Probability of failure of slopes. Application to slopes of a granitic residual soil

Probabilité de rupture des pentes. Application à l'etude des pentes d'un sol résiduel granitique

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ABSTRACT: It is studied the stability of slopes of a granitic residual soil from the city of Braga in the north region of Portugal. This stability is based on probabilities. The confiability index of the factor of safety and the probability of failure are computed. The confiability index is computed as a function of the mean and standard deviation values of the factor of safety. The mean value of the factor of safety is considered the minimum value of this factor obtained using the Bishop method with the average values of the geotechnical parameters (ϕ, c and γ). The probability of failure is the probability of the factor of safety be less than or equal 1 and is obtained assuming a normal probability distribution. It is analyzed the relative weight of the geotechnical parameters in the factor of safety. A parametric study varying the height and the inclination of the slope is also performed.

RÉSUMÉ: On étudie la stabilité des pentes dun sol résiduel granitique de la ville de Braga de la région nord du Portugal. Cette étude est fondée sur probabilités. On calcule l'indice de confiabilité du coefficient de sécurité et la probabilité de rupture. L'indice de confiabilité est calculé comme une fonction des valeurs de la moyenne et de l'écart-type du coefficient de sécurité. La valeur moyenne du coefficient de sécurité est considerée comme la minime valeur de ce coefficient obtenue avec l'usage de la méthod de Bishop avec les valeurs moyennes des paramètres geotechniques (ϕ, c and γ). La probabilité de rupture est la probabilité du coefficient de securité être plus petit ou égal à 1 et est obtenue en supposant une distribution normale de la probabilité. On analyse le poids relatif des paramèters géotechniques sur le coefficient de sécurité. Une étude paramétrique est aussi réalizée en variant l'hauteur et l'inclinaison de la pente.

1 INTRODUCTION

The present paper deals with the probabilistic analysis applied to slope stability. It is based on a presentation (Sayão, 1999) at a short course devoted to slope stability where the basic concepts about the probability of failure were presented. That presentation was illustrated by means of specific examples.

In this paper it is studied the stability of slopes of a granitic residual soil from the city of Braga, in the north region of Portugal. The undisturbed samples were taken some years ago in the place where now it is built the Gualtar Campus of the University of Minho. The unit weight of the samples was directly obtained and the shear strength characteristics were obtained by means of triaxial tests. With the available results it is only possible to have a statistical sample with ten sets of values. These values include the unit weight (γ), cohesion intercept in terms of effective stress (c') and the angle of shearing resistance in terms of effective stress (ϕ').

The analysis performed includes the use of shear strength characteristics corresponding to the peak and to large axial strain in the curve relating principal stress difference and axial strain. It is used the method of Bishop (1955) to compute the factor of safety against the slide of slopes.

2 BASIC CONCEPTS

The basic concepts of probabilistic analysis related to slope stability can be found in Christian et al. (1994) and Sayão (1999). The factor of safety F associated to slope slide has some uncertainty associated with it. The mean or expected value and the standard deviation of the factor of safety are designated E[F] and σ[F], respectively. The reliability index β is defined by

$$\beta = \frac{E[F]-1.0}{\sigma[F]} \qquad (1)$$

The probability of failure P[f] is a function of the reliability index and can be obtained in almost any book on probability and statistics. To perform a probabilistic analysis it is first necessary to have the best estimation of each variable of interest to the stability of the slope being analyzed. Usually are used the mean values $E[x_i]$. Using a computer program based on a method of slices with $E[x_i]$ it is determined the minimum factor of safety which is designated E[F]. The standard deviation of the factor of safety is obtained by

$$\sigma[F] = \sqrt{V[F]} \qquad (2)$$

where V[F] is the variance of the factor of safety which is a function of the variances of the variables, $V[x_i]$, that affect the stability of the slope:

$$V[F] = \sum \left[\frac{dF}{dx_i}\right]^2 V[x_i] \qquad (3)$$

To compute the term dF/dx_i it is necessary to impose a small change in each variable separately and for each change to calculate the change in the factor of safety.

The variables x_i considered in this paper are γ, c', and ϕ'. The changes of the factor of safety are in relation to its value for the critical surface obtained with the average values of the geotechnical parameters γ, c', and ϕ'.

3 EXAMPLES

3.1 *Basic example*

Consider the slope represented in Figure 1. To perform the probabilistic analysis two situations are considered. The shear strength characteristics corresponding to the peak are used in the situation 1 and the shear strength characteristics corresponding to large axial strains are used in the situation 2. The characteristics of the soil for these situations are the following:

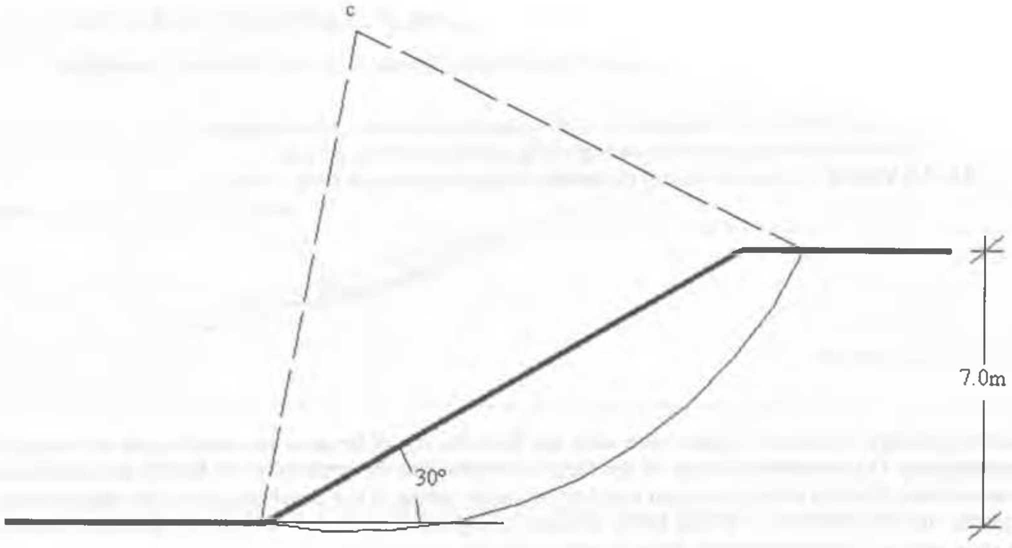

Figure 1. Embankment analyzed

Table 1. Variance composition, peak values

x_i	δx_i	δF	$\delta F/\delta x_i$	$V[x_i]$	$(\delta F/\delta x_i)^2.V[x_i]$	%
γ=20.1kN/m^3	2	-0.093	-0.4653	0.2617	0.0006	0.26
c´=17.9 kN/m^2	1.8	0.1034	0.0575	63.0512	0.2082	95.12
tanϕ´=0.644	0.0848	0.2019	2.3820	0.0018	0.0101	4.62
				V[F]	0.2189	100

Table 2. Variance composition, large axial strains values

x_i	δx_i	δF	$\delta F/\delta x_i$	$V[x_i]$	$(\delta F/\delta x_i)^2.V[x_i]$	%
γ=20.1kN/m^3	2	-0.048	-0.0242	0.2617	0.0001	0.03
c´=9 kN/m^2	0.9	0.0536	0.0596	152.73	0.5418	96.43
tanϕ´=0.620	0.0802	0.1859	2.3189	0.0037	0.0199	3.54
				V[F]	0.5618	100

Situation 1: cohesion intercept, c´=17.9 kN/m^2; angle of shearing resistance, ϕ´=32.8°; unit weight=20.1 kN/m^3
Situation 2: cohesion intercept, c´= 9kN/m^2; angle of shearing resistance, ϕ´=31.8°; unit weight=20.1kN/m^3

The factor of safety for situation 1 is 2.56 and for situation 2 is 1.96.

The determination of the variance of the factor of safety for both situations is presented in Tables 1 and 2.

It can be seen that for both situations the variance of the cohesion intercept, c´, has a strong influence in variance of the factor of safety.

The probability of failure is the probability of the factor of safety be less than or equal 1 and is obtained assuming a normal probability distribution. This probability is very low (2/5000) in situation 1 but it is very high (1/10) in situation 2. This is because the variance of the cohesion intercept, c´, in this case is very high (152.73), which represents a great variability of this parameter. If the same variance of situation 1 is used, the probability of failure is about 2.5/100.

In general the cohesion intercept for large strain values was very low (0 or near 0). However, in some triaxial tests the large axial strains corresponded to peak values and the values of cohesion intercept were high. This justifies the great variability of this parameter.

3.2 *Parametric studies*

The probabilistic analysis is also performed changing the embankment inclination and maintaining its height (7 m) (Figure 2) and changing its high and maintaining the inclination (Figure 3). The results are presented in Tables 3, 4, 5 and 6.

Table 3. Factors of safety, probability of failure and contribution of the parameters in V[F] changing the slope inclination for peak values

Inclination (°)	F	P_f	Contributions (%) γ	c´	tanϕ´
24	3.02	<1/10000	0.25	92.63	7.12
26	2.85	1/10000	0.25	93.68	6.07
28	2.71	1/10000	0.26	94.18	5.56
30	2.56	2/5000	0.26	95.12	4.62
32	2.46	3/5000	0.26	95.19	4.55
34	2.37	9/10000	0.26	95.31	4.43
36	2.26	2/1000	0.26	96.12	3.62
38	2.16	5/1000	0.26	96.76	2.98
40	2.08	2/250	0.26	97.28	2.46
42	2.02	2/250	0.26	96.85	2.89

Table 4. Factors of safety, probability of failure and contribution of the parameters in V[F] changing the slope inclination for large axial strain values

Inclination (°)	F	P_f	Contributions (%) γ	c´	tanϕ´
24	2.34	1/20	0.03	95.51	4.46
26	2.20	1/16	0.03	95.78	4.19
28	2.07	1/12	0.03	96.10	3.87
30	1.96	1/10	0.03	96.43	3.54
32	1.85	1/8	0.03	96.97	3.00
34	1.76	1/6	0.03	97.45	2.52
36	1.68	1/6	0.03	97.55	2.42
38	1.63	1/6	0.03	97.38	2.59
40	1.54	1/5	0.02	97.92	2.06
42	1.48	1/4	0.03	97.99	1.98

It can be seen that for both situations the factor of safety decreases as the inclination increases and the probability of failure increases with the increase of inclination. This is in accordance to the expectations. The contribution of the unit weight to the variance of F is very little and practically doesn't change with the inclination for both situations. The contribution of the cohesion intercept, c´, to the variance of F increases gradually and the contribution of the angle of shearing resistance, ϕ´, decreases gradually as the inclination increases for

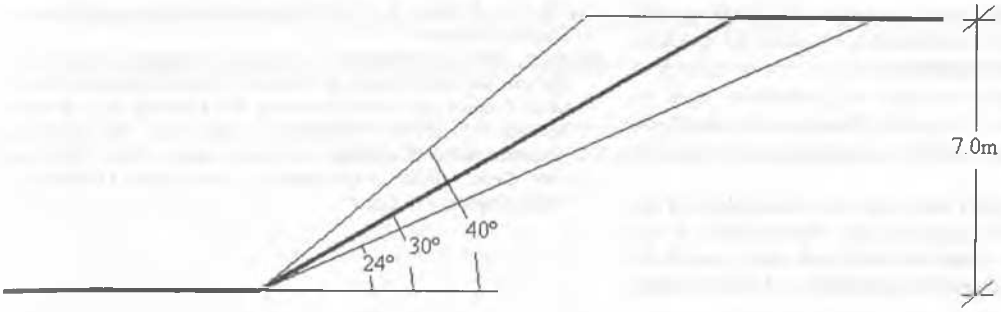

Figure 2. Changing of slope inclination

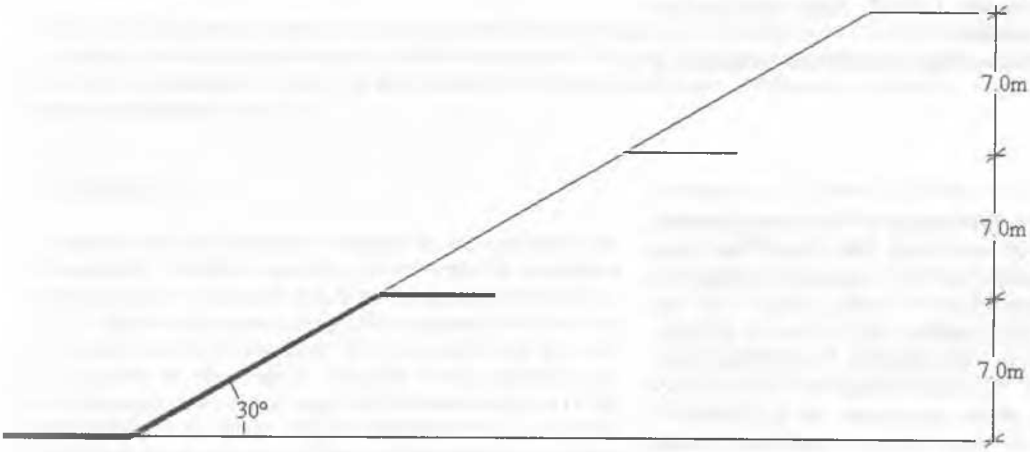

Figure 3. Changing of slope height

Table 5. Factors of safety, probability of failure and contribution of the parameters in V[F] changing the height of the embankment for peak values

			Contributions (%)		
height (m)	F	P_f	γ	c´	tanϕ´
7	2.56	4/10000	0.26	95.12	4.62
14	2.00	1/10000	0.22	87.84	11.94
21	1.79	1/10000	0.22	79.60	20.18
28	1.67	<1/10000	0.20	73.33	26.47
35	1.59	1/10000	0.19	68.43	31.38
42	1.54	1/10000	0.15	61.92	37.93
49	1.50	<1/10000	0.14	54.84	45.02
56	1.47	<1/10000	0.14	49.78	50.08
63	1.45	<1/10000	0.16	46.06	53.78
70	1.42	1/10000	0.12	43.65	56.23

Table 6. Factors of safety, probability of failure and contribution of the parameters in V[F] changing the height of the embankment for large axial strain values

			Contributions (%)		
height (m)	F	P_f	γ	c´	tanϕ´
7	1.96	10/100	0.03	96.43	3.54
14	1.62	11/100	0.03	92.36	7.61
21	1.50	16/100	0.03	86.02	13.95
28	1.42	13/100	0.02	83.21	16.76
35	1.38	15/100	0.02	77.82	22.16
42	1.34	16/100	0.02	72.27	27.71
49	1.32	16/100	0.02	68.03	31.95
56	1.30	16/100	0.02	63.97	36.01
63	1.28	16/100	0.02	60.93	39.05
70	1.27	16/100	0.02	57.46	42.52

both situations. Nevertheless, for situation 1, there is a decrease in the c´ contribution and an increase in the tanϕ´ contribution for the inclination of 42°. For situation 2 there is an exception for tanϕ´ corresponding to 38° of inclination.

For both situations the factor of safety decreases as the height increases. The probability of failure has almost no changes with the variation of height in situation 1. For situation 2 the probability of failure is irregular from 7 m to 35 m and remains the same for higher heights. In relation to the contribution of the parameters to the variance of F it can be seen that the unit weight decreases, the cohesion intercept decreases and the angle of shearing resistance increases with the increase of height. In relation to the contribution of the unit weight for the situation 1 there is an exception for 63 m of height where this parameter increases slightly. It is also important to mention that, for situation 1, the contribution of the angle of shearing resistance is greater than that of the cohesion intercept from 56 m up.

4 DISCUSSION

It is important to mentioned that the residual soils from granite are normally very heterogeneous and many times include disperse balls and the "bedrock" can be strongly irregular. These characteristics affect the sliding surface geometry that frequently is not circular. Therefore it is important to consider the adequate sliding surface for the case in analysis.

In addition the number of sets of parameters used in probabilistic analysis was only 10. Therefore it would be necessary to obtain a large number of samples to perform the triaxial tests.

In the probabilistic analysis were used the mean values of the variables of interest to the slope stability. Nevertheless, if the characteristic values were used the factors of safety would be lower and the calculated value of the probability of failure would be probably higher.

Almost all the soil properties used are inside the range of typical values for the coefficient of variation (standard deviation/mean value) given by Orr & Farrel (1999). Only the cohesion intercept for large axial extensions has a coefficient of variation (1.38) outside the limits presented by that author (0.30-0.50). As we have seen above the scatter of the values of cohesion has affected significantly the results obtained.

Viana da Fonseca et al. (1994) presented for residual soils from granite lower bound limits of 27-30°. These values are less than those used in both situations.

Another parameter of interest that was not here considered is the water table.

5 CONCLUSIONS

This work has been the first contact of the authors with the probabilistic analysis. As was seen, this study has some limitations such as the reduced number of parameters to perform the analysis, the consideration of homogeneous soil, the assumption of circular sliding surfaces and the use of the mean values of the shear strength characteristics. Nevertheless some conclusions can be drawn. The high variance of c' in the case of large axial strain values affects significantly the probability of failure. The probability of failure has not significant changes with the changing of height of the slope. The influence of c' decreases as the height increases and the influence of ϕ' increases as the height increases. The influence of c' increases as the slope inclination increases and the influence of ϕ' decreases as the inclination increases. The probability of failure increases as the inclination increase. The influence of the unit weight is not relevant in the performed analysis.

The probabilistic analysis can be improved if a large number of parameters is used. In addition it is necessary to perform a statistical treatment to study the kind of distribution of the factor of safety.

The use of shear strength values corresponding to large axial strains gives conservative results. However in this study, due to the great variability of c', the probability of failure is high even for high factors of safety.

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