Non-linear space theory and method for excavation engineering and application

Non-linéaire, l'espace théori et méthode, l'application sur l'excavation der ingénierie

X.H.Zhao – *Tongji University, Shanghai, P.R.China*
R.M.Lu – *Guangdong Building Science Research Institute, Canton, P.R.China*
Z.M.Chen – *Shanghai COSCO-Liangwan Property Development Co., Ltd. Shanghai, P.R.China*
J.Gong – *Shanghai No.1 Construction Co., Ltd. Shanghai, P.R.China*

ABSTRACT: In this paper, first, the principle of the non-linear space theory and its method for excavation engineering are described. Then, the software called "SUPER-STAR" based on the non-linear space theory and its method is introduced, which is used in the design of excavation engineering in China. Finally, five case histories of different kinds of excavation are examined, and the comparison of computed values with measured ones in these five cases shows that the theory and its method presented in this paper are reasonable and feasible.

RÉSUMÉ: Dans ce papier, d'abord, on a descrit les principes de l'espace théorie et la méthode non-linéaires sur l'ingénierie d'excavation. Ensuite, on a introduit le logicel avec le nom "SUPER-STAR" qui est utilisé pour les dessins de l'ingénierie d'excavation en Chine sur la base de théorie et la méthode non-linéaires. Enfin, on a examiné cinq cas différentes de l'excavation, et les comparaisons des valeurs calculées et mesurrées pour ces cinq cas montrent que la théorie et la méthode qu'on a représentées sont raisonables et faisables.

1 INTRODUCTION

With a great number of constructions for tall buildings in Shanghai during the past 10 years, deep excavation engineering has been increasing, and the design method for supporting structure of excavation is more and more important. In conventional methods, the earth pressure on the retaining wall is considered as a constant. It is well known that the earth pressure changes with the displacement of the wall.

Now, a non-linear space theory and its method for excavation engineering is presented in this paper taking the retaining wall, the bracing system and soil into consideration as a whole.

2 NON-LINEAR SPACE THEORY & METHOD FOR EXCAVATION ENGINEERING

2.1 *Fundamental Principle*

In this theory and its method, the retaining wall is divided into N vertical beams in the vertical direction. Similarly, it is divided into M horizontal (or curved) beams in the horizontal direction. Thus, the whole retaining wall consists of an intersection beam system with N vertical beams and M horizontal (or curved) beams. The bracing system is also regarded as the horizontal beam system. The relationship between the earth pressure and the displacement of the wall is considered as a non-linearity. Using an iterative method, the condition of static equilibrium and displacement compatibility can be satisfied at each intersection point of the whole structural system. So, the interaction between them can be reflected. As a result, the retaining wall, the soil mass and the bracing system can be analyzed as an integral body.

In analysis, the non-linear FEM for vertical foundation beam is used, which is developed on the basis of the '*m*' method for vertical elastic foundation beam. If replaced by the '*m*' method for vertical elastic foundation beam, the linear space theory & its method for excavation engineering will be obtained.

2.2 *Non-linear vertical foundation beam method*

The core part of the non-linear space theory and its method for excavation engineering is that the non-linear relationship between the earth pressure and the displacement of the wall is supposed to be a hyperbolic type as follows:

$$\Delta p = \frac{u}{\dfrac{1}{\bar{k}} + \dfrac{u}{p - p_0}} = ku \tag{1}$$

or

$$\Delta p = \frac{hu}{\dfrac{1}{\bar{m}} + \dfrac{hu}{p - p_0}} = mhu \tag{2}$$

where

$$k = \frac{1}{\dfrac{1}{\bar{k}} + \dfrac{u}{p - p_0}} \tag{3}$$

and

$$m = \frac{1}{\dfrac{1}{\bar{m}} + \dfrac{hu}{p - p_0}} \tag{4}$$

$p \to p_a$, $k = \bar{k}_a$ and $m = \bar{m}_a$ when $u \le 0.0$,

$p \to p_p$, $k = \bar{k}_p$ and $m = \bar{m}_p$ when $u \ge 0.0$.

Here, Δp is the earth pressure increment and u is the horizontal displacement of the wall. The earth pressure behind the wall is defined as positive when the displacement of the wall occurs in the outward excavation and negative when the displacement of the wall does in the toward excavation. On the contrary, the earth pressure before wall is defined as positive when the displacement of the wall occurs in the toward excavation and negative when the displacement of the wall does in the outward excavation. p_0, p_a and p_p are earth pressures at rest, in active and passive, respectively.

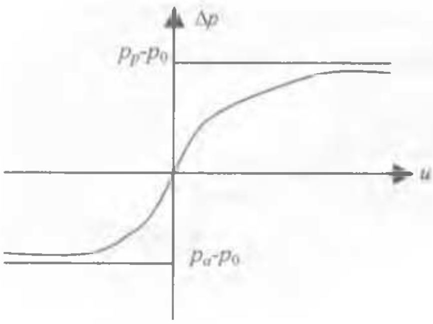

Figure 1. Relationship between earth pressure and displacement of wall

From Eqs.(3) and (4), $k = mh$ and $\bar{k} = \bar{m}h$ can be obtained. Therefore, $\bar{k}_a = \bar{m}_a h$ when $u \le 0.0$ and $\bar{k}_p = \bar{m}_p h$ when $u \ge 0.0$.

For convenience, the Δp - u curve is called the passive curve when $u \ge 0.0$ and the active curve when $u \le 0.0$. $\bar{k}_a$ and $\bar{k}_p$ are the initiative horizontal reaction coefficients of soil of the active and the passive curves, respectively. $\bar{m}_a$ and $\bar{m}_p$ are the proportion coefficients of initiative horizontal subgrade reaction of soil of the active and the passive curves, respectively.

Eqs.(1) and (2) can be expressed by Fig.1. If the displacement of the wall occurs in the outward excavation and is very large, the earth pressure behind the wall tends to be the passive earth pressure and almost unchangeable, i.e. $p = p_0 + \Delta p \rightarrow p_p$. Otherwise, if the displacement of the wall occurs in the toward excavation and is very large, the earth pressure behind the wall tends to be the active earth pressure and almost unchangeable, i.e. $p = p_0 + \Delta p \rightarrow p_a$.

It must be pointed out that the value of m is related to the displacement of the wall and, in turn, the displacement of the wall is dependent on the value of m, and therefore an iterative method has to be used. Iteration will be repeated until the difference between the displacement u_1 of this loop and the proceeding one u_2 is very small. If there are N nodes of the non-linear foundation beams, the iterative accuracy is determined by following equations:

$$D_1 = \sum_{i=1}^{N} |u_{1i} - u_{2i}|$$

$$D_2 = \frac{1}{2}\sum_{i=1}^{N} (|u_{1i}| + |u_{2i}|) \tag{5}$$

$$DJD = \sqrt{D_1 / D_2}$$

The condition of judgement is $DJD < JDX$, where JDX is an iterative accuracy. The iteration will continue until the accuracy is satisfied.

From the above, it can be seen that the non-linear foundation beam theory and its method can reflect that the soil pressure is changed non-linearly with the displacement of the soil mass, and its changing rate decreases with an increase in displacement. Therefore, the non-linear foundation beam method can resolve the main problems existing in FEM for elastic foundation beam.

2.3 *Calculation of earth pressure and water pressure*

The earth pressure at rest and the water pressure must be calculated separately. Rankine's earth pressure formula is used to calculate the active earth pressure, while the simplified Rankine's earth pressure formula with improved Coulomb's formula considering friction between the subsoil and the retaining wall is used to calculate the passive earth pressure as follows:

$$p_p = \sum \gamma_i h_i \cdot K_p + 2c\sqrt{K_{ph}} \tag{6}$$

$$K_p = \frac{\cos^2 \varphi}{\left[1 - \sqrt{\frac{\sin(\varphi + \delta) \cdot \sin \varphi}{\cos \delta}}\right]^2 \cdot \cos^2 \delta} \tag{7}$$

$$K_{ph} = \frac{\cos^2 \varphi \cdot \cos^2 \delta}{[1 - \sin(\varphi + \delta)]^2} \tag{8}$$

where p_p = passive earth pressure at the calculated point; γ_i = unit weight of each soil layer above the calculated point: the natural unit weight is used above the groundwater level; the saturated unit weight is used below the groundwater level if the earth pressure is calculated including the water pressure; the buoyant unit weight is used if the earth pressure and the water pressure are calculated separately. h_i = thickness of each soil layer above the calculated point; δ = friction angle between the subsoil and the surface of the wall (°), $\delta = \left(\frac{2}{3} \sim \frac{3}{4}\right)\varphi$ and $\delta \le 20°$. A bigger value of δ is used for the poor subsoil, such as mucky clay; otherwise, a smaller one is used. $\delta = 0$ when there is no de-watering measures taken during excavation; c and φ = cohesion (kPa) and internal friction angle(°), respectively at the calculated point.

Either the earth pressure including the water pressure is calculated or the earth pressure and the water pressure are calculated separately, depending on the concrete condition.

2.4 *Selection of parameter, $\bar{m}$*

In order to select parameters, $\bar{m}_a$ and $\bar{m}_p$ in the non- linear foundation beam method, a horizontally loaded pile is analyzed first.

As shown in Fig. 2, the earth pressures at rest are in equilibrium on both sides of the pile. Earth pressure increments on both side of the pile are

$\Delta p_1 = m_1 hu$ and $\Delta p_2 = m_2 hu$

thus, the total earth pressure increment is:

$\Delta p = mhu$.

From $\Delta p_1 + \Delta p_2 = hu(m_1 + m_2) = \Delta p = mhu$, we get

$$m = m_1 + m_2 \tag{9}$$

where m is the proportion coefficient of the horizontal soil subgrade reaction when both sides of the pile are considered

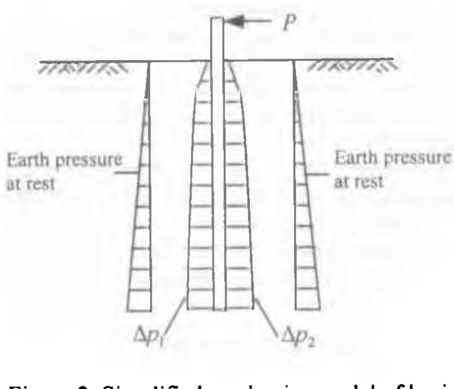

Figure 2. Simplified mechanics model of horizontal loaded pile

When the displacement tends to be zero, and supposing that the curvature of the active curve and the passive curve are continuous at the original point shown in Fig.1, we can obtain

$$m_1 = m_2 = m/2 \tag{10}$$

where m is the proportion coefficient of the horizontal soil subgrade reaction, considering the influence on both sides; m_1 and m_2 are the proportion coefficients of the horizontal soil subgrade reaction on left and right sides, respectively.

From Eq.(10), it can be seen that the initiative values of m_1 and m_2 are all half of the initiative values of m. It should be noted that the values of m in Table 5.4.5 of *Technical Code for Building Pile Foundations* (*JGJ94-94, in Chinese*) are also proportion coefficients of the horizontal soil subgrade reaction considering the influence on both sides of the pile, i.e. it is the same value as that of m in Eq.(9). It is not an initiative value of m but the value of m corresponding to the displacement value shown in that Table. Usually this value is smaller than the initiative value of m. Therefore, the initiative values of m_1 and m_2 are bigger than half of the values in that Table. In the non-linear foundation beam method, the proportion coefficient of initiative horizontal subgrade reaction of soil $\overline{m}$ is equal to the initiative value of m_1 or m_2. That value of m is given in Table 1. The calculation of engineering cases has shown that the value of m in Table 1 is practical and feasible (see Table 5.4.5 of *Technical Code for Building Pile Foundations, JGJ94-94, in Chinese*).

It should be pointed out that in the non-linear foundation beam method, the proportion coefficient of the initiative horizontal subgrade reaction of soil $\overline{m}$ is different from that of m in the 'm' method for elastic foundation beam.

3 "SUPER-STAR" SOFTWARE

Based on the non-linear space theory and its method for excavation engineering, the corresponding software called "SUPER-STAR" has been made and put into the market. This software has 10 functions as follows:

a) It can be applied to all kinds of shapes of excavation and can simulate every step of the excavation process;
b) It can calculate the deformations and interior forces of the whole system as well as the retaining wall in any position;
c) It can consider the horizontal and vertical space effect of the retaining wall as well as the anisotropy of material;
d) It can consider that the earth pressure before or behind the wall changes non-linearly with the displacement of the wall, and the 'm' method can be used also in this software;
e) It can be applied to different types of the retaining wall, such as sheet pile wall, diaphragm wall, steel sheet pile wall, continuous pillar wall, cement-soil wall and a few piles inserted into cement-soil and etc. Especially, these selected types of retaining wall can be used in the different positions in the same excavation;
f) It can consider the displacement compatibility and static equilibrium at connected positions between the bracing system and the retaining wall, which can not be solved using the plane calculation method;
g) It can calculate not only the supporting structure with pre-axial-force to interior braces or with anchor, but also the supporting structure without any bracing system or anchor;
h) It can consider the soil stabilization in the pit and predict the settlement behind the wall;
i) It can display the moment, axial force, shear stress and the displacement of foundation beams, horizontal beams and bracing system, especially, the displacement of the whole supporting structure visually;
j) It can be integrated with the current codes to check all kinds of excavation engineering. Furthermore, the stability of soil nail wall or slope also can be checked overall and dynamically using the new method.

4 ENGINEERING CASES

In order to check the rationality, feasibility and applicability of the non-linear space theory and its method for excavation engineering and its corresponding "SUPER-STAR" software, five different projects are examined for excavation engineering with field measurement data in Shanghai.

Case I: Excavation Engineering for New Shanghai International Mansion. A diaphragm wall with RC bracing is used as a retaining wall, the pit is approximately square and the depth of excavation is 13.4m. Using the non-linear space "SUPER-STAR" software, the calculated results coincide with measured ones, and this software can offer a diagram of the whole space displacement contour. On the contrary, when the linear space 'm' method is used, a considerable outward displacement of the pit is obtained. Obviously, it is irrational, as shown in Fig.3.

Case II: Excavation Engineering for Heng Long Square. A diaphragm wall with RC bracing is also used as a retaining wall, but the pit is of L shape, the depth of excavation 18.2m and its area about 25,000m^2. Using the "SUPER-STAR" software, the calculated displacements accord more with measured results. However, the calculated axial forces using the SAP-5 software are much different. The reason is very obvious, because the pit is of L shape. The former really considers it as a space problem, while the latter considers it only as a plane problem and the earth pressure is taken as a constant value.

Table 1. Values of m

No.	Subsoil	Pre-cast pile or steel pile		Bored pile	
		m (MN/m^4)	Corresponding horizontal displacement of single pile at ground surface (mm)	m (MN/m^4)	Corresponding horizontal displacement of single pile at ground surface (mm)
1	Muck, mucky soil and saturated collapsible loess	2~4.5	10	2.5~6	6~12
2	Clayey soil with I_L>1 or 0.75<I_L≤1; silt with e>0.9, loose silty fine sand, loose or slightly dense fill	5.4~6.0	10	6~14	4~8
3	Clayey soil with 0.25<I_L≤0.75; silt with e=0.75~0.9, collapsible loess, medium dense fill and slightly dense fine sand	6.0~10	10	14~35	3~6
4	Clayey soil with 0<I_L≤0.25 or I_L≤0, collapsible loess, silt with e<0.75, medium coarse sand with medium dense, dense old fill	10~22	10	35~100	2~5
5	Medium dense to gravely dense sand, crashed stone			100~300	1.5~3

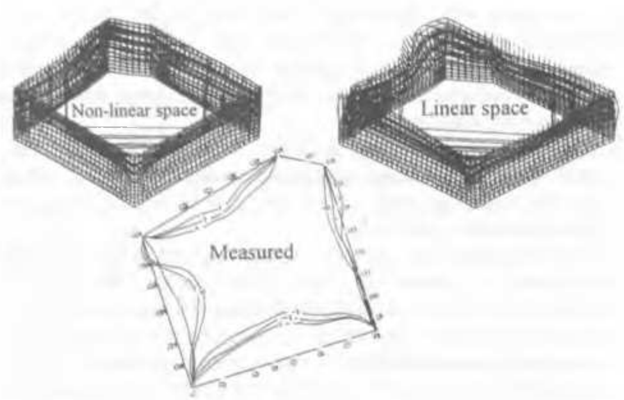

Figure 3. Comparison between measured values and computed ones

Case III: Excavation Engineering for 2nd Phase of Dong Ying Garden. Bored piles with RC bracing are used as the retaining wall. The pit is rectangular and the depth of excavation is 9.67m in tower and 8.67m in podium. This project has more complete measured data for checking the rationality and feasibility of different methods. Now, using software of SAP-5 and SUPER-STAR, the calculated results are in good agreement with the measured data. This shows that the non-linear space "SUPER-STAR" software is totally rational and feasible.

Case IV: Excavation Engineering for Shanghai Sun Square Mansion. Cement mixed piles are used as the gravity retaining wall with 10 different sections, an irregularly rectangular pit. The circumference of the pit is 413m and its depth varying from 4.5m to 6.7m. Different cross sections of the retaining wall are used with different widths of 3500mm~6200mm and different depths of 10m~15m. This "SUPER-STAR" software can calculate such a gravity retaining wall with different depths and different cross sections. Furthermore, the calculated maximum displacement of the wall coincides comparatively with the measured one. To our knowledge, it may be the first time to solve such a complicated problem.

Case V: Excavation Engineering for Kang Ning Mansion. A composite retaining wall is used. The pit is circular and the depth of excavation is 6.3m. This retaining wall consists of three types of the retaining wall at different places of the pit: composite retaining wall, continuous pillar wall and cement soil gravity retaining wall. Using the non-linear space "SUPER-STAR software, this kind of retaining wall with different composite and gravity can be calculated and the displacements are close to measured ones. To our knowledge, it may also be the first time to solve such a complicated problem.

In addition, this "SUPER-STAR" software was also used to design a subway station with a depth of 26.5m in Guangzhou, the calculated results were in good agreement with measured ones and about 1/3 of the number of braces had been saved. Furthermore, it is now being used to calculate a new engineering project of specially deep and big excavation in Shanghai.

5 CONCLUDING REMARKS

Based on the initiated non-linear relationship between the earth pressure and the displacement of the wall and a special iterative method, the non-linear space theory and its method for excavation engineering including corresponding "SUPER-STAR" software are rational and feasible. Obviously, they have three major advantages over other available methods and software as follows:

First, they can reflect the non-linear relationship between the earth pressure and the displacement;

Second, they can also reflect the space effect of the horizontal and vertical directions of the retaining wall as well as the anisotropy for materials in horizontal and vertical directions;

Third, they can indicate the displacements and interior forces at any intersection point of the whole structural system, and create a favorable condition for an optimum design of the whole supporting structure.

Examinations of engineering cases and practice have fully verified the superiority of the non-linear space theory and its method for excavation engineering and the corresponding "SUPER-STAR" software. Because of its unique advantage over other software, this software is used in the design of excavation engineering in China. It can be expected that the SUPER-STAR software will play a more and more important role in engineering practice.