Reliability analysis of progressive failure of slopes and its application

Analyse de la fiabilité des rapture asymptotique d'accolement et application

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ABSTRACT: In this paper, the approach of probabilistic analysis of progressive failure of slope is presented. The proposed procedure is based on the 2-D reliability model of progressive slope failure and Monte-Carlo method. As an illustrative form, the surplus-thrust method of slope stability analysis is incorporated in the calculation method of progressive slope failure and a computer program is developed according to the above analysis. Then the results of progressive failure probabilistic analysis of one case study are presented.

1 INTRODUCTION

The fact that movements of the slope can occur for many months or years before the slope finally collapses indicates that the failure process is progressive rather than instantaneous as assumed in most forms of stability analyses. In fact, both internal and external mechanical parameters such as stresses, strength of rock masses and water pressure or seepage force are non-uniformly distributed along the sliding surface. Hence it is impossible for the whole slope to be simultaneously in the state of failure and it is undoubted that the failure of slope will be initiated from the zone with the driving stress higher than shear strength available. Therefore progressive failure process should be considered in the stability analysis. In the progressive failure process, the failure starts from the local area which has the biggest failure probability and then may spread and propagate to the neighbor area and finally the whole slope reach eventual collapses, or the failure stops to propagate and the whole slope is also stable.

In the process of the analyses, when the local area is in the state of failure, the strength is reduced to residual strength and the surplus of shear stresses are undertaken by the neighbor area units. The unfailing area still poses the peak strength. The failure probability is used to evaluate the slope stability and the conditional probability is used to express the possibility of the failure propagation. The initial failure slice is defined as the slice of which failure probability is the largest and more than the given failure threshold in all of the slices. It is thought that the failure of slope starts from this slice and expands to its neighbor slices one by one. And the failure of slope will not take place if the largest failure probability is less than the given failure threshold.

The threshold value of the progressive failure probability is determined by considering many factors. At present, the failure probability is determined as $3\times10^{-2}\sim1\times10^{-2}$ for the whole slope and 10×10^{-2} for the bench slope. The level of risk undertaken can not be taken higher. Because of the lower technological conditions and the lower ability of compensation to the risk, and considering the factor that the slope of Baiyunebo Open Pit Mine is higher and the most rock masses have not been exposed, the engineering geology is of indetermination. However in the 2-D condition, the failure probability is conservative, therefore the failure probability may take higher in some extent. In this paper, the failure probability is taken as 5×10^{-2} for the whole slope and 8×10^{-2} for the slice failure probability because the importance of the slice is less than that of the whole slope and higher than that of the bench slope.

In this paper, the approach of probabilistic analysis of progressive failure of slope is presented. The proposed procedure is based on the 2-D reliability model of progressive slope failure and Monte-Carlo method. A computer program is developed according to the above concepts and the results of progressive failure probabilistic analysis of the slope of the Baiyunebo Open Pit Mine using this program are presented.

2 MATHEMATICAL MODEL

Taking the surplus-thrust method of slope stability analysis as an example, the calculation method of progressive slope failure is illustrated as below.

Suppose that the slope be divided into a number of vertical slices, which are numbered 1 to n from beginning at the toe. Figure 1 shows the force condition of the i-th slice. In the analysis, the following hypotheses have been made: ① the slice's weight W_i, the normal force N_i and the seepage force U_i are all through the slice's barycenter; ② the equivalent horizontal force SUIPQ_i caused by blasting acts on the slice's barycenter; ③the shear strength of sliding plane is generated by the Mohr-Coulomb criterion; ④ $\psi_i=\alpha_i$, $\psi_{i+1}=\alpha_{i+1}$. Then, the force equilibrium equations of the i-th slice are written as follows.

$$S_i = W_i \sin\alpha_i + Q_{i+1}\cos(\alpha_{i+1}-\alpha_i) - Q_i + \mathrm{SUIP}Q_i\cos\alpha_i \qquad (1)$$

$$N_i = W_i \cos\alpha_i + Q_{i+1}\sin(\alpha_{i+1}-\alpha_i) - U_i - \mathrm{SUIP}Q_i\sin\alpha_i \qquad (2)$$

In which α_i and α_{i+1} are represented the dip angle of the sliding faces of the i-th slice and the $(i+1)$-th slice respectively; Q_i and Q_{i+1} stand for the inter-slice thrusts of these two neighboring slices respectively, and given by

$$Q_i = \sum_{j=1}^{i-1}[c_jL_j - \mathrm{SUIP}Q_j\cos\alpha_j + (W_j\cos\alpha_j - U_j \\ -\mathrm{SUIP}Q_j\sin\alpha_j)\mathrm{tg}\phi_j - W_j\sin\alpha_j] \qquad (3)$$

$$Q_{i+1} = \sum_{j=i+1}^{n}[W_j\sin\alpha_j - \mathrm{SUIP}Q_j\cos\alpha_j - c_jL_j \\ -(W_j\cos\alpha_j - U_j - \mathrm{SUIP}Q_j\sin\alpha_j)\mathrm{tg}\phi_j] \qquad (4)$$

It is necessary to point out the problem about the value of Q_{i+1}. The case that $Q_{i+1}<0$ suggests that the sliding resistance of the slices from $(i+1)$ to n is more than the sliding force, i.e., the i-th slice undertakes certain upward tension. Considering that

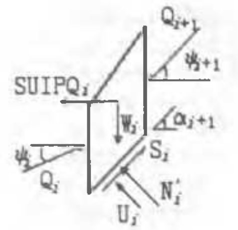

Figure 1. Force analyses of the i-th slice.

the compressive strength of rock mass is much more than the tensile strength, so it is adopted simply as zero, that is $Q_{i+1} = 0$.

The margin of safety SM is defined as the difference of the shear resistance and the shear force. Based on Mohr-Coulomb criterion, the margin of safety of the i-th slice can be written as:

$$SM_i = c_i L_i + N_i \mathrm{tg}\phi_i - S_i \tag{5}$$

where L_i is the length of the sliding plane of the i-th slice; c_i and ϕ_i respectively represent the cohesion and internal friction angle in the length of L_i. Under the condition that the i-th slice is not failure, these two strength parameters are the means of the peak strength parameters c_p, ϕ_p, otherwise, they will be the means of the residual strength parameters c_r, ϕ_r. The margin of safety indicates the stability of slice or slope. If it is less than zero, the slice or slope is in an unreliable state.

2.1 *Determination of the initial failure slice*

At beginning, each slice undertakes the peak shear strength. The initial margin of safety of the i-th slice is $SM_i = c_{pi}L_i + N_i \mathrm{tg}\phi_{pi} - S_i$, and its initial failure probability is $P_{fi} = P[SM_i \leqslant 0]$. The initial failure slice is defined as the slice which failure probability is the largest and more than the given failure threshold in all of the slices. It is thought that the failure of slope starts from this slice and expands one by one. And the failure of slope will not take place if the largest failure probability is less than the given failure threshold.

2.2 *Calculation of progressive failure probability*

Suppose that the slices from j to k are failure, then their strengths are reduced to their residual strengths. In order to undertake the surplus shear, the failure probabilities of their neighbor slices have to increase.

$$SM_{jk} = \sum_{i=j}^{k}[c_{ri}L_i - \mathrm{SUIP}Q_i\cos\alpha_i - W_i\sin\alpha_i + (W_i\cos\alpha_i - U_i - \mathrm{SUIP}Q_i\sin\alpha_i)\mathrm{tg}\phi_{ri}] \tag{6}$$

$$SM_{j-1} = c_{p\,j-1}L_{j-1} + N_{j-1}\mathrm{tg}\phi_{p\,j-1} - S_{j-1} \tag{7}$$

$$SM_{k-1} = c_{p\,k+1}L_{k-1} + N_{k-1}\mathrm{tg}\phi_{p\,k+1} - S_{k+1} \tag{8}$$

After the slices from j to k have been failed, the progressive failure probability of expanding to the $(j-1)$-th slice or the $(k+1)$-th slice can be written as follows. And the failure will expand to the one of both slices of which failure probability is higher.

$$P_{jk\cdot\,j-1} = P[SM_{j-1} \leqslant 0 \mid SM_{jk} \leqslant 0] = P[SM_{j-1} \leqslant 0 \cap SM_{jk} \leqslant 0]/P[SM_{jk} \leqslant 0] \tag{9}$$

$$P_{jk\cdot\,k+1} = P[SM_{k+1} \leqslant 0 \mid SM_{jk} \leqslant 0] = P[SM_{k+1} \leqslant 0 \cap SM_{jk} \leqslant 0]/P[SM_{jk} \leqslant 0] \tag{10}$$

For example, if $P_{jk\cdot\,k+1} > P_{jk\cdot\,j-1}$, then the failure will expand to the $(k+1)$-th slice and the slices from j to $k+1$ will constitute a new failure system. Through the above method, the progressive failure probabilities of expanding to the $(j-1)$-th slice and to the $(k+2)$-th can also be calculated respectively. The rest may be deduced by analogy until all of slices have been calculated.

2.3 *Failure probability of whole slope in local failure state*

In the process of the failure propagation, one of two following cases is likely to take place. First, the progressive failure probability is less than the failure threshold, so the progressive failure stops and the slope reaches a new stable state. Second, the progressive failure develops continuously, as a result, the failure probability of whole slope increases gradually and the slope is thought to be failure when its probability exceeds the threshold. When the slices from j to k are failing, the margin of safety and the failure probability of the whole slope can be expressed as follows respectively.

$$\begin{aligned} SM_z = SM_{jk} &+ \sum_{i=1}^{j-1}[c_{pi}L_i - \mathrm{SUIP}Q_i\cos\alpha_i - W_i\sin\alpha_i \\ &+ (W_i\cos\alpha_i - U_i - \mathrm{SUIP}Q_i\sin\alpha_i)\mathrm{tg}\phi_{pi}] \\ &+ \sum_{i=k+1}^{n}[c_{pi}L_i - \mathrm{SUIP}Q_i\cos\alpha_i - W_i\sin\alpha_i \\ &+ (W_i\cos\alpha_i - U_i - \mathrm{SUIP}Q_i\sin\alpha_i)\mathrm{tg}\phi_{pi}] \end{aligned} \tag{11}$$

$$P_{fz} = P[SM_z \leqslant 0] \tag{12}$$

Taking the surplus thrust method as an example, the analysis of the progressive slope failure is shown as above. For other methods for slope stability analysis, such as the simple Bishop method, the expressions of the various failure probabilities can be deduced with the relevant mechanical model. In the analysis of failure propagation, two following acceptable hypotheses are implied. First, there is only one initial failure slice from which the failure expands to other slices. Second, the failure of one slice or mass expands only to its neighbor slices. When the progressive failure probability is calculated using Monte-Carlo method, the sample number N is taken 5000 based on the engineering precision. The computer program is developed according to the above analysis.

3 CASE STUDY

In this paper, to two plans of the Baiyunebo Open Pit Mine (the original plan and the revised plan in which slope angle increases 2° relative to the original one), the reliability of the slopes have been evaluated.

The unit weight and the strength parameters of rock masses are listed in the Table 1.

Because Beyunebo locates in the zone of seismic intensity of 6 degree, the earthquake force is not taken into consideration according to the relevant criteria (SDJ10-78, 1978, BJ13-89,1989). For blasting force, it is supposed that the blasting-induced force acts on the barycenter of slice and points to the slope face. Based on the pseudo-static method, the expression of static force equivalent to blasting force is written as:

Table 1. The parameters of rock mass.

Lithology	Weathering	Unit weight (kN/m^3)	C (MPa)	ϕ
Dolomite	Slight	26.5	0.462	40°
Feldspathic slate	Middle	25.5	0.217	38°
Feldspathic slate	Slight	25.5	0.406	38°

$$F_i = \beta_0 \cdot K_i \cdot W_i \tag{13}$$

Table 3. Comparison of Reliability of Two Slope Plans.

Slope partition	Slope occurrence	Plan	Ground water	Failure probability of whole slope (%)	Serial number of initial failure slice	Initial failure probability (%)
A	168/45	Original	No	1.565	87	3.275
			Yes	4.300	6	4.541
		Revised	No	3.130	87	4.780
			Yes	5.884	6	6.069
B	199/45	Original	No	1.200	81	3.521
			Yes	3.507	14	4.521
		Revised	No	2.900	81	5.095
			Yes	5.149	14	6.055
C	281/43	Original	No	1.325	87	3.235
			Yes	4.433	16	4.631
		Revised	No	2.725	87	4.735
			Yes	6.246	16	6.202
D	011/44	Original	No	1.350	68	2.970
			Yes	4.107	13	4.489
		Revised	No	2.831	82	4.425
			Yes	5.904	16	6.202
E1	008/43	Original	No	1.120	56	2.465
			Yes	4.324	13	4.595
		Revised	No	2.650	56	3.696
			Yes	5.831	13	6.002
E2	102/43	Original	No	1.075	56	2.745
			Yes	4.189	16	4.556
		Revised	No	2.475	56	4.245
			Yes	5.589	16	6.056

In which F_i is the equivalent static force of blasting force; β_0 is the coefficient of blasting force which is taken 0.2 here; W_i is the weight of the i-th slice; K_i is the blasting-induced seismic coefficient. The expression of K_i is inferred as follows:

$$K_i = \frac{a_i}{g} = \frac{2\pi fV_i}{g} = 2\pi f\frac{K(\sqrt[3]{Q}/R_i)\alpha}{g} \tag{14}$$

in which a_i is the maximum acceleration of particle vibration; g is the acceleration of gravity taken as 980cm/s^2; f is the frequency of blasting vibration taken 15Hz in the research; V_i is the vibration velocity of particle; K is an empirical constant; α is the horizontal earthquake coefficient; Q is the dynamite weight; R_i is the distance from the explosion spot to the barycenter of the i-th slice. The values of parameters K, α, Q and R_i are listed in Table 2.

Table 3 shows the calculation results of two plans for each partition slope of Beyunebo Open Pit Mine.

It can be seen from Table 3 that groundwater has great influence on slope stability which is reflected in two following aspects. First, groundwater can make failure probability increase by 2%~3%. Second, groundwater can change the stress distribution of slope remarkable and make the position of the initial failure slice near the slope toe greatly.

Table 2. The values of parameters K, α, Q and R_i

Lithology	Slope Partition	K	α	Q (kg)	R_i (m)
Dolomite	A、B	57.25	1.34	2000	30
Feldspathic Slate	C、D、E1、E2	134.45	2.49	2000	30

From Table 3, it also can be seen that each failure probability of whole slope in the original plan is less than the probability threshold 5%, i.e., the slope is stable and its gradient may be increased reasonably. In the revised plan, when groundwater is disregarded, failure probability is still small though it is about two times as large as that of the original plan. And when groundwater is regarded, failure probability increases obviously and exceeds the threshold. However, because the mine lies in the area which has simple hydrogeological condition and lower groundwater table, the failure probability of the slope can be reduced to the threshold or less under the conditions that the water drainage and slope management are paid attention. It is uneconomical and unadvisable to ask for lower failure probability excessively.

4 CONCLUSIONS

It is more reasonable to take the failure process of slope as progressive rather than instantaneous in the slope stability analyses. From the case study, it can be also seen that groundwater has noticeable influence on slope stability which is reflected in two following aspects. First, groundwater can make failure probability increase by 2%~3%. Second, groundwater can change the stress distribution of slope remarkably and make the position of the initial failure slice near the slope toe significantly.

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