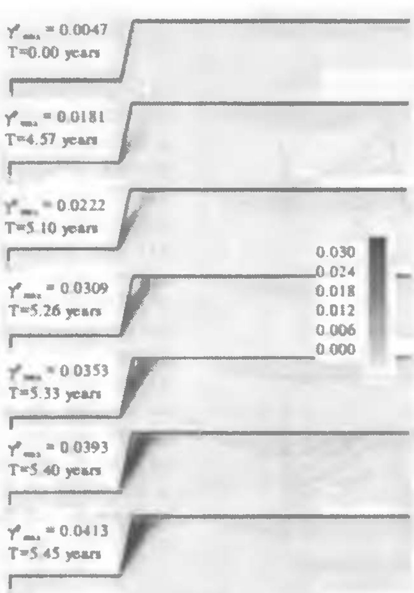


Figure 5. Changes in the plastic shear strain.

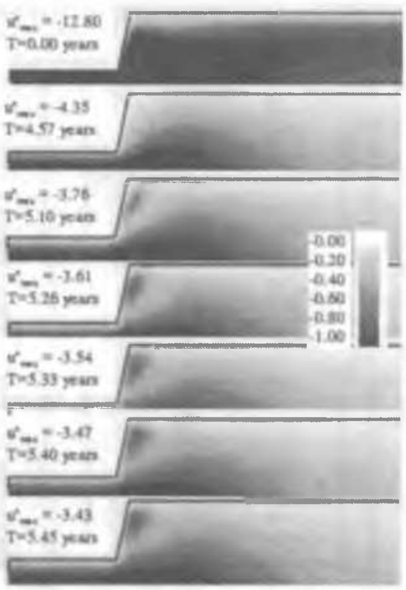
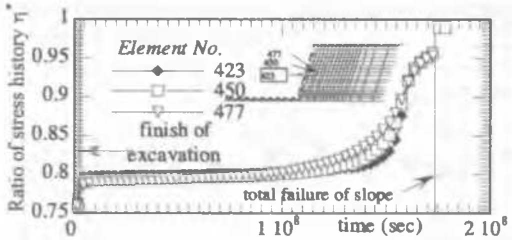
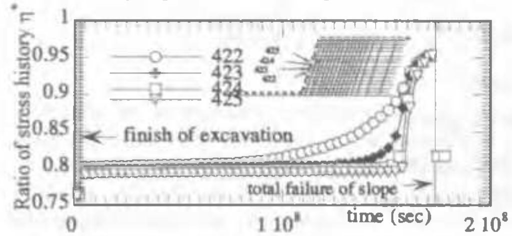


Figure 6. Change of excessive pore-water pressure.

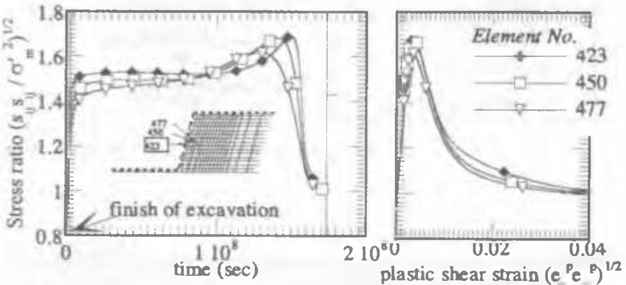



(a) group A, elements along a horizontal line

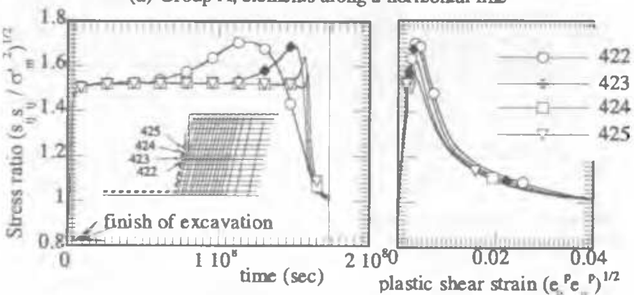



(b) group B, elements along slope surface

Figure 7. Changes in the stress-history ratio with time.

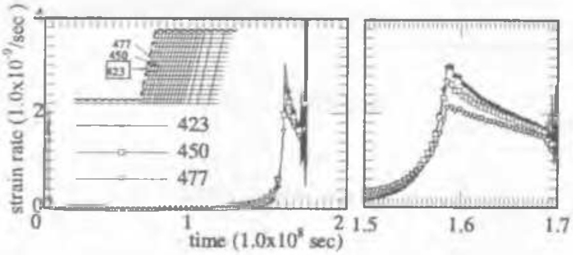



(a) Group A, elements along a horizontal line

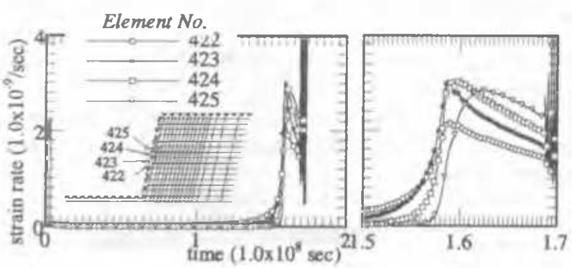



(b) Group B, elements along slope surface

Figure 8. Time changes in the stress ratio and stress-strain relations.




(a) group A, elements along a horizontal line




(b) group B, elements along slope surface

Figure 9. Relation of strain rate and time.

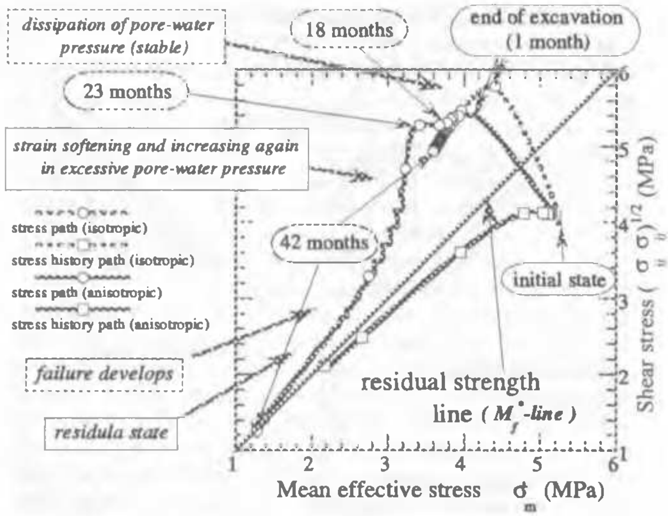



Figure 10. Stress and stress-history paths of element 423.

different elements, the creep failure which is usually marked by an acceleration in strain rate occurs at the same time for all elements. This implies that global failure does not depend on a single element, but depends on the deformation of the surrounding ground.

## 4 INFLUENCE OF THE ANISOTROPIC GROUND

An analysis of the progressive failure of the anisotropic ground is also conducted using the same mesh as that shown in Figure 3.

Figure 10 shows the stress and stress-history paths of element 423 in the isotropic ground and the anisotropic ground whose parameters are listed in Table 2. It is found that the cut slope in the anisotropic ground loses its stability faster than that in the isotropic ground. This is thought to be a result of the effects of the anisotropic behavior of the ground in a stress-strain-dilatancy relation and strength.

Table 2. Anisotropic parameters of the ground.

| $E_x$(MPa) | $E_y$(MPa) | $G_{yx}$(MPa) | $\nu_{yx}$ | $\nu_{zx}$ | $\alpha$ | $\beta$ | $\gamma$ |
|---|---|---|---|---|---|---|---|
| 200.0 | 100.0 | 20.0 | 0.1 | 0.3 | 0.95 | 1.0 | 1.05 |

## 5 CONCLUSION

(1) A cut slope in soft rock may remain stable for a long time after the completion of a rapid excavation. Sometime, however, a failure band may form abruptly in the slope, and then the slope may fail overwhelmingly after only a few months.
(2) The propagation of the shear zone in a cut slope takes the same form as the propagation of the failure zone.
(3) The progressive failure of a cut slope is caused by the redistribution of stress due to stain softening.
(4) The anisotropic behavior of the ground may greatly affect the progressive failure behavior and long-term stability of a cut slope.
(5) By conducting a soil-water coupling analysis, it is possible to simulate the time dependent behavior of geologic materials due to pore water-soil interaction.
(6) The progressive failure of a cut slope can be simulated with a soil-water coupling analysis based on an elasto-plastic model with strain softening.

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