On the bending of a beam resting on elastic base with consideration for large deflections

Sur le repli de la poutre se trouvant à la raison élastique, compte tenu de grandes flèches

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ABSTRACT: The problems on flexures of the hingefixed and free on the ends of elastic base taking into account large deformations are solved. Possibility of influence of existence of boss of the carrying capacity of the base for large flexures in the critic condition is found out. Analysis of this condition is given.

RÉSUMÉ: On décide les tâches sur les flèches fixé aves les articulqtions et libre sur les extrémités des poutres élastiques à la fondation raison élastique compte tenu de grands déplacements. On découvre la possibilité de l'existence de grandes flèches des situations critiques de la perte de la capacité portant de la fondation. On donne l'analyse de cette situation.

Large deflections at which consideration for geometrical non-linearity is necessary are observed in thin-walled structural members. Moreover, Winkler linear relationship which relates reaction pressure to the deflection, $q_R = kw$, must be substituted for more general one. Here, the following expression for it at large deflections is proposed:

$$q_R = kw(1 - k_1 w)(1 - k_2 w) \tag{1}$$

It is obtained from the solution of a geometrically non-linear problem on the motion of the upper surface of an elastic layer resting on a rigid base. When the layer is loaded with a uniformly distributed load. In (1) k_1 and k_2 will be called the secondary Winkler's coefficients as distinct from the ordinary Winkler's coefficient k.

For small deflections w of the beam at the ordinary Winkler's relationship $q_k = kw$ the deflection of the beam is proportional to the intensity of exterior uniformly distributed pressing force q:

$$w = \frac{w_0}{\Delta}\{(\mathrm{ch}\,\beta l \sin\beta l - \mathrm{sh}\,\beta l\cos\beta l)(\mathrm{sh}\,\beta x\cos\beta x - \mathrm{ch}\,\beta x\sin\beta x) - \mathrm{ch}\,\beta x\cos\beta x + 1\} \tag{2}$$

$$w_0 = \frac{q}{k};\beta = \sqrt[4]{\frac{bk}{2EJ}};\Delta = \mathrm{sh}^2\beta l\cos^2\beta l + \mathrm{ch}^2\beta l\sin^2\beta l \tag{3}$$

E - Young's modulus of elasticity, J - moment of inertia of cross section, h - thickness, b - width, l - length of the beam.

In this way with the use of the soid Winkler's linear relationship for small deflections we shall obtain monotonous growth of deflection with the growth of the outer load q, that is, stability of the process of deflection resting on elastic base takes place. However, for large deflections the situation will be different. To clarify this it is necessary to have a solution of the problem on the deflection of a beam resting on elastic base at large displacements and with consideration for non-linear Winkler's law (1).

In case of presence of friction being considered for by introducing reactive frictional force $q_\tau = k_\tau u$ proportional to horizontal shift of points of surface of the beam contacting the base full system of equations within the framework of the hypothesis of flat sections will have the form:

$$\begin{cases} N_{,x} - q_\tau = 0; (Nw_{,x})_{,x} + M_{,xx} + (q - q_R)b = 0 \\ N = EF\left(u_{,x} + \frac{1}{2}w_{,x}^2\right); M = -EJw_{,xx} \end{cases} \tag{4}$$

where N - longitudinal force in beam, M - bending moment, F - cross section area of the beam.

The boundary conditions are of the form:

A) for hinged restraint of ends:
$w=0$; $N=0$; $M=0$ when $x=0$; l (5)

B) for free ends: $w=0$; $N=0$; $M=0$ when $x=0$; l (6)

Due to considerable mathematical complexity of the analytical solution of the system of equations (4) the variational method of solution with the use of Reisner's functional was employed for it:

$$\mathrm{K} = \int_0^l \left\{ N\left[u_{,x} + \frac{1}{2}w_{,x}^2\right] - Mw_{,xx} - \frac{1}{2E}\left(\frac{N^2}{F} + \frac{M^2}{J}\right) + \frac{1}{2}k_\tau u^2 + \frac{1}{2}bkw^2 - \frac{1}{3}k(k_1 + k_2)bw^3 + \frac{1}{4}kk_1k_2bw^4 - qwb \right\}dx + K_c \tag{7}$$

where x - longitudinal axial coordinate, K_c - expression dependent on the boundary conditions which has the following form for the conditions (6):

$$K_c = -(M_{,x}w)\Big|_0^l \tag{8}$$

and the following form for the conditions (5):

$$K_c = -(Nu + Nww_{,x} + M_{,x}w)\Big|_0^l \tag{9}$$

Realization of the Reisner's variational principle is made for the following approximations depending on the type of boundary conditions:

$$\text{A)}\ w = w_0\sin\frac{\pi x}{l};\quad M = M_0\sin\frac{\pi x}{l};$$
$$N = N_1 + N_0\sin^2\frac{\pi x}{l};\quad u = u_0\cos\frac{\pi x}{l}\sin\frac{\pi x}{l}$$
$$\text{B)}\ w = w_0\sin\frac{\pi x}{l};\quad M = M_0\sin\frac{\pi x}{l};$$
$$N = N_0\sin\frac{\pi x}{l};\quad u = u_0\cos\frac{\pi x}{l} \tag{10}$$

Finally, we shall obtain the relationship between a dimensionaless maximum deflection $\nu = w_0/h$ and a load $\tau = 2bh^3q/EY$ of the form:

$$\tau = C_{A(B)}\nu^3 - C_1\nu^2 + C_2\nu \tag{11}$$

where

$$C_1 = \frac{4}{3}g(g_1 + g_2),\ C_2 = \frac{\pi}{2}(g + \pi^4\gamma^4)$$

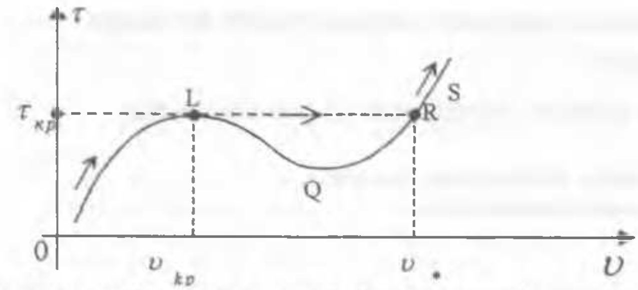

Figure 1. The dependence curve of bending from load.

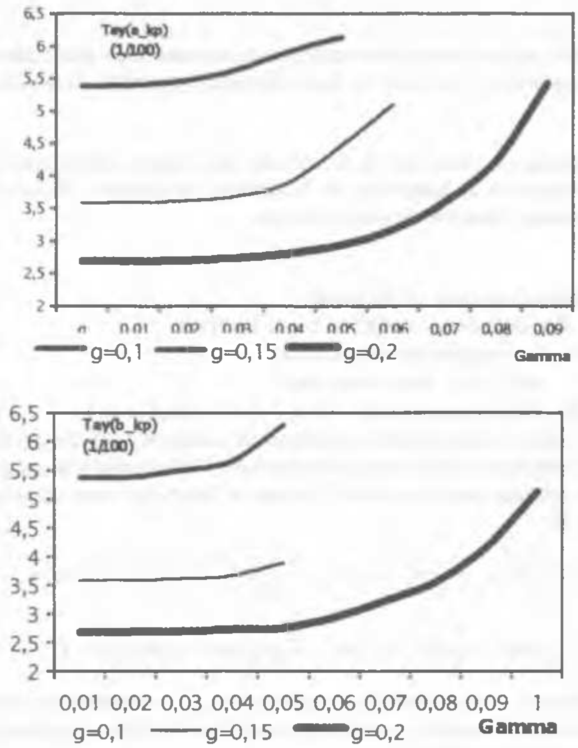

Figure 2. The dependence curves of critical load from relative beam length.

$$C_A = \frac{4\pi}{9} g_r \gamma^2 \left(1 + \frac{g_r}{3\pi^2\gamma^2}\right)^{-n} + \frac{3\pi}{8} g g_1 g_2;$$

$$C_B = \frac{3\pi}{8}\left[g g_1 g_2 + \frac{3\pi^2\gamma^4\left(g_r + 8\pi^2\gamma^2\right)}{2\left(g_r + 12\pi^2\gamma^2\right)}\right]; \qquad (12)$$

$$\frac{h}{l} = \gamma; k_1 h = g_1; \quad \frac{K_r h^4}{EJ} = g_r; \quad \frac{kbh^4}{EJ} = g$$

Given that $D = C_1^2 - 3C_2 C_{A(B)} > 0$ the "load-deflection" curve has the form shown in figure 1.

According to it, with growth in the load up to aparticular τ_{kp} the deflection also increases monotonously up to v_{kp} after which the deflection increases instantaneously in an abrupt way up to v^* at the same

load τ_{kp} and then again increases monotonously. The portion LQR on the curve in Figure 1. is not realized. It corresponds to the loss in stability of the process of deflection of a beam on elastic base manifesting itself in catastrophic instantaneous subsidence of soil base under foundation. Critical deflection is determined form the expression:

$$v_{kp} = \frac{1}{3C_{A(B)}}\left(C_1 - \sqrt{D}\right) \qquad (13)$$

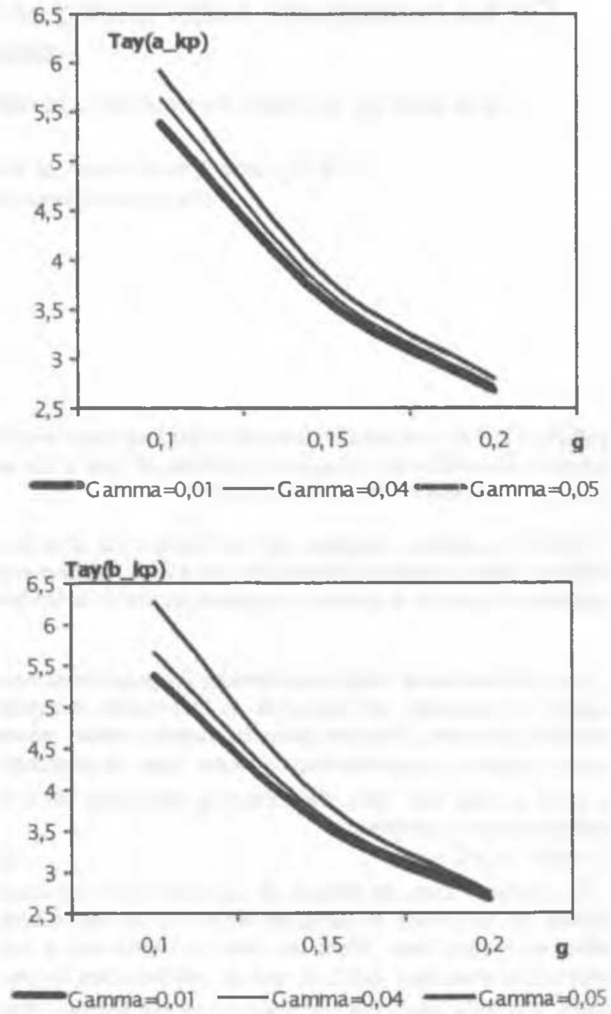

Figure 3. The dependence curves of critical load from secondary Vinkler coefficients.

Critical load is determined from the formula (11) with consideration for the relationship (13) in it.

Figure 2. shows the curves of dependence of the critical load on relative length of a beam and fig. 3 displays the curves of dependence of it on Winkler's secondary coefficients. Tay (a - cr.) and Tay (b - cr.) in these Figures denote critical loads for fixation condition A and B respectively.

CONCLUSIONS

1. The suggested formulas for reactive contact force between the bar and elastic base is explained. This formula generalizes. Vinkler' s formula for large flexures. Vinkler' s coefficients of the second type are introduced which consider nonlinearity of flexures.
2. The problems on flexure of the elastic bar on the elastic base for large flexures are solved in the frame of the hypothesis of plane sections for two cases of fixing of the end of the bar. Comparative analysis with solutions of similar problems for small displacements is given. It is shown that only for large flexures the critic condition of jump type growth of the flexure (instantenous slump) is possible.
3. It is shown that calculations of flexures of the bar on the elastic base which are carried out by Vinkler' s classic scheme excluding possibility of loss of the carrying capacity of base and that has place for large flexures.