Sand drains and stone columns-general unifying theory

Drains de sable et colonnes ballastées-théorle générale d'unification

A.Saha – *Irrigation & Waterways Dte., Govt of West Bengal, India*

ABSTRACT: The efficacy of sand drains in accelerating consolidation settlements of soft cohesive deposits has been proved beyond doubt. The study of consolidation of fine grained soils by drain wells (Barron, 1948) is a classical example of the underlying mathematics governing the radial flow through the installed drain well, usually consisting of sand as a backfill material. Till today, the most frequently used equation to account for radial drainage is due to his pioneering work. Barron equation account for the factors affecting the drainage flow through the drain well without navigating into the strength factor that the well or column incorporates to the virgin ground due to its inherent higher rigidity. The present paper addresses this very issue and examines the consolidation of the composite system due to radial drainage in relation to the stress and strain tensors and predicts the performance of a stone column reinforced ground loaded by a rigid raft (equal vertical strain) at upper boundary. The strain rate and the settlement reduction factor have been determined.

INTRODUCTION

Reinforcement of ground by granular "pinning" in the form of columnar inclusions to support structural load beneath foundations have become increasingly popular due to its ability to sustain increased structural loads under reduced settlements. The field application of the technology has gained momentum than the intricate design methodology due to its simplicity in installation and intuitive performance. This very improvement is due to the complex interaction within the 'unit-cell' comprised of the installed column in ambient ground under sustained vertical stress and the interdependence of problem geometry and material properties of the two different soils beneath foundations-one 'native' and the other 'backfilled'. The qualitative beneficial effects of installation in weak or difficult subsoil deposits are manifested mainly in the form of increased load carrying capacity and significant reduction in total and differential settlements. Moreover, minimization of post construction settlements effected by accelerating consolidation, reduced risk of liquefaction, aiding in improvement of slope stability of embankments and natural slopes and effecting changes in the dynamic response are the other benefits that are well appreciated. The problem becomes complicated as one tries to quantify them. The 'unit-cell' concept of a tributary soil cylinder encapsulating an installed column with some reasonable assumptions and boundary conditions (Goughnour & Bayuk, 1979; Balaam & Brooker, 1981; Balaam & Poulos, 1983; Goughnour, 1983; Van Impe & De Beer, 1983; Madhav & Miura, 1994; Saha & De, 1994, 1995; Saha & Das, 1998) deals with the load-settlement behavior of the composite system. Though radial pore water pressure dissipation through the stone column has been given due qualitative acknowledgement, the same factor has not been included in the analytical model (except in Poorooshasb & Meyerhof, 1996, though they have assumed Poisson's ratio equal to zero) due to reasons of over complication of the underlain mathematics. The present paper ventures into this very issue of radial drainage and functional output behavior of the reinforced ground with Fig.1 depicting the problem geometry and Fig.2 the problem definition.

MATHEMATICAL FORMULATION

The mean effective soil stress at any position and time p' is defined by

$$p' = {}^{1}/_{3}\,[(\sigma_r - u) + (\sigma_\theta - u) + (\sigma_z - u)] \quad \ldots\ldots (1)$$

Where σ_r, σ_θ, σ_z are the components of total radial, tangential and vertical stress tensors expressed in polar-cylindrical space and ε_r, ε_θ, ε_z are the components of strain tensors in the same space and 'u' is the excess pore water pressure (p.w.p.).
The strain tensors are expressed as:

$$\varepsilon_r = (1/E)\{\sigma_r - \mu(\sigma_\theta + \sigma_z) - u(1-2\mu)\} \quad \ldots\ldots (2a)$$

$$\varepsilon_\theta = (1/E)\{\sigma_\theta - \mu(\sigma_r + \sigma_z) - u(1-2\mu)\} \quad \ldots\ldots (2b)$$

$$\varepsilon_z = (1/E)\{\sigma_z - \mu(\sigma_r + \sigma_\theta) - u(1-2\mu)\} \quad \ldots\ldots (2c)$$

Where, E, μ are the Young's modulus & Poisson's ratio of in-situ soil. Moreover, the strain tensors are related to the radial displacement $\hat{u}$ as follows:

$$\varepsilon_r = \partial\hat{u}/\partial r \quad \ldots\ldots (2a') \quad ; \quad \varepsilon_\theta = \hat{u}/r \quad \ldots\ldots (2b')$$

Regardless of the elastic versus plastic state, the horizontal or radial stress field can be analyzed by the equation of radial equilibrium:

$$\partial\sigma_r/\partial r = (\sigma_r - \sigma_\theta)/r \quad \ldots\ldots (3)$$

If it is further assumed that the soil stresses and deformations are related by the theory of elasticity, the elastic behavior of the material being fully described by:

$$\sigma_r = (\lambda + 2G)\varepsilon_r + \lambda(\varepsilon_\theta + \varepsilon_z) \quad \ldots\ldots (4a)$$

$$\sigma_\theta = (\lambda + 2G)\varepsilon_\theta + \lambda(\varepsilon_r + \varepsilon_z) \quad \ldots\ldots (4b)$$

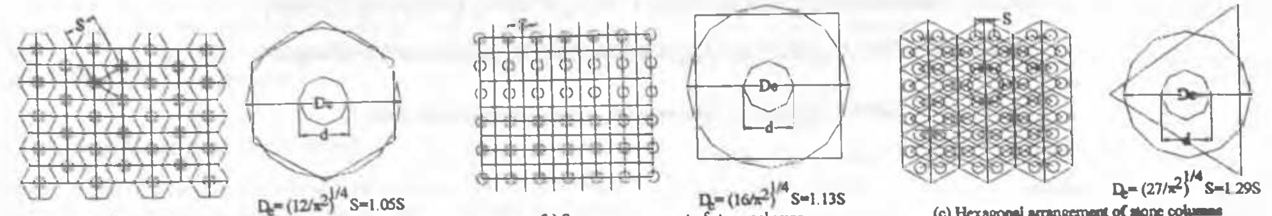
$$\sigma_z = (\lambda + 2G)\varepsilon_z + \lambda(\varepsilon_r + \varepsilon_\theta) \quad \ldots\ldots (4c)$$

Where, $G = E/2(1+\mu)$ ......(5a) & $\lambda = E\mu/(1+\mu)(1-2\mu)$ ......(5b)

With this hypothesis the equilibrium condition (3) becomes a differential equation of radial displacement $\hat{u}(r)$ or $\hat{u}$:-

$$(\partial^2\hat{u}/\partial r^2) + (1/r)(\partial\hat{u}/\partial r) - \hat{u}/r^2 = 0 \quad \ldots\ldots (6)$$

The integration constants are determined by the boundary conditions. Assuming a regular hexagonal, triangular or squared plan pattern of column arrangement, and further assuming that the mid-point between adjacent columns ('unit-cell' boundary) will not move (frictionless roller joint) and represent the radius



$D_e = (12/\pi^2)^{1/4} S = 1.05S$
(a) Triangular arrangement of stone columns

$D_e = (16/\pi^2)^{1/4} S = 1.13S$
(b) Square arrangement of stone columns

$D_e = (27/\pi^2)^{1/4} S = 1.29S$
(c) Hexagonal arrangement of stone columns

Figure 1. Problem geometry showing the domain of influence of each column

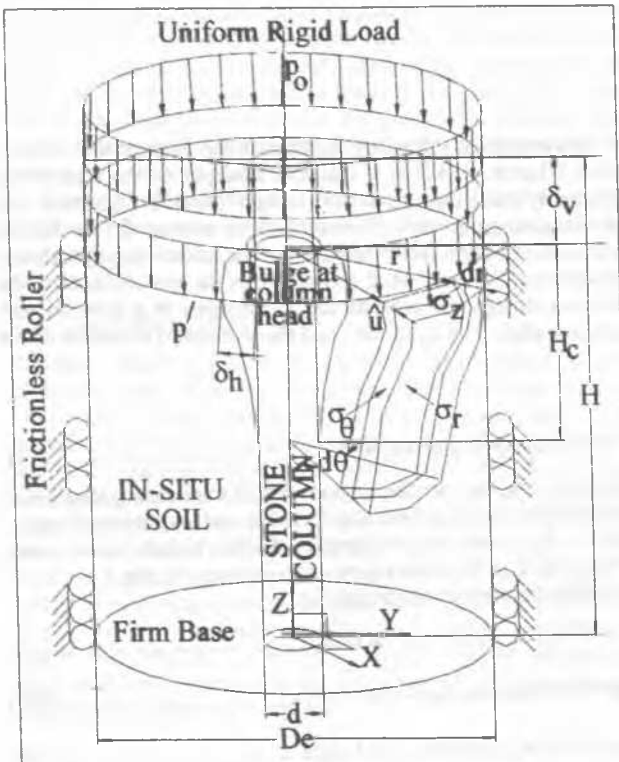


Figure 2. Axi-symmetric model of unit-cell with cylindrical co-ordinates showing boundary conditions & stress-deformations

of influence ($D_e/2$) or effective cell diameter and $^d/_2$ be the initial radius of stone column. The boundary conditions become:

$\hat{u}(D_e/2) = 0$ ....(7a) & $\hat{u}(^d/_2) = \delta_h$.............(7b)

in which δ_h is the radial expansion of the cavity or the column 'bulge'. The solution to equation (6) can be shown to be:

$\hat{u}/r = \psi[\{1-(^{De}/_{2r})^2\}/\{1-(^{De}/_d)^2\}]$................................(8)

Where, $\psi = 2\delta_h/d$ = Diametrical strain

Comparing equations (2b′) & (8) at the column-soil interface (inner boundary) i.e., at $r = {}^d/_2$, $(\varepsilon_\theta)_{d/2} = \psi = \varepsilon_3$............(9a)

Comparing equations (2a′) & (6) at the 'unit-cell' boundary (outer boundary), i.e., at $r = {}^{De}/_2$, and putting $(\varepsilon_\theta)_{De/2} = 0$ and denoting $A_r = (^d/_{De})^2$ = Area ratio or replacement factor,

$(\varepsilon_r)_{De/2} = -2\psi\{A_r/(1-A_r)\} = -2\varepsilon_3\{A_r/(1-A_r)\}$......................(9b)

The negative sign in the above expression implies that the column and soil strains are directionally opposite to each other. Giving due cognizance of the fact of ***equal vertical strain*** of soil and column, let us denote the vertical strain of the composite system as ε_1 instead of ε_z since the column resembles a large tri-axial test specimen, where ε_1 is the vertical column strain. Hence, putting $(\varepsilon_z)_{De/2} = \varepsilon_1$ & $(\varepsilon_\theta)_{De/2} = 0$, at 'unit-cell' boundary, $(\sigma_z)_{De/2}$ becomes,

$(\sigma_z)_{De/2} = (\lambda+2G)\varepsilon_1 - 2\lambda\varepsilon_3\{A_r/(1-A_r)\}$ ....(10)

For radial flow to a central drain well the basic partial differential equation for consolidation in stratified soil is,

$-(\partial p'/\partial t) = (^1/_3)\, c_h[(\partial^2 u/\partial r^2) + (1/r)(\partial u/\partial r)]$ ....(11)

Where, c_h is the coefficient of consolidation for horizontal flow

$c_h = k_h(1+e)/a_v\gamma_w = k_h/m_v\gamma_w = E\,k_h/\gamma_w(1-2\mu K_o)$...............(12)

Where e, k_h, a_v, m_v are the void ratio, coefficient of permeability for horizontal flow, coefficient of compressibility and of volume compressibility of in-situ soil respectively and γ_w is the unit weight of water and $K_o = \mu/(1-\mu)$. Further it can be shown that,

$\partial\varepsilon_\theta/\partial r = (1/r)(\varepsilon_r - \varepsilon_\theta)$ ....(13) ; $\partial\sigma_r/\partial r = (1/r)(\sigma_\theta - \sigma_r)$.......(14)

Differentiating equation (2b) w.r.t. r, and further simplification of equations (2a) and (2b) results in

$(\partial\sigma_\theta/\partial r) - (\partial u/\partial r)(1-2\mu) = (1/r)(\sigma_r - \sigma_\theta)$ ....(15)

Adding equations (14) & (15) one gets

$\partial(\sigma_r+\sigma_\theta)/\partial r = (1-2\mu)(\partial u/\partial r)$ ....(16)

Integration of equation (16) yields

$(\sigma_r+\sigma_\theta) = (1-2\mu)u + f(t)$ ....(17)

Where, f(t) is a function of time only. Simplifying equations (2b) & (9) & by boundary condition at outer periphery of unit-cell,

$f(t) = (\sigma_r)_{De/2} + \mu\{(\sigma_z)_{De/2} + (\sigma_r)_{De/2}\}$ ....(18)

Putting back the value of f(t) in equation (17)

$(\sigma_r+\sigma_\theta) = (1-2\mu)u + (1+\mu)(\sigma_r)_{De/2} + \mu(\sigma_z)_{De/2}$ ....(19)

Comparing equations (1) & (19),

$p' = {}^1/_3[\{E\varepsilon_1 + \mu(\sigma_r+\sigma_\theta) + (1+\mu)(\sigma_r)_{De/2} + \mu(\sigma_z)_{De/2}\} - (1+4\mu)u]$...(20)

Putting the value of p' in equation (11)

$$\frac{\partial}{\partial t}[(1+4\mu)u - \{E\varepsilon_1 + \mu(\sigma_r + \sigma_0) + (1+\mu)(\sigma_r)_{De/2} + \mu(\sigma_z)_{De/2}\}] = c_h\{\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\} \text{......... ....(21)}$$

Equation (21) can be solved once $(\sigma_r)_{De/2}$ & $(\sigma_z)_{De/2}$ are determined. It may be noted that at column boundary excess p.w.p., u=0. Furthermore, the cylindrical strain compatibility at soil-column interface necessitates $\varepsilon_\theta = \varepsilon_3$, where ε_3 is the lateral column strain. On substituting ε_3 in place of ε_θ in eqn. (2b),

$(\sigma_\theta)_{d/2} = E\varepsilon_3 + \mu\{(\sigma_r)_{d/2} + (\sigma_z)_{d/2}\}$ ....(22)

And simplification with equation (19) yields

$(\sigma_r)_{d/2} = (\sigma_r)_{De/2} - \{E/(1+\mu)\}\varepsilon_3$ ....(23)

Equation (19) in conjunction with equation (14) yields the first order differential equation

$$(\partial\sigma_r/\partial r)+(2\sigma_r/r)=(1/r)(\sigma_r+\sigma_\theta)\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(24)$$

Which can be simplified to

$$(D_e/2)^2\times(\sigma_r)_{De/2}-(d/2)^2\times(\sigma_r)_{d/2}=(1-2\mu)\int_{d/2}^{De/2} r u\,\partial r+\left\{\frac{(D_e/2)^2-(d/2)^2}{2}\right\}\times$$
$$\{(1+\mu)(\sigma_r)_{De/2}+\mu(\sigma_z)_{De/2}\}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(25)$$

Substituting for $(\sigma_r)_{d/2}$ from eqn.(23) and rearranging one gets,

$$(\sigma_r)_{De/2}=\frac{2}{(1-\mu)\{(D_e/2)^2-(d/2)^2\}}\Big[(1-2\mu)\int_{d/2}^{De/2} r\,u\,\partial r-$$
$$\frac{E}{(1+\mu)}\varepsilon_3(d/2)^2+\frac{\mu}{2}\{(D_e/2)^2-(d/2)^2\}(\sigma_z)_{De/2}\}\Big]\ldots\ldots(26)$$

Comparing equations (26), (19) & (10), simplifying & using normalized co-ordinates, $R=r/0.5D_e$, $T=c_h t/(0.5D_e)^2$, the ***governing equation of the problem*** reduces to

$$\boxed{\begin{array}{l}(1+2\mu)\dfrac{\partial u}{\partial T}-\dfrac{2(1-2\mu K_o)}{(1-A_r)}\times\dfrac{\partial}{\partial T}\displaystyle\int_{\sqrt{A_r}}^{1}Ru\,\partial R=\dfrac{1}{(1+\mu)}\left(\dfrac{\partial^2 u}{\partial R^2}+\dfrac{1}{R}\dfrac{\partial u}{\partial R}\right)-F(A_r,\dot{\varepsilon}_1)\\ \qquad\qquad\qquad\qquad\qquad\qquad\ldots\ldots(27)\\ \text{Where, } F(A_r,\dot{\varepsilon}_1)=E\left\{1-\dfrac{K_o}{(1-2\mu K_o)}\right\}\times\left\{\dfrac{\dot{\varepsilon}_1}{(1+\mu)}+\dfrac{2A_r\dot{\varepsilon}_3}{(1-\mu)(1-A_r)}\right\}\ldots(28)\end{array}}$$

Where, $\dot{\varepsilon}_1=\partial\varepsilon_1/\partial t$ & $\dot{\varepsilon}_3=\partial\varepsilon_3/\partial t$ are the vertical & diametrical strain rates of the stone column.

Boundary conditions are:- at $R=\sqrt{A_r}$, $u=0$ & at $R=1$, $\partial u/\partial R=0$ & at $T=0$, $u=u_o$, where, =initial uniform pwp. Further, equation (28) represents a boundary condition since it involves *vertical strain rate* which when multiplied by H, is equal to the rate of settlement of foundation.

ANALYSIS

Equal vertical strain at top of 'unit' (i.e., upper boundary) causes diametrical strain of column. The stresses are transferred by arching action to the in-situ soil and the dispersion angle in arching varies depending upon soil strength and problem-geometry. Below a certain stress level, consolidation through the column tends to completion without plastic deformation of the stone, whence the granular particles get reoriented and packed in the best possible fashion warranted by the size and shape of particles. In other words, elastic behavior of the columns is expected when the applied load is below a certain level, and the total settlement of the composite system is relatively small. On further increase of load, when the strain at column top exceeds a certain level - the *'threshold strain'*, stone particle to particle slippage occurs marked by formation of bulge at column head. It is this bulge that dictates the state of plastic behavior along the critical height of stone column. Eventually consolidation tends near completion for critical column depth and the stone in this zone is left with such internal stresses entrapped that a state of impending (or constrained) plastic equilibrium will exist. Below the critical depth, the strains are below the threshold level and the column behaves elastically. This is so, because the overburden and confining stresses applied to the stone column by the in-situ soil increases with depth and the ability of the unit cell to sustain load without plastic deformation or bulging of the column, increases with depth. Hence at higher depths, the total load on the unit-cell components may reasonably be assumed to obey Boussinesq's theory. In the above light, incompressibility or constant volume deformation of stone column remains a valid proposition within the domain of its *'limits of applicability'* in ground improvement. Constant volume deformation of column yields:

$$(\pi d^2/4)\,H=\{\pi(d+2\delta_h)^2/4\}H_c+(\pi d^2/4)\{H-(\delta_v+H_c)\}\ldots\ldots\ldots(29)$$

Where, δ_h & δ_v are the horizontal & vertical displacements under loaded condition, H_c the critical height & H the length of the stone column. On simplification,

$(\delta_v/H)=(H_c/H).\psi.(2+\psi)\ldots(30a)$; or, $\varepsilon_1=h_f.\,\varepsilon_3.(2+\varepsilon_3)\ldots.(30b)$

Where, $h_f=H_c/H$ =Critical height factor; Now, since $\varepsilon_1=f(\varepsilon_3)$, Differentiating equation (30b) w.r.t. T and transposing,

$$\dot{\varepsilon}_3=\left\{\frac{\dot{\varepsilon}_1}{2h_f(1+\varepsilon_3)}\right\}\ldots..(31)$$

Assuming, $u(R,T)=f_1(R)\times f_2(T)$, eqn. (27) becomes,

$$(1+\mu)(1+2\mu)\left\{\frac{\partial f_2(T)}{f_2(T)\partial T}\right\}=$$
$$\frac{\left[\dfrac{\partial^2 f_1(R)}{\partial R^2}+\dfrac{1}{R}\dfrac{\partial f_1(R)}{\partial R}\right]-\dfrac{(1+\mu)F(A_r,\dot{\varepsilon}_1)}{f_2(T)}}{\left[f_1(R)-\dfrac{2(1-2\mu K_o)}{(1+2\mu)(1-A_r)}\displaystyle\int_{\sqrt{A_r}}^{1}Rf_1(R)dR\right]}=-\alpha_1^{\,2}(\text{say})\ldots.(32)$$

Differentiating both sides w.r.t. R

$$(1+\mu)(1+2\mu)\frac{\partial f_2(T)}{f_2(T)\partial T}=$$
$$\frac{\left[\dfrac{\partial^3 f_1(R)}{\partial R^3}+\dfrac{\partial^2 f_1(R)}{R\partial R^2}-\dfrac{1}{R^2}\dfrac{\partial f_1(R)}{\partial R}\right]}{\left[\dfrac{\partial f_1(R)}{\partial R}-\dfrac{2(1-2\mu K_o)}{(1+2\mu)(1-A_r)}\left\{\displaystyle\int_{\sqrt{A_r}}^{1}\dfrac{R\partial f_1(R)}{\partial R}dR+\int_{\sqrt{A_r}}^{1}f_1(R)dR\right\}\right]}$$
$$=-\alpha_2^{\,2}(\text{say})\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(33)$$

Assuming, $f_1(R)=\sum_{i=0}^{\infty}a_i R^{(m+i)}$, the R.H.S. of eqn. (32) reduces

$$\text{to}\ \frac{\displaystyle\sum_{i=0}^{\infty}a_i.(m+i)^2.R^{(m+i-2)}-\frac{(1+\mu)F(A_r,\dot{\varepsilon}_1)}{f_2(T)}}{\displaystyle\sum_{i=0}^{\infty}a_i.R^{(m+i)}-\frac{2(1+2K_o)}{(1-A_r)}\sum_{i=0}^{\infty}\frac{a_i}{(i+2)}\left\{1-(\sqrt{A_r})^{(m+i+2)}\right\}}=-\alpha_1^{\,2}$$

& equation (33) reduces to ....(34)

$$\frac{\displaystyle\sum_{i=0}^{\infty}a_i.(m+i)^2.(m+i-2).R^{(m+i-3)}}{\displaystyle\sum_{i=0}^{\infty}a_i.(m+i).R^{(m+i-1)}-\frac{2(1-2\mu K_o)}{(1+2\mu)(1-A_r)}\sum_{i=0}^{\infty}a_i\left\{1-(\sqrt{A_r})^{(m+i+1)}\right\}}=-\alpha_2^{\,2}$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots(35)$$

From recurrence relation, i.e., equating coefficients of like powers of R^m on both sides, one gets the root of the indicial equation, $m=0$ & $a_1=a_3=a_5=a_7=\ldots\ldots=0$. And further assuming $a_o=1$,

$$a_2=\frac{(-\alpha^2)^1}{2^2}.1\ ,a_4=\frac{(-\alpha^2)^2}{2^2.4^2}.1,a_6=\frac{(-\alpha^2)^3}{2^2.4^2.6^2}.1,a_8=\frac{(-\alpha^2)^3}{2^2.4^2.6^2.8^2}.1..(36)$$

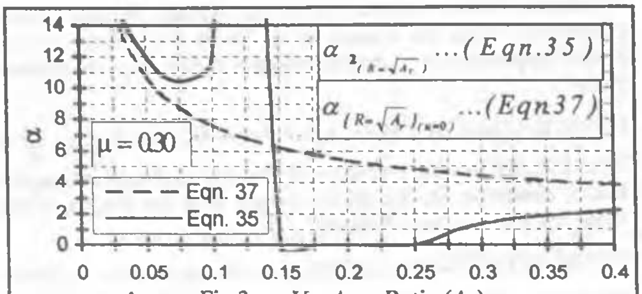


Fig.3. α Vs. Area Ratio (Ar)

The boundary conditions: R=√Ar, u_r =0 & R=1, ∂u/∂R=0 yields respectively

$$\sum_{i=0}^{\infty}\frac{(-1)^i\left\{\frac{\alpha_{(R=\sqrt{A_r})}}{2}\sqrt{A_r}\right\}^{2i}}{(i!)^2}=0\ \ldots(37)..\&\ldots\sum_{i=0}^{\infty}\frac{(-1)^i\,2i\left\{\frac{\alpha_{(R=1)}}{2}\right\}^{2i}}{(i!)^2}=0\ \ldots(38).$$

The variation of α versus A_r is plotted in Fig.3.

If $\bar{u}_r$ represent the average excess pwp of the entire soil cylinder,

$$\pi\overline{u_r}\left\{(D_e/2)^2-(d/2)^2\right\}=\int_{d/2}^{D_e/2}2\pi u_r r\,dr\ ;\text{on simplification,}$$

$$\bar{u}_r=\frac{2f_2(T)}{(1-A_r)}\sum_{i=0}^{\infty}\frac{(-1)^i\left(\frac{\alpha_{2(R=\sqrt{A_r})}}{2}\right)^{2i}}{(i+2)(i!)^2}\left\{1-(\sqrt{A_r})^{(i+2)}\right\}..(39)$$

Comparing equation (39) & L.H.S. of equation (33) & noting that at T=0, $\bar{u}_r=\bar{u}_o$, the following expression results:

$$f_2(T)=\frac{u_o}{\left\{\frac{2}{(1-A_r)}\sum_{i=0}^{\infty}\frac{(-1)^i\left\{\left(\frac{\alpha_{2(R=\sqrt{A_r})}}{2}\right)^{2i}\{1-(\sqrt{A_r})^{(i+2)}\}\right\}}{(i+2)(i!)^2}\right\}}\exp\left\{\frac{-(\alpha_{2(R=\sqrt{A_r})})^2T}{(1+2\mu)(1+\mu)}\right\}...(40)$$

Putting the value of $f_2(T)$ in equation (39)

$$\bar{u}_r=\bar{u}_o\exp.\left\{\frac{-(\alpha_{2(R=\sqrt{A_r})})^2T}{(1+2\mu)(1+\mu)}\right\}...(41)$$

The average degree of consolidation=$\bar{U}$=1-($\bar{u}_r$ / $\bar{u}_o$)

$$\bar{U}=1-\exp.\left\{\frac{-(\alpha_{2(R=\sqrt{A_r})})^2T}{(1+2\mu)(1+\mu)}\right\}...(42)$$

The excess p.w.p. due to radial flow, u_r is

$$u_r=u_o\sum_{i=0}^{\infty}\left\{\frac{(1-A_r)(i+2)R^{2i}}{2\{1-(\sqrt{A_r})^{(i+2)}\}(i!)^2}\right\}\exp.\left\{\frac{-(\alpha_{2(R=\sqrt{A_r})})^2T}{(1+2\mu)(1+\mu)}\right\}...(43)$$

Figure 4 depicts the performance of the reinforced ground and compares the results with Barron theory.

CONCLUSIONS

An effort has been made in the present paper to quantify the **load-time-settlement** behavior of a foundation system composed of a rigid raft at upper boundary supported by a number of stone 'columns' or 'wells' arranged in a regular array in the host soil. A mathematical axi-symmetric model of unit cell has been analyzed taking into account the geometry of stone column installation and the strength and deformation properties of the ambient subsoil and backfilled granular column. The governing equation for rate of settlement, degree of consolidation excess pwp has been obtained in terms of relevant parameters. The output results have been superimposed on the Barron theory to highlight the relative merits of its applicability.

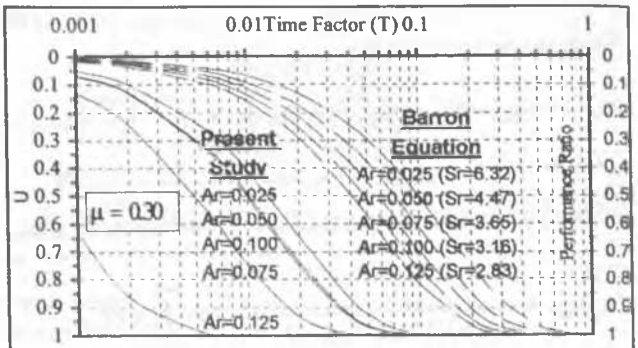


Fig.4 Degree of consolidation(U) Vs. Time Factor(T)

REFERENCES

1.Balaam, M.P. & Poulous, H.G. 1983. The Behaviour of Foundations Supported by Clay Stabilised by Stone Columns. *Proc. Speciality session 5, VIII Euro. Conf. on Soil Mech.& Fdn. Engg.*, Helsinki, 2: 199-204.

2.Barron, R.A.1948. Consolidation of fine-grained soils by drain wells. *Transactions, A.S.C.E.* 113: 718-742.

3.De P. K. and Saha Abhijit. 1995. Area Ratio Optimisation in Soil-Stone Column Interaction. *Proc. 10th Asian Regional Conf. on SMFE*, Beijing, China.1:395-398.

4.Goughnour, R.R. 1983.Settlement of Vertically Loaded Stone Columns in Soft Ground. *Proc. Speciality session 5, VIII Euro. Conf. on Soil Mech. and Fdn. Engg.*, Helsinki, 2: 235-240.

5.Madhav, M.R. and Miura, N. 1994. Stone Columns. *Proc. XIII International Conf. on Soil Mech. and Fdn. Engg.*, India, 5:163-164.

6.Poorooshasb. H.B. & Meyerhof G.G. 1996. Consolidation settlement of rafts supported by stone columns. *Geotechnical Engineering, Dec.1996, 27(2):83-92.*

7.Saha Abhijit and De P. K. 1994. Field and Analytical Study of Stone Columns by Non -Linear Strain. *Proc. XIII International. Conf. on Soil Mech. & Fdn. Engg.* , New Delhi, 3:1211-1214,.

8. Saha Abhijit & Das S.C. 1998. The Strain Function and Failure Mechanics of Stone Columns. *Proc. Indian Geotech. Conf.*, New Delhi , India ,1: 255-260.

9. Van Impe, W. and De Beer, E. 1983. Improvement of Settlement Behaviour of Soft Layers by means of Stone Columns." *Proc. 8th Euro. Conf. on Soil Mech.& Fdn. Engg.*, Helsinki,1: 309-312.

10. Vesic, A.S. 1972. Expansion of Cavities in Infinite Soil Mass. *Journal of the SMFE Div., ASCE*, 3: 265-290.