# Strain-controlled testing for subgrade resilient modulus

## L'Essai de déflexion-réglé pour subgrade modulus elastique

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ABSTRACT: Research has shown that stress-based testing for soil resilient modulus can lead to widely varying results. This study demonstrates the use of a strain-based approach and presents corroborating laboratory data. Results from 23 different resilient modulus test sequences (a total of about 200 different load stages) indicate strain levels are much easier to correlate to resilient modulus than either deviator or bulk stress. Further treatment of the data show that application of threshold strain concepts lead to a more reliable prediction of resilient modulus under varying conditions

RÉSUMÉ: La recherche a montré cet essai tension-basé pour le sol modulus élastique peut mener aux résultats qui varient largement. Cette étude démontre l'usage d'une approche déflexion-basé et présente les données de laboratoire qui confirment. Les résultats de 23 séquences d'examen de modulus élastiquees différentes (un total d'environ 200 étapes de chargement différentes) indique que les niveaux de tension sont beaucoup plus faciles à correspond à modulus élastique que deviator ou la tension de masse. Egalement le traitement des données montre cette application de plomb de concepts de tension de seuil à une prédiction plus fiable de modulus élastique sous varier de conditions

## 1 INTRODUCTION

Resilient modulus is an important, and often elusive, measure of soil stiffness. Its application is primarily in the design of pavements, however, its use need not be confined to only this. Several studies (Lenker 1992, TRB 1994) have examined the difficulty in obtaining consistent, unambiguous results from the standard testing methods. This paper offers a combination of testing method and analysis approach that has shown great promise. The lack of ambiguity in data analysis, repeatability of results, ease of numerical fitting, and theoretical underpinning, make it worthy for consideration.

## 2 OUTLINE OF APPROACH

The strain-based approach consists of three components: a deflection-controlled testing method, a strain-based data analysis method, and a strain-based modulus normalization procedure to enable numerical curve fitting.

Most resilient modulus testing methods (AASHTO T292-911) subject a soil specimen, confined in a triaxial cell, to repeated cycles of prescribed load levels, based on anticipated deviator stress. The process is usually repeated at progressively higher deviator stress stages, and then redone at progressively higher confining stresses. The inherent nature of a stress-controlled test implies that a specimen may fail at any time, yielding less data (or none at all) than if tested under strain control. A soil tested under strain control may reach a point of failure, but is far less likely to collapse. In our experimental program of 164 deflection controlled loading stages, no collapses occurred.

Applying a strain-based method to determine resilient modulus proved far more consistent than stress-based methods. Typical stress-based methods have been available for the past 30 years and consist primarily of correlating either deviator or bulk stress to resilient modulus (AASHTO T292-911). The forms of two stress-based and the strain-based model discussed herein are listed below.

$M_R = a\,\sigma_d{}^b$ stress-based, clay samples (1)

$M_R = a\,\upsilon^b$ stress-based, sand samples (2)

$M_R = a\,\varepsilon_R{}^b$ strain-based, sand and clay samples (3)

where a,b = curve-fitting constants

$M_R$ , $\varepsilon_R$ = resilient modulus, resilient strain

$\sigma_d$, $\upsilon$ = deviator and bulk stress, respectively

Note that the curve-fitting constants a and b, for equations (1), (2), and (3) will not be the same value.

The modulus normalization procedure is based on threshold strain concepts, specifically applied to resilient modulus testing. The use of normalized values allows the engineer to generalize test data and compare resilient modulus results with a wider variety of soil types and testing conditions. While the derivation of the relationship is too lengthy to present here, the final outcome may be written as

$$\frac{M_R}{M_{RMAX}} = \frac{2}{3}\left(\frac{1-\left((1-\frac{\hat{a}}{\hat{a}_t})\right)}{\frac{\hat{a}}{\hat{a}_t}}\right) \qquad (4)$$

where $M_{RMAX}$ = maximum calculated $M_R$

$M_R$ = resilient modulus at strain $\varepsilon$

$\varepsilon_t$ = threshold axial strain

Threshold axial strain is derived from manipulation of the threshold shear strain along a hypothetical failure plane in a triaxial test. As derived by Dobry et al. (1982), threshold shearing strain in an array of cubic packed spheres of quartz may be written

$$\tilde{a}_t = \frac{2.08(2-\grave{ı})(1+\grave{ı})f}{(1-\grave{ı}^2)^{1/3}E^{2/3}}\acute{O}_c^{2/3} \qquad (5)$$

where $\gamma_t$ = threshold shear strain

E,$\mu$ = Young's modulus, Poisson's ratio of mineral (e.g. quartz) = $1.1\text{x}10^6$ psi, 0.31

f = friction coefficient between spheres = 0.5

$\sigma_c$ = confining stress of array

Transforming this to triaxial conditions where the shearing strain takes place along an incipient failure surface,

$$\mathring{a}_t = \tilde{a}_t / \left(\sin(45 + \ddot{o}/2)\right) \qquad (6)$$

Finally, the expression for $M_{RMAX}$ is based on empirical formulations by Hardin for shear modulus $G_{MAX}$, modified for triaxial conditions (Montemayor 1995).

$$M_{RMAX} = 625 \frac{OCR^k}{0.3 + 0.7e^2} (P_a \acute{o}_0)^{0.5} \qquad (7)$$

where OCR = over-consolidation ratio,

k = function of plasticity, 0 for sand, 0.14 for clay

e = void ratio

$P_a, \sigma_0$ = atmospheric pressure, mean effective stress

When combined together, the components discussed above form a straightforward and reliable method for modeling resilient modulus behavior.

## 3 LABORATORY MATERIALS

The experimental basis for strain-controlled testing consisted of a series of tests on local sand and clay soils. Pertinent index properties are listed below. The clay is brownish red sandy silty clay, residual in origin, found throughout the lower Piedmont region of South Carolina. Although its UCS classification is ML, there is a significant fine sand component that accounts for the reduction in plasticity index. While this soil is a three-component mixture, the clay tends to dominate its behavior. The light tan sand originates from the Sand Hills region, an ancient coastal dune formation adjacent to the lower Piedmont. It has a small percentage of mica and kaolin present as well.

Following excavation, the clay was carefully blended and water added to generate a uniform set of specimens. The specimens were extruded from a Vac-Aire extrusion machine, employing a vacuum pressure of -6.0 psi. After extrusion, material was cut into manageable sizes, wrapped, and stored for six weeks. Following this time, specimens were trimmed to a diameter of 3.0 in. and repackaged. Total storage time for a specimen ranged from 5-10 months. Immediately prior to testing the specimens were trimmed to their final length, approximately 6.2 in.

Once in the triaxial cell, the specimens were isotropically consolidated with radial drainage provided, for 24 hours. The consolidation pressure was equal to the eventual confining pressure. All tests were performed with pore pressure drainage open.

Sand specimens were formed directly on the triaxial base pedestal by pluviating from a beaker, through a #16 sieve, falling within a clear plastic pipe from a height of 30 in. Specimen dimensions were the same as for clay soil. Tests were performed on dry specimens.

## 4 TESTING PROGRAM

In order to investigate the differences between load-controlled and deflection-controlled testing, several sets of parallel tests were performed. Table 2 lists the various testing sequences. Control refers to either a load (stress) controlled or deflection (strain) controlled test. Connection refers to the method of connecting the loading ram with the piston of the triaxial cell. A threaded connection could transfer both tension and compression. A ball and plate arrangement could only transfer compression. This was necessary and desirable for the deflection-controlled test. The specimens would not be pulled upward in tension during the test.

Table 1. Index properties of soils tested

| Soil | UCS | Test Density $lb/ft^3$ | LL/PL or $D_{min}/D_{max}$ | w/c % | e |
|---|---|---|---|---|---|
| Clay | ML | 103-107 | 46/30 | 42-48 | 1.14-1.31 |
| Sand | SP | 96-99 | 93/110 | -- | 0.64-0.69 |

According to ASTM D 2487, 4253, 4254, 4318, 2216

Table 2. Testing Matrix, Clay and Sand Specimens

| Sequence* | Control | Confining Pressure, psi** | Connection | Cycles per Stage | Stages |
|---|---|---|---|---|---|
| C-DC-1 | Deflec. | 4.0 | Thread | 50 | 12 |
| C-DC-2 | | 4.0 | Ball-Plate | 50 | 14 |
| C-DC-3 | | 4.0 | Ball-Plate | 5000 | 14 |
| C-DC-4 | | 4.0 | Ball-Plate | 50 | 11 |
| C-DC-5 | | 12.0 | Ball-Plate | 50 | 16 |
| C-DC-6 | | 8.0 | Ball-Plate | 50 | 16 |
| C-DC-7 | | 4.0 | Ball-Plate | 50 | 16 |
| C-LC-1 | Load | 4.0 | Thread | 50 | 5 |
| C-LC-2 | | 4.0 | Thread | 5000 | 5 |
| C-LC-3 | | 12.0 | Thread | 50 | 8 |
| C-LC-4 | | 8.0 | Thread | 50 | 7 |
| C-LC-5 | | 4.0 | Thread | 50 | 5 |
| S-DC-1 | Deflec. | 4.0 | Thread | 50 | 4 |
| S-DC-2 | | 4.0 | Ball-Plate | 50 | 4 |
| S-DC-3 | | 4.0 | Ball-Plate | 5000 | 9 |
| S-DC-4 | | 12.0 | Ball-Plate | 50 | 16 |
| S-DC-5 | | 8.0 | Ball-Plate | 50 | 16 |
| S-DC-6 | | 4.0 | Ball-Plate | 50 | 16 |
| S-LC-1 | Load | 4.0 | Thread | 5000 | 4 |
| S-LC-2 | | 12.0 | Thread | 50 | 9 |
| S-LC-3 | | 8.0 | Thread | 50 | 4 |
| S-LC-4 | | 4.0 | Thread | 50 | 7 |
| S-LC-5 | | 8.0 | Thread | 50 | 5 |

* C- is clay, S- is sand. ** 1 psi ~7kPa

Only four sequences used 5000 cycles per stage. Its purpose was to investigate the effects of high cycles of loading on resilient modulus behavior. Since the soils reached an equilibrium hysteresis much sooner, high numbers of loading cycles were not deemed necessary. Deflection stages ranged 0.004-0.700 in. and load stages 5-70 lb. Note also that deflection control allowed for a greater number of stages, especially at low strain levels. This was done primarily to study behavior below and near predicted threshold strain levels. None of the tests used conditioning stages.

Chronologically, the extended tests were first performed with load and deflection recorded for cycles 1, 2, 4, 6, 8, 10, 20, 30, 40, 50, 100, 200, 300, 400, 500, 1000, 2000, 3000, 4000, 5000. Based on observations of the initial tests, later tests recorded cycles 1, 10, 20, 30, 40, 50. Each cycle was composed of about 200 readings.

## 5 SUMMARY OF RESULTS

In general, the deflection-controlled tests allowed for adjustments during a test sequence so that more data (more stages) could be gathered. Typically, one could approach the point of failure or collapse with more confidence since failure would not be catastrophic.

Figure 1 shows $M_R$ vs. resilient strain for a single deflection-controlled test sequence on clay. The value for $M_R$ was taken as the slope of the stress-strain curve at equilibrium hysteresis, about cycle 50. The solid line represents a least-squares fit of equation 3. Figure 2 illustrates the same data, plotted as $M_R$ vs. deviator stress with a least squares fit of equation 1. Listed on both figures are the constants as well as the coefficient of determination, $R^2$ (1.0 = perfect fit).

Figure 3 illustrates $M_R$ vs. resilient strain for a typical deflection-controlled test on sand. Note again the best fit for equation 3. Figure 4 is $M_R$ vs. bulk stress for the same test. Although the improvement is not as striking as before, the $R^2$ values indicate a better fit. Table 3 summarizes all tests.

In general, equation 3 provided a better representation of both load-controlled and deflection-controlled tests. However, for sands, the load-controlled test data show a better fit than the de

Table 3. Summary of fitting equations 1,2,3

| Test Sequence | Curve-fit coefficients, correlation Eq. (1) or (2)* | | | Eq. (3) | | |
|---|---|---|---|---|---|---|
| | a | b | $R^2$ | a | b | $R^2$ |
| C-DC-2 | 6924 | -2.163 | 0.304 | 6.4 | -0.847 | 0.966 |
| C-DC-3 | 2042 | -0.442 | 0.094 | 24.8 | -0.636 | 0.771 |
| C-DC-4 | 6282 | -1.697 | 0.566 | 15.0 | -0.716 | 0.955 |
| C-DC-5 | 132400 | -2.621 | 0.497 | 14.5 | -0.820 | 0.974 |
| C-DC-6 | 7766 | -1.778 | 0.723 | 18.2 | -0.689 | 0.968 |
| C-DC-7 | 12440 | -2.589 | 0.612 | 9.1 | -0.790 | 0.978 |
| C-LC-1 | 24420 | -1.325 | 0.903 | 69.2 | -0.585 | 0.982 |
| C-LC-2 | 31710 | -2.045 | 0.895 | 26.7 | -0.688 | 0.990 |
| C-LC-3 | 24370 | -0.767 | 0.719 | 218.0 | -0.473 | 0.916 |
| C-LC-4 | 15670 | -0.903 | 0.755 | 119.9 | -0.510 | 0.937 |
| C-LC-5 | 10830 | -1.090 | 0.819 | 68.1 | -0.549 | 0.960 |
| S-DC-1 | 111800 | -0.740 | 0.595 | 2393 | -0.221 | 0.736 |
| S-DC-2 | 43840 | -0.424 | 0.285 | 5514 | -0.108 | 0.317 |
| S-DC-3 | 0 | 13.099 | 0.030 | 1.0 | -1.001 | 0.914 |
| S-DC-4 | 11470 | -0.922 | 0.001 | 3.0 | -0.869 | 0.863 |
| S-DC-5 | 1030000 | -2.245 | 0.876 | 325.1 | -0.288 | 0.943 |
| S-LC-1 | 1.15 E07 | -2.525 | 0.280 | 380.7 | -0.411 | 0.726 |
| S-LC-2 | 1.34 E10 | -3.909 | 0.916 | 836.8 | -0.276 | 0.935 |
| S-LC-3 | 1.72 E37 | -23.917 | 0.612 | 34.3 | -0.597 | 0.929 |
| S-LC-4 | 2.89 E05 | -1.476 | 0.762 | 850.2 | -0.237 | 0.897 |
| S-LC-5 | 3.53 E11 | -5.633 | 0.539 | 250.4 | -0.351 | 0.869 |

* Eq. (1) used for clay, Eq. (2) used for sand

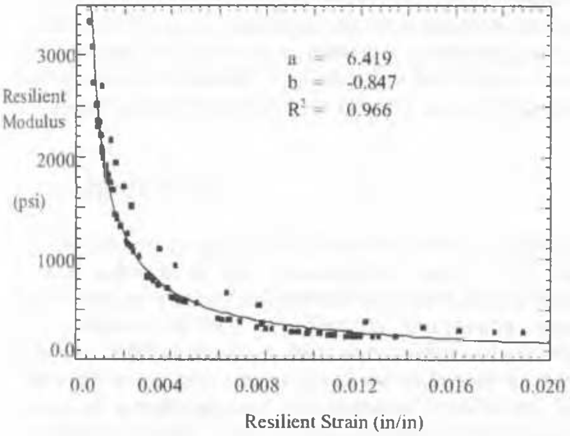

Figure 1. Resilient Modulus vs. Resilient Strain, Test C-DC-2

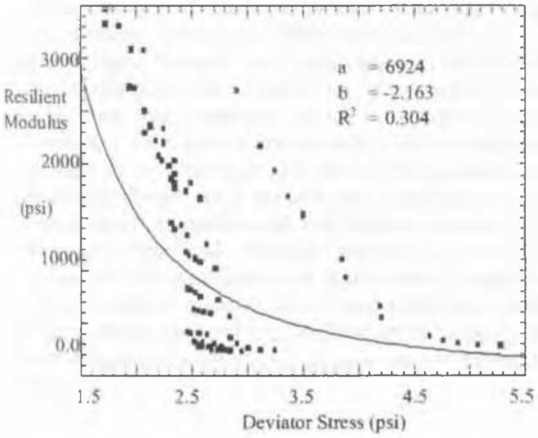

Figure 2. Resilient Modulus vs. Deviator Stress, Test C-DC-2

deflection-controlled data. This may be due to the fact that there were fewer stages (fewer data points to fit) for the load-controlled tests.

Figures 5 and 6 illustrate normalized curves of resilient modulus versus resilient strain. While not yet fully developed, the approach does tend to collapse the effects of confining stress, especially in sand (Fig. 6). These curves may be fit by equations 1, 2 or 3 as the previous data. Clay data, (Fig. 5) collapses nicely on the empirical curve for equation 3.

Figure 6 shows the advantage of normalizing on sand. Compare with the same data in figure 3, especially the increase in $R^2$.

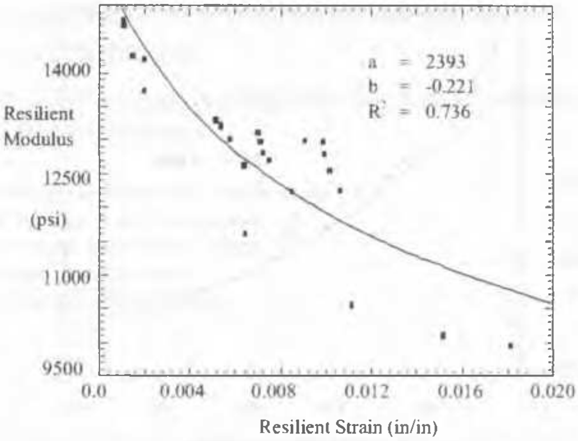

Figure 3. Resilient Modulus vs. Resilient Strain, Test S-DC-1

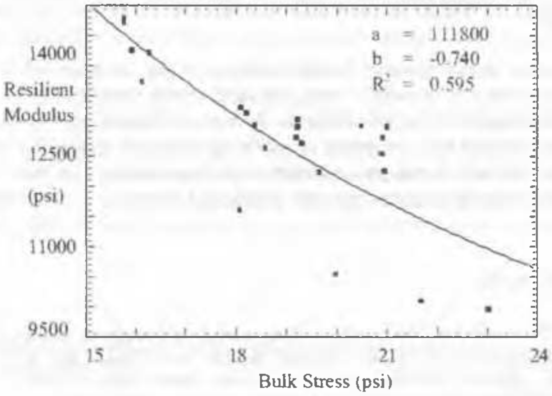

Figure 4. Resilient Modulus vs. Bulk Stress, Test S-DC-1

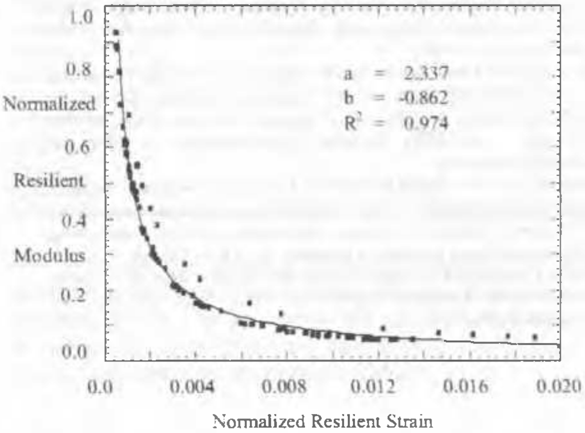

Figure 5. Normalized Resilient Modulus vs. Normalized Resilient Strain, Test C-DC-2.

While this approach is still in its developmental stages, the trends are promising.

## 6 CONCLUSIONS

Use of a deflection/strain approach to resilient modulus testing and data analysis has led to a more accurate modeling capability. Deflection-controlled testing allows for better control of the test, reducing the likelihood of specimen collapse. Typically more stages can be performed at critical levels than with a load-controlled test.

Basing the data reduction and analysis procedures on strain rather than deviator stress or bulk stress has its advantages as well. Results tend to correlate more closely to simple models. A simple and accurate method to summarize test data then follows. Normalizing data to allow for generalization of results beyond a prescribed confining stress and for other sample densities (or

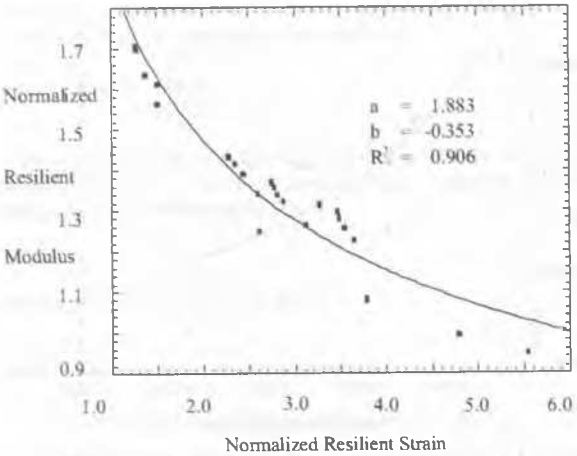

Figure 6. Normalized Resilient Modulus vs. Normalized Resilient Strain, Test S-DC-1.

void ratios) shows promise. Further analysis of the influence of confining stress is presently being pursued. Other statistical hypothesis testing will be an additional avenue of research on this data. It is hoped that a method of modeling behavior similar to that used in soil dynamics and earthquake engineering can be adapted for use in resilient modulus testing and analysis.

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