A simplified approach to dynamic analysis of pile groups in sand

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ABSTRACT: Pile foundations supporting bridge piers, offshore platforms and marine structures are required to resist not only static loading but also lateral dynamic loading. The static p-y curves are widely used to relate pile deflections to the nonlinear soil reactions. The p-multipliers are used to account for the group effect by relating the load transfer curves of a pile in a group to the load transfer curves of a single pile. Some studies have examined the validity of p-multipliers concept for the static and cyclic loading cases. However, the concept of p-multipliers has not been considered yet for the dynamic loading case. This is attempted in the current study. An analysis for the dynamic lateral response of pile groups is described. The proposed analysis incorporates the static p-y curve approach and the plane strain assumptions to represent the soil reactions within the frame of a Winkler model. The model accounts for the nonlinear behavior of the soil, energy dissipation through the soil and the pile group effect. A parametric study was performed employing the proposed analysis and the results were used to establish dynamic soil reactions for single piles and pile groups for different types of sand and harmonic loading with varying frequencies applied at the pile head. Dynamic p-multipliers were established to relate the dynamic load transfer curves of a pile in a group to the dynamic load transfer curve for a single pile. The dynamic p-multipliers were found to vary with the spacing between piles, soil type, peak amplitude of loading and the angle between the line between any two piles and the direction of loading.

RESUME: Les pieux supportant les piles des ponts, les plateformes en mer et les structures marines doivent résister non seulement aux charges statiques mais aussi aux charges dynamiques latérales. Le courbes statiques p-y sont largement utilisées utilisé pour relier les déflexions aux réactions non-linéaire du sol. Les multiplicateurs-p sont utilisés pour prendre en compte l'effet de groupe en reliant les courbes de transfert de charge d'un pieu d'un groupe aux courbes de transfert de charge d'un seul pieu. Quelques études ont examiné la validité du multiplicateur-p pour les cas de charges statiques et cycliques. Cependant, le concept du multiplicateur-p n'a pas encore été considéré pour les cas de charge dynamique. Ceci est tenté dans cette étude. Une analyse pour la réponse dynamique laterale de groupes de pieux est décrite. L'analyse proposée incorpore l'approche de la courbe statique p-y et les suppositions de deformations planes pour représenter les réactions du sol dans le cadre d'un modèle de Winkler. Le modèle prend en compte le comportement de non-linéaire du sol, la dissipation d'énergie à travers le sol et l'effet du groupe de pieu. Une étude parametrique a été effectuée en employant l'analyse proposée et les résultats ont été utilisés pour établir les réactions d'un seul pieu et les groupes de pour différents types de sable et charges harmoniques avec différentes fréquences appliquées à la tête du pieu. Les multiplicateurs-p dynamique ont été établis pour relier les courbes de transfert de charges dynamiques d'un pieu dans un groupe à la courbe de transfert de charges dynamiques pour un pieu seul. On a trouvé que les multiplicateurs-p dynamique varient selon l'espacement entre les pieux, le type de sol, l'amplitude de la charge et l'angle entre la ligne entre n'importe lesquels de deux pieux et la direction de charge.

1 INTRODUCTION:

Piles are subjected to various types of loading. Dynamic loading such as wind, wave and earthquake forces has a severe effect on piles supporting offshore structures, towers, bridge piers and buildings. The response of these structures and their integrity are in many cases governed by the lateral response of the pile foundations.

Various approaches have been developed for the static and dynamic lateral response of piles. Finite element analyses were conducted to evaluate the static and dynamic response of pile groups (e.g. Wolf and Arx, 1978). However, this approach requires large computational effort and is not suitable for regular design jobs. The boundary element approach has been also studied but the inclusion of the soil nonlinear behavior in this approach is difficult. The Winkler model has been used to approximately model the response of single piles and pile groups to lateral dynamic loads and has proved to be a powerful technique. Matlock et al. (1978) developed a unit load transfer curves approach, also known as p-y curves, to be used in Winkler model analysis of nonlinear response of piles in the time domain. Novak et al. (1978) developed a frequency dependent pile-soil interaction model assuming linear or equivalent linear soil properties. Gazetas and Dobry (1984) introduced a simplified linear method to predict fixed head pile response accounting for both geometric and radiation damping and using available formulae for static stiffness of piles. Both methods are not suitable for the seismic response analysis because of the linearity assumptions and also because they did not account for permanent deformation or gapping at the pile-soil interface.

Nogami et al. (1991) have developed a time domain analysis for single piles and pile groups by integrating plane strain solutions with a nonlinear zone around each pile using p-y curves. El Naggar and Novak (1996) also developed a computationally efficient model for evaluating the lateral response of single piles and pile groups based on the Winkler hypothesis accounting for nonlinearity using a hyperbolic stress-strain relationship, and slippage and gapping at the pile-soil interface. The model also accounts for the propagation of waves away from the pile and energy dissipation through both material and geometric damping.

One reasonable approach to account for pile-soil-pile interaction for piles in a group would be to predict the reduction in soil resistance relative to that of an isolated single pile. Poulos and Davis (1980) introduced the interaction factors concept to reduce the soil stiffness in the context of linear elastic analysis. Focht and Kock (1973) introduced the nonlinearity of soil into the evaluation of group interaction factors by applying a y-multiplier to stretch p-y curves. Cox et al. (1984) described an alternate approach to account approximately for the group effect, in which a "p-factor" would be used to shrink the p values on the p-y curve rather than to stretch the y values.

Brown et al. (1988, 1996) introduced the concept of p-multipliers. This concept represents the response of the pile group to lateral loading in terms of the response of the single pile to the same value of lateral loading. The p-multiplier concept states that the group effect reduces the p-value on the p-y curve at every point on every p-y curve for a given pile (based on its geometric position in the group) by the same amount. The p-multiplier assumes a different value depending upon whether a pile is in a leading or in a trailing position and the angle between the line connecting two piles and the load direction. The p-multipliers found in the literature are given either by row (i.e. same p-multipliers value for all piles in the same row) or assuming different values for each pile. In the latter case, the total p-multiplier for any pile is obtained by multiplying (rather than summing) the p-multipliers due to all the piles in the group.

These studies, however, do not represent the dynamic loading conditions caused by wind, wave or earthquake events. Therefore, it is necessary to check the validity of the p-multiplier concept under dynamic loading and develop p-multipliers from dynamic loading events.

2 MODEL DESCRIPTION

The soil along the pile shaft is divided into a number of layers; each layer has different soil properties according to the soil profile considered. Within each layer, the soil medium is divided into two annular regions as shown in Figure 1. The first region is an inner zone adjacent to the pile and accounts for the soil nonlinearity. The second region is the outer zone that allows for wave propagation away from the pile and provides for the radiation damping in the soil medium. The soil reactions at both sides of the pile are modeled separately to account for the state of stress and discontinuity conditions such as slippage and gapping at both sides as the load direction changes.

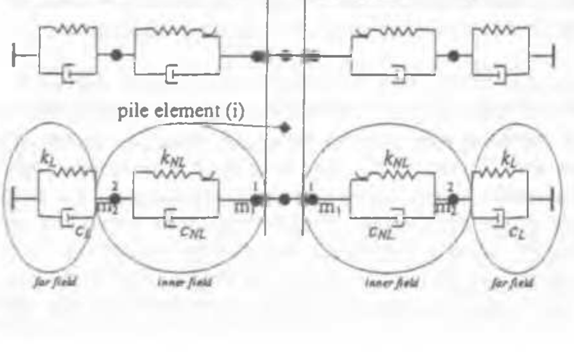

Fig. 1. Element representation of proposed model

2.1 Soil Model

The stiffness is established using the p-y curve approach and the damping is established from analytical solutions that account for wave propagation. The dynamic p-y curve represents the total soil response to the pile deflection (i.e. the inner and outer zones responses combined). The flexibility of the model can be represented as:

$$\frac{1}{k_{py}}=\frac{1}{k_L}+\frac{1}{k_{NL}} \tag{1}$$

where k_{py} is the static stiffness of the total soil medium, k_L is stiffness of the far field and k_{NL} is the stiffness of the near field. The p-y curves are established using empirical equations (Bhushan and Haley 1980; Bhushan et al. 1981). The static stiffness, k_{py}, represents the relationship between the static soil reaction, p, and the pile deflection, y, for a given p-y curve at a specific load level and is given by the slope of the p-y curve, i.e.

$$k_{py}=\frac{P_{(y_2)}-P_{(y_1)}}{y_2-y_1} \tag{2}$$

where y_1 and y_2 are the lateral pile displacement at points 1 and 2 on the p-y curve and $P_{(y2)}$ and $P_{(y1)}$ are the soil reactions at points 1 and 2. Thus, k_{py} represents the slope of the p-y curve between these two points.

The far field is assumed to behave in a linear fashion all the time and allows for the wave propagation away from and towards the pile. The soil reactions in the far field are represented by a spring (stiffness) and a dashpot (including both material and geometric damping) connected in parallel. The constants of the spring and the dashpot are evaluated the plane strain solutions developed by Novak et al. (1978), i.e.

$$k_L=G_{\max}S_{u1}(\nu) \tag{3}$$

$$c_L=\frac{2G_{\max}r_1}{V_s}S_{u2}(a_o=0.5,\nu) \tag{4}$$

where $G_{\max}$ is the initial shear modulus of the soil and S_{u1} and S_{u2} are constants obtained from the plain strain solution and are taken as 3.0 and 9.0, respectively.

The nonlinear stiffness (near field) is then calculated as:

$$k_{NL}=\frac{k_{py}k_L}{k_L-k_{py}} \tag{5}$$

Further details can be found in El Naggar and Bentley (2000).

2.1.1 *Discontinuity conditions*

The discontinuity conditions of the motion between pile and soil are caused by slippage and gapping at the soil-pile interface. The soil reactions to the pile motion at both sides are modeled separately as shown in Fig. 1. Pile and soil nodes in each layer are connected using a no-tension spring. This spring is disconnected to allow a gap to develop if tension stresses are detected in the spring. Cohesionless soils, however, tend to cave in the developed gap. To account for this behavior in the analysis, the pile and soil nodes remain connected at all times.

2.2 Pile Model

The pile shaft is assumed to be elastic, vertical and has a circular cross-section. It is subdivided into a number of elements equal to the number of soil layers. The standard structural stiffness matrix relating the translation and rotation to load and moment at both ends is used. To reduce the computational efforts, only degrees of freedom of interest are maintained and the rest are eliminated through a static condensation process.

2.3 Group Effect

The dynamic stiffness of a group of piles is greatly affected by the interaction between piles. For the lateral vibration, interaction between piles depends on the angle between the lines of the two piles, the direction of the horizontal applied force, θ, and the spacing between the two piles. Gazetas and Dobry (1984) found that the passive pile 2 with $\theta = 90°$ is affected essentially only by S-waves which emanate from active pile 1 and which have a phase velocity V_s. They also found that passive pile 2 with $\theta = 0°$ is affected by compression-extension waves coming from the active pile and propagating with an apparent phase velocity which is equal to the so-called Lysmer's analog velocity V_{La} given by:

$$V_{La} = \frac{3.4\,V_s}{\pi(1-\nu)} \tag{6}$$

Assuming that waves propagate in the horizontal direction only and also assuming a Winkler soil model, the displacement at any point in the elastic soil domain may be given by (Makris and Gazetas, 1992)

$$u(a_o,S,\theta) = u_o\,\Psi_u(a_o,S,\theta) \tag{7}$$

where u_o is the amplitude of the disturbance at the source, Ψ_u is an attenuation function accounting for the wave propagation away from the source and the radiation damping and S is the spacing between the center lines of two piles. It is sufficient to compute Ψ_u only for $\theta = 0°$ and $\theta = 90°$ and use the approximation by Dobry and Gazetas (1988) to evaluate Ψ_u at any angle θ, i.e.

$$\psi_u(a_o,S,0^o) = \psi_u(a_o,S,\theta^o)\cos^2\theta + \psi_u(a_o,S,90^o)\sin^2\theta \tag{8}$$

in which

$$\psi_u(a_o,S,0^o) = \sqrt{\frac{r}{S}}\,e^{-ia_o\frac{\pi(1-\nu)(s-r)}{3.4r}} \tag{9a}$$

$$\psi_u(a_o,S,90^o) = \sqrt{\frac{r}{S}}\,e^{-ia_o\frac{(S-r)}{r}} \tag{9b}$$

The interaction effect is assumed to vary linearly through each time step Δt. The soil displacement at the axis of pile l due to a disturbance at pile *m* is given by

$$u_l(t_i,S_{lm}) = P(t_i-t_{lm-1})H_1(t_i,r_{lm}) + P(t_i)H_2(t_i,r_{lm}) \tag{10}$$

where i is the number of the time step, S_{lm} is the distance between the piles l and m, t_{lm-1} is the travel time between the two piles minus one time step, H_1 and H_2 are the convolution integrals over the period Δt as described in El Naggar and Novak (1996). P is the interaction force between the two piles and can be calculated as

$$P = -K_f u_f \tag{11}$$

where K_f is the complex stiffness of the far field, and u_f is the soil displacement in the far field.

3 EQUATIONS OF MOTION

To include all aspects of nonlinearity, the governing equations of motion were formulated in the time domain. The mass of the inner field is lumped in two halves: m_1 at the node adjacent to the pile, and the other one, m_2, at the node adjacent to the outer field as shown in Fig.1. The equations of motion for the inner field and outer field are written, and the compatibility and equilibrium between the two fields are introduced. The linear acceleration assumption and the Newmark β method for direct time integration are used to develop the equations of motion. The modified Newton-Raphson iteration scheme (Bathe 1995) is used to solve the nonlinear system of equilibrium equations. Further details on the model can be found in El Naggar and Novak (1996).

4 DYNAMIC PILE GROUP ANALYSIS USING p-y CURVES

The proposed dynamic analysis of pile group response is based on the dynamic *p-y* curves for single piles and dynamic p-multipliers. El Naggar and Benley (2000) introduced the dynamic p-y curves for a single pile as:

$$P_d = P_S\left[\alpha + \beta a_o^2 + 2\kappa a_o\left(\frac{\varpi y}{d}\right)^n\right] \qquad P_d \le P_u \tag{12}$$

where P_d = dynamic soil reaction at depth x (N/m), P_s = static soil reaction obtained from the static *p-y* curve at depth x (N/m), a_o is dimensionless frequency = $\omega d/V_s$, ω = frequency of loading (rad/sec), d = pile diameter (m), y = lateral pile deflection at depth x (m), and α, β, κ, and n are constants determined from curve fitting Eq. 14 to dynamic soil reaction obtained from the model described above.

5 DYNAMIC p-MULTIPLIERS

Using the p-multipliers allows the analysis of the lateral response of a pile group as an ensemble of individual piles. The soil resistance to the movement of each of these individual piles can be represented by p-y curves with p values reduced by properly chosen p-multipliers.

The analytical model described above is used to analyze the response of a single pile and groups of two piles to a prescribed harmonic displacement at the pile heads. Thus, dynamic p-multipliers could be established by comparing the soil response for a pile in a group of two piles to that of a single pile. The parameters whose influence on p-multipliers is investigated in this study for a given pile and soil profile include:

1- The ratio of pile spacing to pile diameter, S/d.
2- The ratio of pile head displacement to pile diameter, y/d.
3- The dimensionless frequency, a_o.
4- The angle between the line connecting the two piles and the load direction, θ.

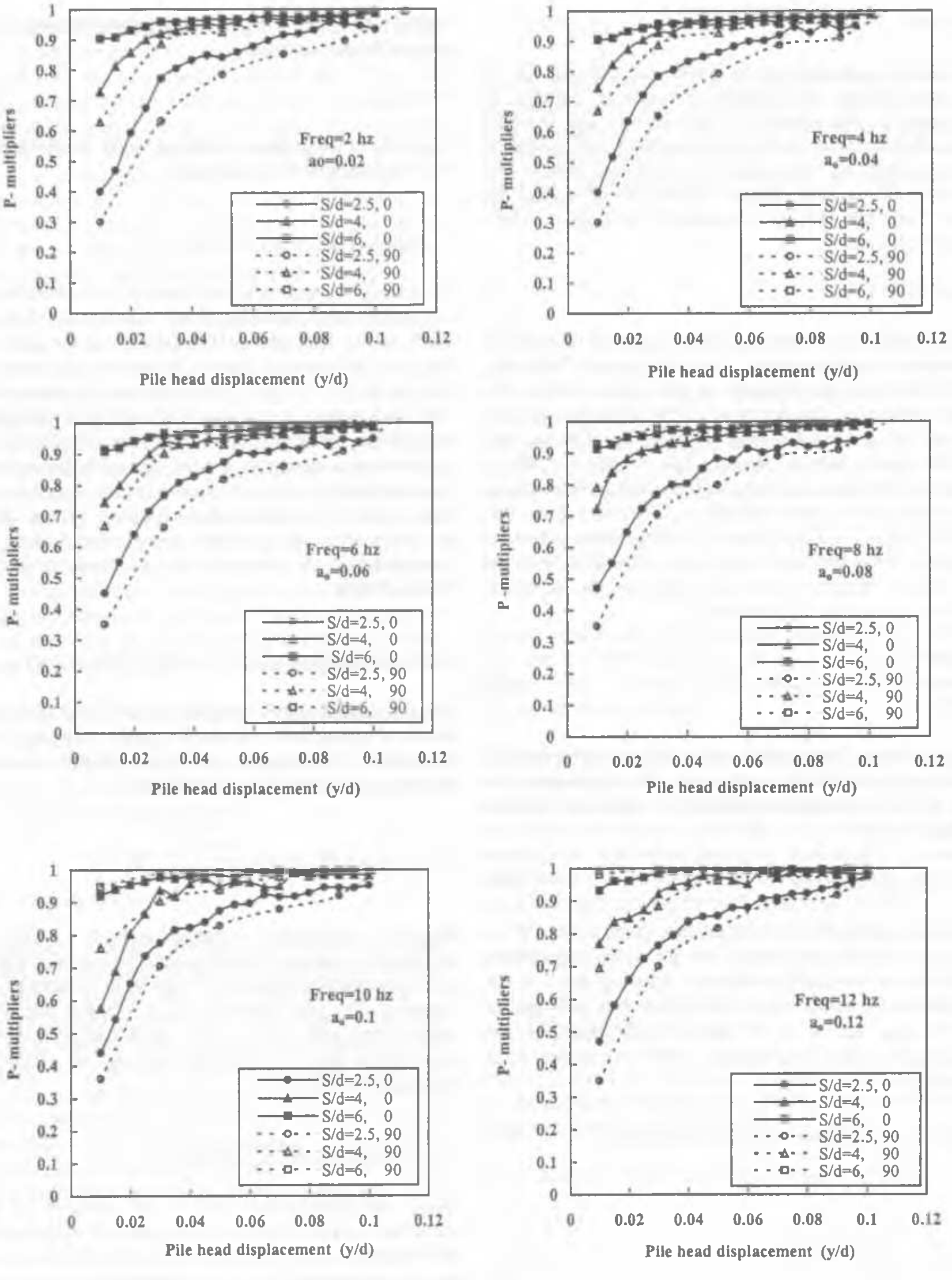

Fig. 2. P-multipliers versus pile head displacement for loose sand

Table 1 Pile and soil characteristics used in the analysis

Sand Type	Dr	ϕ	v	d (m)	L/d	E_p/E_s	G_{max} (Pa)	V_s (m/s)
Loose	35%	30°	0.3	0.5	20	6300	1.20E+07	100
Medium	50%	34°	0.3	0.5	20	3800	2.00E+07	150
Dense	90%	40°	0.3	0.5	20	1580	4.70E+07	250

where:

Dr = relative density of the sand

ϕ = angle of internal friction v = poison's ratio

E_p, E_s = Young's modulus for pile and soil, respectively

To establish the p-multiplier, two loading cases were considered separately, the first case is a pile loaded individually, and the second case is a group of two identical piles. In both loading cases, a prescribed harmonic displacement with specific peak amplitude and frequency was applied at the pile head, the response was analyzed and the p-multiplier was approximated by the peak pile head force at one pile in the two-pile group divided by the peak head force for the single pile. The forces are established after five loading cycles where the response was found to stabilize almost completely.

The p-multipliers for piles installed in cohessionless soil were computed and are plotted versus the peak of the applied

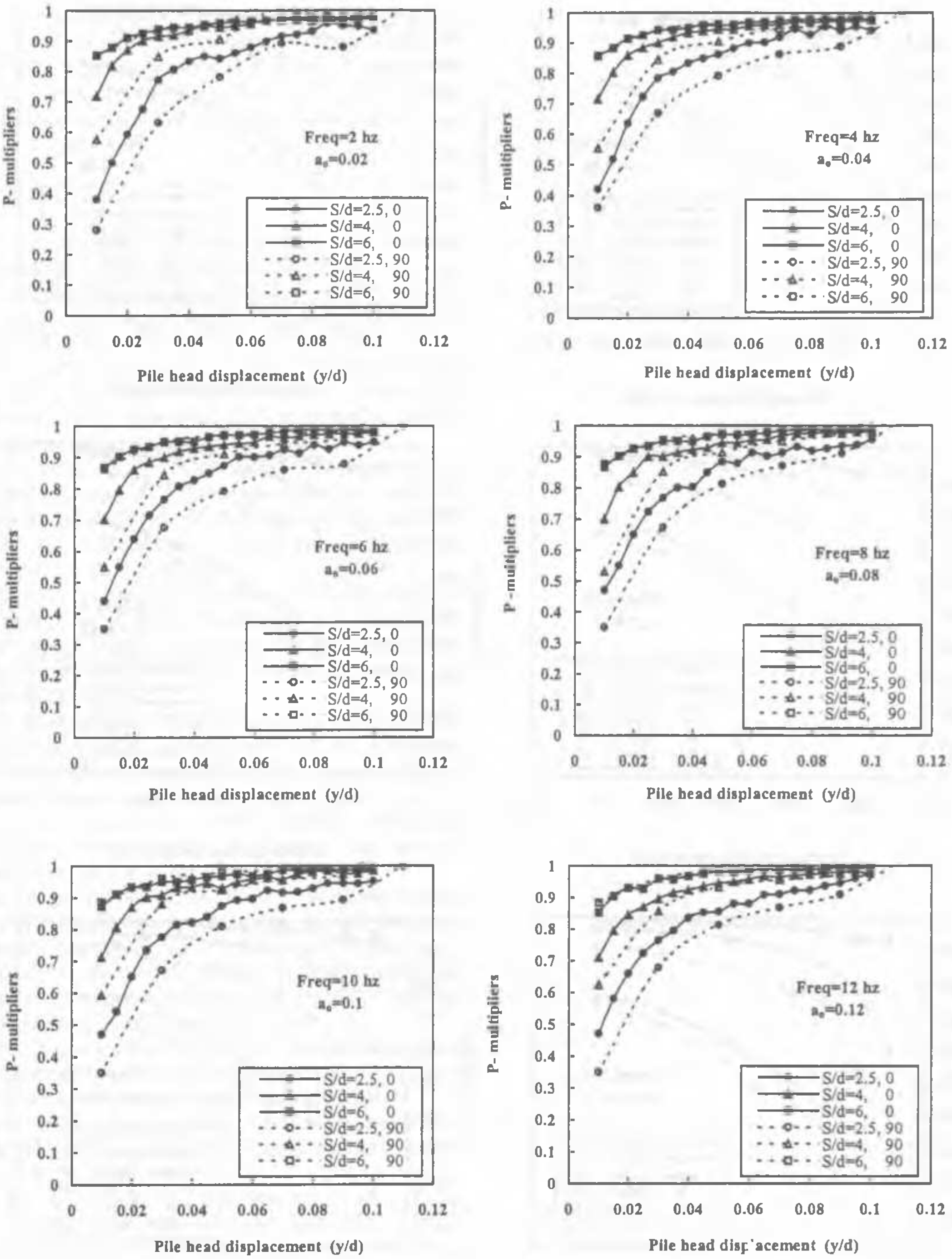

Fig. 3. P-multipliers versus pile head displacement for medium dense sand

harmonic displacement with different frequencies as a ratio of the pile diameter, y/d, in Figures 2, 3, and 4 for loose, medium dense and dense sand, respectively. The pile head is assumed to be pinned (i.e. rotation is allowed at the pile head). Table 1 shows the pile and soil characteristics used in this analysis.

Figures 2-4 show variation of the p-multipliers with the main factors that affect their values and considered in this study. The p-multipliers (pm) increase with an increase in S/d, meaning that the group effect decreases. Also, the pm increases with an increase in y/d. This means that during a dynamic loading event that is characterized by large pile head displacement, the pile-soil-pile interaction decreases and the piles tend to behave as individual piles. This may be attributed to the concentration of soil deformations in the vicinity of the pile at higher displacements. Comparing the pm in Figures 2-4 obtained for different loading frequencies, it can be noted that the effect of frequency on the p-multipliers is small and has no clear trend.

The effect of the angle between the line connecting the two piles and the load direction, θ, was studied for two cases, $\theta = 0^0$ (i.e. the line connecting the two piles is coinciding the direction of loading) and $\theta = 90^0$ (i.e. the line connecting the two piles is perpendicular to the direction of loading). It can be noted from Figures 2-4 that the p-multipliers for the case of $\theta = 90^0$ is less than those for the case of θ = 0°, especially for S/d=2.5 and

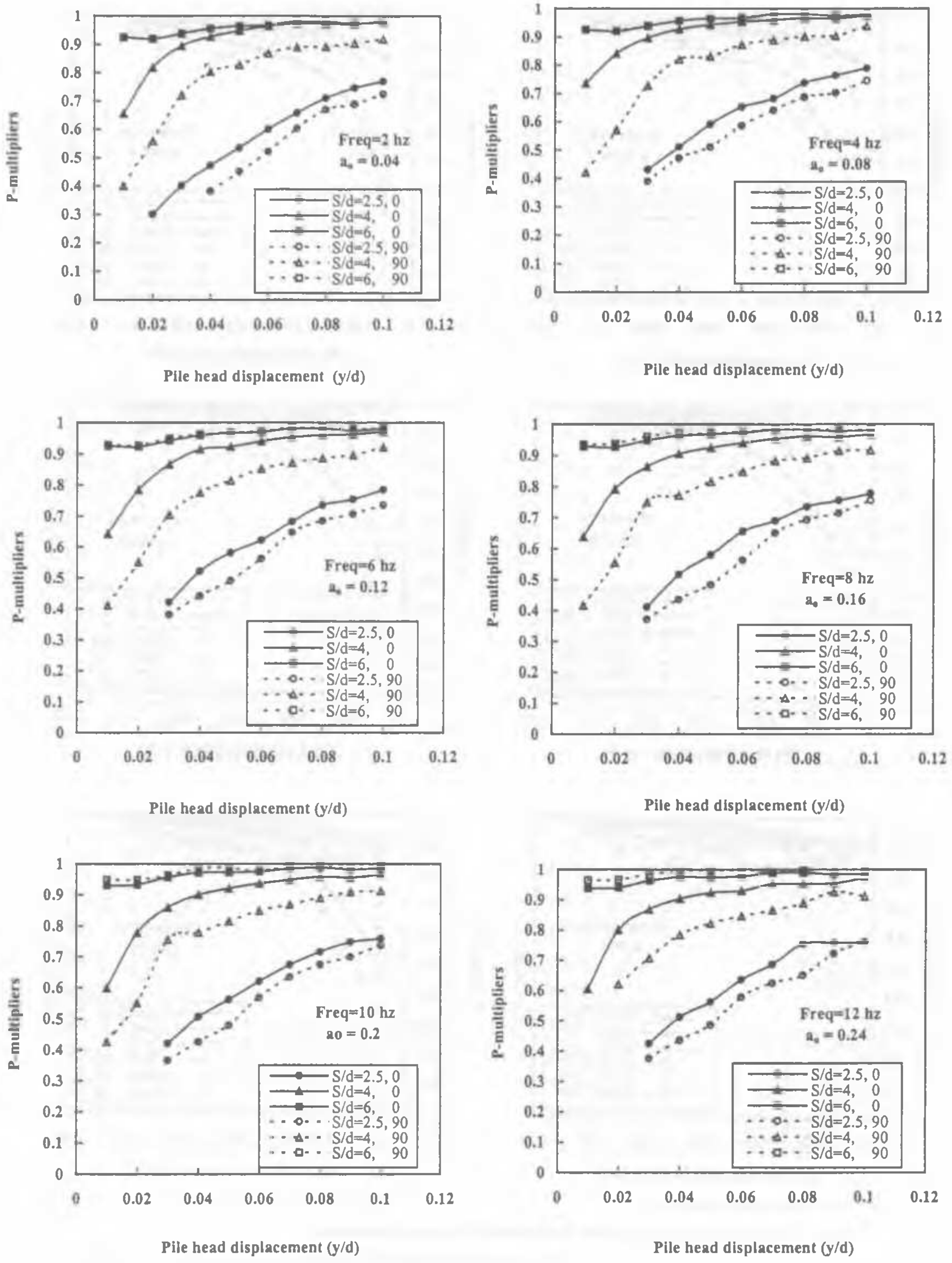

Fig. 4. P-multipliers versus pile head displacement for dense sand

S/d=4. This may be attributed to the increased nonlinearity along the loading direction. It was also found that the p-multipliers for case of $\theta = 90^0$ for dense sand are less than the corresponding p-multipliers for loose and medium dense sand.

6 CONCLUSIONS

A method for the analysis of transient nonlinear response of piles to dynamic loads is presented. The analysis is formulated in the time domain and accounts for pile-soil-pile interaction. A procedure for the development of dynamic p-multipliers that can be used for pile group analysis is proposed. It was shown that the p-multipliers are a function of S/d, y/d, a_0, and θ. The p-multipliers increase as S/d increases and as y/d increases.

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