The volume change of unsaturated soils

Le changement de volume des sols non saturés

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ABSTRACT: Volume changes for unsaturated soils are described by relations between the stress state variables, net total stress and matric suction, and the void ratio. To illustrate the influence of each stress state variable on the volume change of expansive soils, undisturbed unsaturated samples of Madrid grey clay have been tested.The results of suction controlled tests have been reported and it is shown the lateral stress induced under oedometric condition (K_o). A mathematical relationship is introduced which provides a framework for predicting deformations produced by altering the soil suction and the stress path in a unsaturated clay. A constitutive surface is presented.

RESUME: On décrit les changements de volume des soils non satures sont descrit pour variables d'etat qui relie l'indice des vides aux le contrainte totale nett et la succion matricielle. Pour illustrer l'effect de chaque variable d'etat ou les variations de volume des sols gonflant, on été réalisé essais sur des echantillons non saturées et inaltérée d'argile de Madrid. Les resultats d'essais de succion contrôlée montrent le contrainte lateral induite par la condition oedometriquee (K_o). Une relation analytique a été presentée pour predire les déformations produit par les changements de chemin de succion et de contrainte sur une argile non saturée. Une surface constitutif est présentée

1 INTRODUCTION

Many problems associated with foundations built in unsaturated clays are climate controlled response to wetting or drying of the soil. Loss or gain of water directly affects the soil in proportion to the energy loss or gain associated with the water content change. Soil water may be characterized by its matric potential or suction, which is a macroscopy property that indicates the intensity or energy level wich a soil attracts water.

Distress and distortion of structures due to heave of expansive clays is often attributed to the vertical component of swelling pressure. However, the lateral component of swelling pressure may also cause damage even with little evidence of vertical swelling.

It has been shown (Fredlund and Morgenstern, 1977), that the stress state of a unsaturated soil, can be defined by the applied net normal stress $(\sigma-u_a)$ and the matric suction (u_a-u_w), where: $\sigma=(\sigma_y+2\sigma_x)/3$, $\sigma=$ mean total normal stress, $\sigma_y=$ total normal stress in the y-direction, $\sigma_x=$ total normal stress in the x-direction, $u_a=$ pore air pressure, and $u_w=$ pore water pressure. The x-, y- and z- directions are assumed to be the three principal stress direction. Therefore, $\sigma_y=\sigma_1$ and $\sigma_x=\sigma_3$.

The oedometer is often the most appropiate method to simulate vertical displacements of shallow foundations on expansive unsaturated soils. Under K_o-loading, a total normal stress increment of $d\sigma_y$ is applied to the sample and only the vertical deformations $d\varepsilon_y$ are allowed, while the soil is not permitted to deform laterally, $d\varepsilon_x=d\varepsilon_z=0$ . .

In saturated soil mechanics for purpose of finite element analysis, in order to investigate the effects of volume changes, linear and nonlinear constitutive models for an isotropic elastic soil structure, can be formulated in acordance with the generalized Hooke's law using the effective stress, $(\sigma-u_w)$ and the material elastic parameters, the modulus of elasticity or Young's modulus E and the Poisson ratio ν. In unsaturated soils mechanics, the constitutive relations have been formulated as an extension of the equations used for saturated soils, using the stress state variables $(\sigma-u_a)$ and (u_a-u_w), and the material elatic parameters, the modulus of elasticity with respect to a change in the net normal stress, E, the Poisson ratio ν, and the modulus of elasticity with respect to a change in matric suction, H (Biot, 1941).

Changes in the matric suction in unsaturated expansives clays causes changes in mechanical properties of the soil, resulting in changes in both, E, and ν. Equally H, varies with the external applied vertical and horizontal normal stresses.

This paper describe a method of predicting the amount of expansive soil heave by taking into account the effect of the total normal stress increment of $d\sigma_x=d\sigma_z$ in the horizontal direction due to lateral swelling pressure, as a result of changes in matric suction under constant total vertical stress.

Test results show the void ratio changes, Δe, associated with mean stress and the matric suction changes. The relationship for the Madrid grey clay, is presented graphically in the form of a constutive surface.

2 TEST PROCEDURES AND MATERIAL PROPERTIES

The swelling tests reported in this investigation were performed using an oedometer developed by Escario (1967) and later modified by Cuellar (1978). This apparatus allows under controlled suction, the measurement of lateral swelling pressure of a soil specimen in a special confining ring (Sabbagh, 1995).

Tests were conducted on a set of three undisturbed a miocen Madrid grey clay, with index properties: liquid limit, w_L =70 %, Plastic limit, w_p =39 %, shrinkage limit, w_C =22 %, passing No. 200 sieve 100 %, clay fraction 42 %, specific gravity of solid particles, G_s=2.80. The initial condition in situ, typical in summer, were: natural water content, w=17.6%, matric sution, (u_a-u_w) =21 MPa, and dry unit weight, γ_s =15.134kN/m^3.

The test samples, with 70mm in diameter and 25mm in high, were subjected to 10, 150 or 500 kPa constant vertical surcharge pressures respectively, and their vertical swelling and lateral swelling pressure, under suction controlled of 1000, 300, 50 and 0 kPa was recorded.

3 BEHAVIOUR AND MATERIAL PARAMETERS

The stress-strain behaviour of saturated soils given by the generalized form of Hooke's law and extended for unsaturated soils, when is written in terms of principal stresses and principal strains by considering the soil loaded or unloaded under constant matric suction, become:

$$d\varepsilon_y = \frac{1}{E}\Big(d(\sigma_y - u_a) - 2\nu d(\sigma_x - u_a)\Big) \quad (1)$$

$$d\varepsilon_x = \frac{1}{E}\Big(\big(1-\nu\big)d(\sigma_x - u_a) - \nu d\big(\sigma_y - u_a\big)\Big) \quad (2)$$

where:

$d\varepsilon_y$, $d\varepsilon_x$ and $d(\sigma_x - u_a)$, are the vertical and lateral strain changes, and the net lateral stress change respectively, both associated to any variation in the vertical loading (Fredlund and Rahardjo, 1993). They are written for the special case of axial symetry where the application of a net total stress increment, $d(\sigma_y - u_a)$, is in the vertical direction, $d(\sigma_x - u_a) = d(\sigma_z - u_a)$, and $d\varepsilon_x = d\varepsilon_z$.

Now, in the case of one-dimensional compression test with $d(\sigma_x - u_a)$ = lateral swelling pressure, recorded by strain gauges attached to the containing ring, $d\varepsilon_x = 0$, and $d\varepsilon_y = d\varepsilon_v$, Eqs.(1) and (2) become:

$$d\varepsilon_v = \frac{1}{E}\Big(d\big(\sigma_y - u_a\big) - 2\nu d(\sigma_x - u_a)\Big) \quad (3)$$

$$d(\sigma_x - u_a) = \frac{\nu}{\big(1-\nu\big)} d\big(\sigma_y - u_a\big) \quad (4)$$

From Eq. (2), the change in lateral stress to the change in vertical stress ratio defined as the coefficient of earth pressure at rest, K_o, may be written:

$$K_o = \frac{\nu}{1-\nu} - \frac{E d(u_a - u_w)}{\big(1-\nu\big) H d\big(\sigma_y - u_a\big)} \quad (5)$$

From Eqs. (3) and (4) the material elastic parameters E and ν, can be expressed under constant controlled suction, $d(u_a - u_w) = 0$, as:

$$\nu = \frac{K_o}{1+K_o} \quad (6)$$

$$E = \frac{d\big(\sigma_y - u_a\big)}{d\varepsilon_v}\left(\frac{\big(1-K_o\big)\big(1+2K_o\big)}{1+K_o}\right) \quad (7)$$

where:

$$K_o = \frac{d(\sigma_x - u_a)}{d\big(\sigma_y - u_a\big)} \quad (8)$$

At this stage, under these circumstances, we could consider practically that the lateral pressure is a result of the change in the applied vertical stress. However, following the path of the test with a variation in the matric suction while now the vertical stress remains constant, the sample will have change in volumen, and because it is limited by the lateral confining of the oedometer, the specimen also will exhibit change in lateral swelling pressure. In fact, until equilibrium at this suction, changes in ε_v and $(\sigma_x - u_a)$, will occur. Hence, by introducing these recorded new values in the Eqs. (5) and (6), we will have account nonlinear changes in the parameters ν and E, if the vertical stress is varied under this new matric suction, wich again must remain constant.

The corresponding equations representing the stress-strain behaviour of the soil under the matric suction change, and constant vertical load, $d(\sigma_y - u_a) = 0$, become:

$$d\varepsilon_v = \frac{d\big(u_a - u_w\big)}{H} - \frac{2\nu d\big(\sigma_x - u_a\big)}{E} \quad (9)$$

$$\frac{d\big(u_a - u_w\big)}{H} + \frac{d\big(\sigma_x - u_a\big)\big(1-\nu\big)}{E} = 0 \quad (10)$$

The elastic moduli due to matric suction changes, H may therefore, be calculated :

$$H = \frac{d\big(u_a - u_w\big)}{d\varepsilon_v}\Big(1 - 2K_o\Big) \quad (11)$$

Therefore, the proposed constitutive equation for volume changes in unsaturated soils in terms of deformations and considering the lateral pressure, can be written as:

$$d\varepsilon_v = \frac{\big(1-K_o\big)\big(1+2K_o\big)}{E\big(1+K_o\big)} d(\sigma_y - u_a) + \frac{\big(1-2K_o\big)}{H} d(u_a - u_w) \quad (12)$$

The summation of the volumetric strain changes give the final volumetric strain of the soil:

$$\varepsilon_v = \Sigma d\varepsilon_v \quad (13)$$

The amount of total heave can be computed from the difference between the present in situ stress state and some future stress state.

These constitutive equation formulated using a elasticity form, can be written in a compresibility form more common to soil mechanics.The compresibility for of the constitutive equation for the soil structure of a saturated soil is written as:

$$d\varepsilon_v = m_v(\sigma - u_a) \quad (14)$$

where:

m_v = coefficient of volume change.

The compressibility form for the soil structure constitutive equation for an unsaturated soil under K_o-loading or one dimensional loading condition, is as follow:

$$d\varepsilon_v = m_1 d(\sigma_y - u_a) + m_2 d(u_a - u_w) \quad (15)$$

where:

m_1 = coefficient of volume change with respect to net vertical normal stress change. From Eq. (12):

$$m_1 = \frac{\big(1-K_o\big)\big(1+2K_o\big)}{E\big(1+K_o\big)} \quad (16)$$

m_2 = cofficient of volume change with respect to matric suction change. From Eq. (12):

$$m_2 = \frac{(1-2K_o)}{H} \quad (17)$$

The m_1 and m_2 coefficients of volume change can be called coefficient of compresibility, since they are essentially a ratio between changes in volumetric strain and stress variables.

The coefficients of volume change, m_1 and m_2, have negative signs, and it is the result of the E and H moduli being negative for typical soils. The m_1 and m_2 coefficients vary in a nonlinear manner, but can be constant for a small variation of stress or strain.

The volume- mass properties of a soil can also be used in the formulation of the constitutive equation for unsaturated soils. A change in void ratio de is commonly used as the deformation state variable for a saturated soil, being expressed by the constitutive equation:

$$de = a_v d\Big(\sigma - u_w\Big) \quad (18)$$

Table 1. Stress state and volume change under ko–wetting conditions

Test No	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Matrix suction, (kPa)	21000	21000	21000	21000	1000	1000	1000	300	300	300	50	50	50	0	0	0
Matrix suction change, (kPa)		0	0	0	-20000	0	0	-20700	0	0	-20950	0	0	-21000	0	0
Net vertical normal stress, (kPa)		10	150	500	10	150	500	10	150	500	10	150	500	10	150	500
Net vertical normal stress change, (kPa)			140	350		140	350		140	350		140	350		140	350
Net lateral normal stress, (kPa)		3.67	22.02	68.72	27.6	64.3	145.04	44.98	85.35	184.44	82.12	129.83	243.6	102.76	154.14	275.25
Net lateral normal stress change, (kPa)			18.35	36.7		36.7	80.74		40.37	99.09		47.71	113.77		51.38	121.11
Net normal or octahedral stress, p' (kPa)		5.78	64.68	205.8133	21.73333	92.86667	263.36	33.32	106.9	289.6267	58.08	138.5533	329.0667	71.84	152.76	350.1667
Net normal or octahedral stress change, dp' (kPa)			58.9	141.1333		71.13333	170.4933		73.58	182.7267		78.47333	192.5133		80.92	197.4067
Initial volumetric strain		-0.0032	-0.0192	-0.0269	0.0027	-0.0142	-0.0258	0.0062	-0.0112	-0.0247	0.0151	-0.0035	-0.023	0.0229	0.00171	-0.0222
Initial volumetric strain change			-0.016	-0.0077		-0.0169	-0.0116		-0.0174	-0.0135		-0.0186	-0.0195		-0.02119	-0.02391
Normal stress change ratio, Ko			0.131071	0.104857		0.262143	0.230686		0.288357	0.283114		0.340786	0.325057		0.367	0.346029
Poisson's ratio			0.115883	0.094906		0.207697	0.187445		0.223818	0.220646		0.254169	0.245316		0.268471	0.257074
Oedometric modulus, Em (kPa)			-8750	-45454.5		-8284.02	-30172.4		-8045.98	-25925.9		-7526.88	-17948.7		-6606.89	-14638.2
Young's modulus, E (kPa)			-8484.19	-44549.9		-7381.96	-27563.1		-7007.41	-22686.8		-6222.97	-15086.2		-5304.95	-12033.9
Octahedral modulus, Eoct (kPa)			-3681.25	-18329		-4209.07	-14697.7		-4228.74	-13535.3		-4219	-9872.48		-3818.78	-8256.24
Total volume of soil, V (cm3)	92.3628	92.06724	90.58943	89.87824	92.61218	91.05125	89.97984	92.93545	91.32834	90.08144	93.75748	92.03953	90.23846	94.47791	92.52074	90.31235
Dry volume of soil, Vs (cm3)	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912	50.8912
Void volume, Vv (cm3)	41.4716	41.17604	39.69823	38.98704	41.72098	40.16005	39.08864	42.04425	40.43714	39.19024	42.86628	41.14833	39.34726	43.58671	41.62954	39.42115
Void ratio of soil, e	0.814907	0.809099	0.780061	0.766086	0.819807	0.789135	0.768082	0.82616	0.79458	0.770079	0.842312	0.808555	0.773164	0.856468	0.818011	0.774616
Specific volume, v=1+e	1.814907	1.809099	1.780061	1.766086	1.819807	1.789135	1.768082	1.82616	1.79458	1.770079	1.842312	1.808555	1.773164	1.856468	1.818011	1.774616
Specific gravity of the soil solid, Gs	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8
Shear stress, q (Kpa)		3.165	63.99	220.84	-8.8	42.85	177.48	-17.49	32.325	157.78	-36.06	10.085	128.2	-46.38	-2.07	112.375
Average stress, p (Kpa)		6.835	86.01	279.36	18.8	107.15	322.52	27.49	117.675	342.22	46.06	139.915	371.8	56.38	152.07	387.625
dq/dp ratio			0.768235	0.810189		0.584607	0.62511		0.552365	0.558708		0.491663	0.509369		0.463058	0.485853
Mean stress, q' (Kpa)		6.33	127.98	441.28	-17.6	85.7	354.96	-34.98	64.65	315.56	-72.12	20.17	256.4	-92.76	-4.14	224.75
dq'/dp' ratio			2.065365	3.809891		1.452202	2.709944		1.354036	2.298833		1.176068	2.071466		1.095156	1.964946

where:

a_v = coefficient of compressibility.

For an unsaturated soil, void ratio, e can be used also as the deformation state variable for the soil structure. Using the soil mechanics terminology, the change in void ratio, de, of an unsaturated soil under K_o or one dimensional loading condition can be expressed as:

$$de = a_t d(\sigma_y - u_a) + a_m\left(u_a - u_w\right) \qquad (19)$$

where:

a_t = coefficient of compressibility with respect to a change in net vertical normal stress. The coefficient a_t, can be related to the cofficient m_1, (Lambe and Whitman, 1979), as follow:

$$a_t = m_1\left(1 + e_o\right) \qquad (20)$$

where:

e_o = initial void ratio

a_m = coefficient of compressibility with respect to a change in matric suction. Also the coefficient a_m, can be related to the cofficient m_2, as follow:

$$a_m = m_2\left(1 + e_o\right) \qquad (21)$$

The constitutive surfaces can be plotted with respect to the logarithm of the stress state variables (i. e., $\log(\sigma - u_a)$ and $\log\left(u_a - u_w\right)$). The logarithmic plots are linear over a relatively large stress range.

The slopes of the curves are the volumetric deformation indices associated with the void ratio surface, C_t and C_m. These indices can be obtained from the same test data used to obtain a_t and a_m coefficients.

The difference the soil properties lies in the manner in wich the result are plotted. The void ratio versus net normal stress curve can be linearized in a logarithmic scale.

4 TEST RESULTS AND DISCUSSION

The constitutive equation or surface for unsaturated soils have been formulated in the elasticity form. The soil has been considered homogeneous and isotropic. Therefore, the E, H and ν parameters are required to define the volumetric deformations associated with the soil structure constitutive equation. A direct method to determine these parameters at a specific state point is to compute their magnitud at the stress point under consideration, through laboratory experiments.

It should be noted that, although in practice as the soil is assumed to be linear elastic, the values of E, and ν are asumed remain constant over small increments of stress and strain, however, examination of the test data in Table 1, indicates that these parameters vary from one state point to another along a non linear constitutive surface.

The void ratio curves for the unsaturated expansive clay, are presented in Figs. 1 and 2. These show the void ratio changes when the soil is subjected to a decreasing matric suction and a variation in vertical total stress. We can see in Fig. 1 the lateral pressure change curves due to both changes in vertical surcharge and in matric suction. This shows the vertical total stress path, together to the horizontal stress path. Initially during the swelling stages at a small vertical stress, the lateral pressure and the void ratio changes are maximum. However when the vertical stresses increase, the lateral stresses and void ratio changes decrease. In other words, with the increase of the vertical surcharge, changes in matric suction will not be as effective as a total vertical stress in producing volume and lateral pressure changes.

It can observed in Fig. 2 that the void ratio increasing as the degree of saturation increase under a constant surcharge, becomes smaller when the net total or octahedral stress increase. Basically, this combination of vertical and lateral stress changes, becomes more effective in changing void ratio than is a change in matric suction.

In Fig. 3 is illustrated that, for all range of matric suction changes, at lower vertical stress changes, the variation obtained in E is lesser than under higher vertical stresses.

In Fig. 4 and 5 are shown the paths followed for the soil during the vertical total stress and matric suction changes.The stress path represented in a plot with the axes deviator stress, $(\sigma_1 - \sigma_3)$ versus net normal stress, $(\sigma_1 + 2\sigma_3)/3$, and another one plot with the axes shear stress, $(\sigma_1 - \sigma_3)/2$ versus the average normal stress $(\sigma_1 + \sigma_2)/2$, respectively.

The Fig. 6 presents the soil structure constitutive surface for the Madrid grey clay as plotted with respect to the $\log(\sigma - u_a)$ and $\log(u_a - u_w)$ axes. The constitutive surface was traced following different test paths through various combinations of

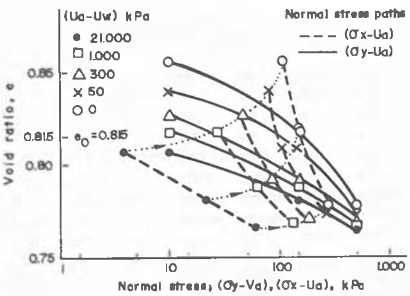

Figure 1. Compression curve.

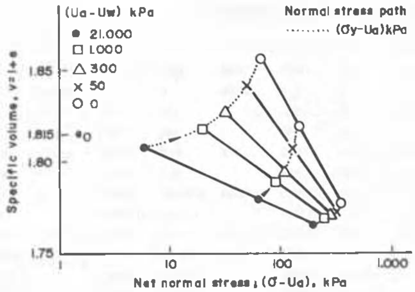

Figure 2. Specific volume versus octahedral stress relationship.

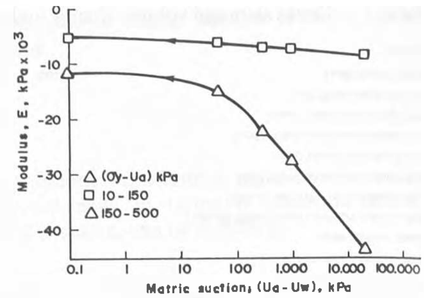

Figure 3. E versus suction relationship during loading.

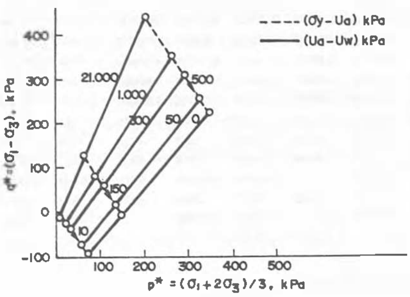

Figure 4. Stress paths in q*: p* space.

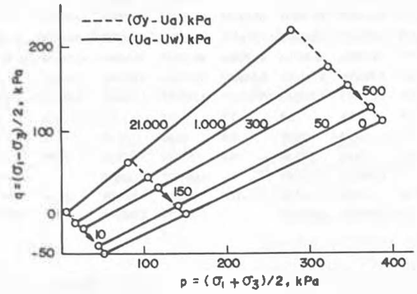

Figure 5. Stress paths in q : p space.

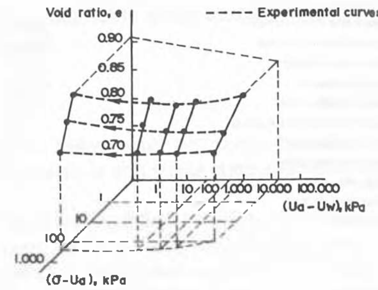

Figure 6. Soil structure constitutive surface for monotonic wetting.

stress state variables, in all cases the pore-water pressure was held constant, while the total normal stress and the pore-air pressure were varied.

5 FINAL CONSIDERATIONS AND CONCLUSIONS

An expansive clay is generally unsaturated due to desiccation. Expansive soils are found in many parts of the world. In particular, in areas with a semi-arid climate such as in Spain, soil matric suction change has been considered to be an important factor in the design and construction of building foundations, embankments and highway or airport pavements on expansive clays, in order to control and minimise (i) total and differential heave (ii) roughness or loss of shape (iii) longitudinal cracking, (Rodriguez Ortiz, 1975; Jimenez Salas, 1990; Juca, 1990; Oteo et al, 1995). The soil structure constitutive equation proposed and the elasticity parameters evalued experimentally in this paper, can be used to calculate volume changes in the Madrid grey clay.

An unsaturated soil will undergo change when the net normal stress or the matric suction variable changes in magnitud. Under a constant total vertical stress, an unsaturated soil will experience swelling and shrinking as a result matric suction variations associated with enviromental changes. In general, the net normal stress remains constant, while the matric suction stress changes during the heave process. Therefore, total heave can be predicted by measuring the in situ matric suction and estimating or predicting the future matric suction in the field under specific environmental conditions.

In finite element analysis, consequently with some problems involving soil-structure interaction, a procedure using small increments of stress state variables and strain can be used to model the nonlinear character of the soil. The required level of accurancy for the problem at hand must be born in mind when defining the above incremental procedure.

Finally can be concluded:

1. For unsaturated expansive soils, the investigation of lateral pressure under a constant surcharge and controlled suction, appears to be appropiated to simulate the expected field conditions.

2. In unsaturated expansive clays the vertical swelling phenomenon is primarily related to the reduction of the matric suction.

3. Both vertical pressure and suction matric changes results in lateral pressure changes.

4. Increasing in surcharge pressure and reduction in matric suction results in raising of lateral pressure. However, for light loads, the lateral pressure due to a reduction in matric suction change, is greater than the lateral pressure transfer due to a vertical pressure.

5. When the loads become heavy, the influence of reduction in matric suction on the lateral pressure, become progressively neutralized.

6. The elastic modulus E, is decreased, when the matric suction is reduced and increased when the matric suction is increased.

7. The constitutive equation proposed is suitable to reflects the elastic behaviour of unsaturated soils, in numerical analysis, both linear and nonlinear.

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