Comparisons between soil stiffnesses in laboratory tests using dynamic and continuous loading

Comparaisons entre des rigidités de sols en tests de laboratoire utilisant des pressions dynamiques et continuelles

M. R. Coop, V. Jovičić & J. H. Atkinson – *City University, London, UK*

ABSTRACT: Recent developments in local instrumentation for triaxial tests allow stiffnesses to be measured under monotonic loading at similar strain levels to those imposed by dynamic methods so that the elastic shear modulus at very small strains G_0 may be measured. In parallel, developments in the bender element technique mean that dynamic measurements of G_0 may also be made in the stress path apparatus. The paper presents comparisons for three soils between measurements made using the two techniques illustrating the effects of stress and strain level on the values of stiffness obtained and examining the elastic ranges of the soils.

RESUME: Des dévelopements récents en jauge locale pour des tests triaxial permettent à la rigidité d'être mesurée sous pression continuelles de tensions similaires à celles imposées par les pression dynamiques afin de mesuer la rigidité élastique de petite tension G_0. De même, des dévelopements dans la technique "bender element" permettent que des measures dynamiques de G_0 puissent être faits sur de machines "stress path". Le dossier présente des comparisons sur des mesures effectuées avec les deux techniques sur trois sols.

1. INTRODUCTION

The stiffness:strain relationship for most soils is generally assumed to have an S-shape so that the initial part of loading in shear is assumed to be linear and elastic within a very small range of strains after which the behaviour becomes highly non-linear. The stiffness in the elastic range is usually referred to as G_0 (or G_{max}) and apart from being an input parameter for many non-linear soil models is important as an indicator of the starting point of the stiffness:strain curve. It is usually measured by means of dynamic tests either in situ or in the laboratory.

Comparisons between resonant and torsion shear tests in the resonant column apparatus have most frequently been used both to show that G_0 is not rate dependant and to establish the general shape of the stiffness:strain curve (e.g. Iwasaki et al., 1978; Ni, 1987; Bolton & Wilson, 1989). There has been some uncertainty however as to the extent of the elastic range measured by these tests. Georgiannou et al. (1991) proposed that it was dependant on the plasticity of the soil, the yield point being at about 0.001% for sands and silts and at 0.01% for plastic clays. Lo Presti (1995) suggested that in some cases the larger elastic limit strains resulted from rate effects in the dynamic tests and that the yield point should be around 0.001% for all soils except where cementing was known to be present when the range would be increased significantly (e.g. Cuccovillo & Coop, 1997).

More recently emphasis has been given to the comparison of dynamic and continuous loading measurements of stiffness in the triaxial apparatus, as the mode of shearing is more relevant to most engineering applications and the apparatus is much more widespread. Early work of this type (e.g. Georgiannou et al., 1991) suffered from the fact that the local instrumentation used in the triaxial testing had relatively poor accuracy so that the first values of stiffness could only be established at strains of about 0.01%, which was well beyond the elastic range. There are however now new developments in the local measurement of axial strain in the triaxial apparatus such as the LDT (Goto et al., 1991) and a system based on miniature LVDTs (Cuccovillo & Coop, 1996) which allow the stiffness of the soil to be established at strains about two orders of magnitude smaller. The value of G_0 can now therefore be defined from a continuous loading test in the triaxial apparatus. Comparisons between such measurements and dynamically measured stiffnesses for the same soil have been made by Tatsuoka & Kohata (1995) using in situ seismic data and Lo Presti (1995) using the resonant column apparatus. The development more recently of the bender element method for use in the stress path apparatus (Viggiani & Atkinson, 1995a) now also allows the possibility of comparing continuous loading and dynamic measurements of stiffness both in the triaxial cell.

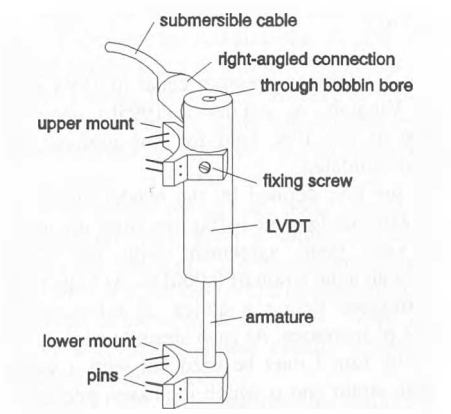

Figure 1. The mounts to hold the miniature LVDTs on the sample (after Cuccovillo & Coop, 1996)

2. TEST PROGRAMME AND PROCEDURES

The tests were conducted in hydraulic triaxial apparatus which were computer controlled. The samples were isotropically compressed and undrained shearing probes were conducted at increasing values of the mean effective stress, p'. The probes were stopped at an axial strain of only 0.1% so that the strain that the sample underwent between successive probes was sufficient to erase any influence on the stiffness measured from the memory of the previous probe. The probes were carried out at a constant rate of strain which was about 0.01-0.02%/hour for the clays and 0.1%/hour for the sand. The probes were not started until the rate of strain of the sample following the isotropic compression had

fallen to a rate which was insignificant by comparison with these values.

Measurements of the axial strains during shearing were made using a new system of local strain transducers developed by Cuccovillo & Coop (1996). A pair of miniature submersible LVDTs was attached to the sample using the mounts illustrated in Fig.1. These mounts permit the transducers to rotate due to the barrelling of the sample at large strains so that measurements may be made up to the ultimate state. The smallest strain measurable by the system is dependant on the quality of the power supply, signal conditioning and data-logging systems used. In this case the first reliable values of the soil stiffness were at about 0.0001%. For each probe about 500 data points were collected and the undrained tangent stiffness (G) was calculated by means of a linear regression through 21 successive points for most of the test but 11 at the start where the data are sparser. The tangent stiffness has been defined as:

$$3G = dq/d\varepsilon_a \qquad (1)$$

The bender element tests were conducted with the standard arrangement of elements with the transmitter in the top platen and the receiver in the base pedestal. A sinusoidal input wave was used with a frequency which was sufficiently high to avoid a near field effect (Jovicic et al., 1996) while for the sands the frequency was also kept low enough to avoid the error identified by Gajo (1996) which can result from the inertial coupling of the two phases for a coarse grained soil. The travel time was read directly off the oscilloscope and used to calculate the shear wave velocity (V_s) and hence the elastic shear modulus:

$$G_0 = \rho V_s^2 \qquad (2)$$

where ρ is the mass density of the sample.

Tests were carried out on three soils which were selected for their diverse characteristics. Each of them was reconstituted so that the comparisons of the stiffness measurements could be made in the absence of any natural bonding or cementing. The first soil was Dogs Bay sand which was tested extensively by Coop (1990). It is a biogenic carbonate sand and is poorly graded with a D_{50} of about 0.2mm, its particles consisting predominantly of mollusc and foraminifera shells. Two clays were also tested; Speswhite kaolin and Boom clay. Kaolin was chosen as an example of a low plasticity clay (PI = 24) while the Boom clay is of much higher plasticity (PI = 47). The Boom clay is of Oligocene age and is of

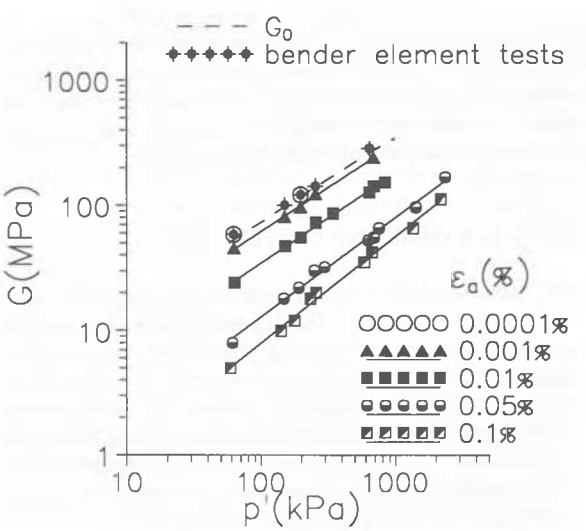

Figure 2. Bender element and LVDT stiffnesses for Dogs Bay sand under isotropic first loading (adapted from Jovicic & Coop, 1997)

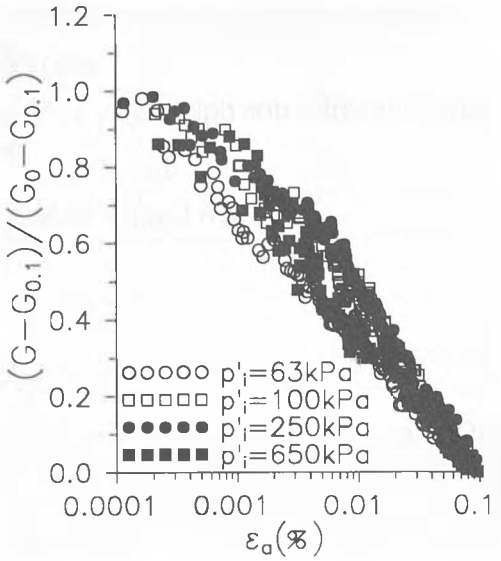

Figure 3. Normalised stiffness:strain curves for Dogs Bay sand under first loading

marine origin. The samples were retrieved from a depth of 223m from the face of a test drift at Mol, Belgium.

The clays were reconstituted into slurries and then one-dimensionally compressed in consolidometers to a vertical stress of around 50kPa before being transferred to the triaxial apparatus in which they were isotropically compressed. The sand samples were created in as loose a state as possible using the wet compaction method. Carbon dioxide was circulated through the samples prior to flooding them with distilled water, thus ensuring full saturation when the back pressure was subsequently applied. The tests on the sand were the last to be conducted and used an apparatus fitted with both the bender element and miniature LVDT systems. The work on the clays was carried out in earlier versions of the apparatus so that the two measurements were done on two samples in different apparatus.

3. DISCUSSION OF RESULTS

Figure 2 shows a comparison between the G_0 data from the bender element tests on the Dogs Bay sand during isotropic first loading with tangent shear moduli from the LVDT data for a variety of different strain levels. The bender element test measurements of G_0 define a unique relationship between logG and logp' so that the G_0 of these loose samples under first loading may be defined by an equation of the form:

$$G_0/p_r = A(p'/p_r)^n \qquad (3)$$

where p_r is a reference pressure equal to 1kPa and A and n are constants. Viggiani & Atkinson (1995b) demonstrated that a relationship of this type held for fine grained soils which are normally consolidated.

In Fig.2 the line defined by the bender element tests may be compared with the tangent stiffnesses from the undrained probes. There is very good agreement with the LVDT measured stiffnesses at an axial strain of 0.0001%. At higher strain levels the LVDT stiffnesses define a series of sub-parallel lines which converge as p' increases. At each strain level a relationship of the type given by Eqn.3 may be used but with a value of A which reduces with strain and n which increases, another feature which had been identified for fine grained soils by Viggiani & Atkinson (1995b). To investigate this comparison between dynamic and continuous loading measurements of stiffness in a greater detail, a

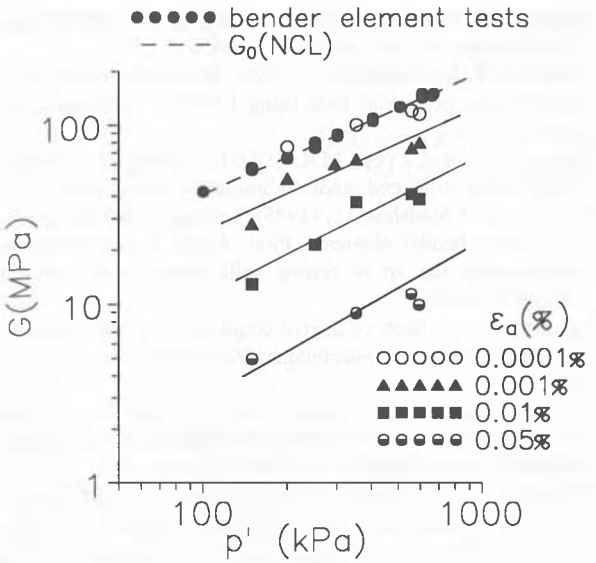

Figure 4. Bender element and LVDT stiffnesses for normally consolidated kaolin

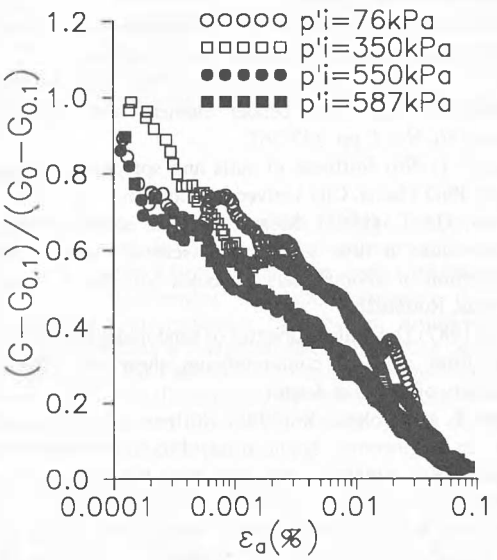

Figure 5. Normalised stiffness:strain curves for normally consolidated kaolin

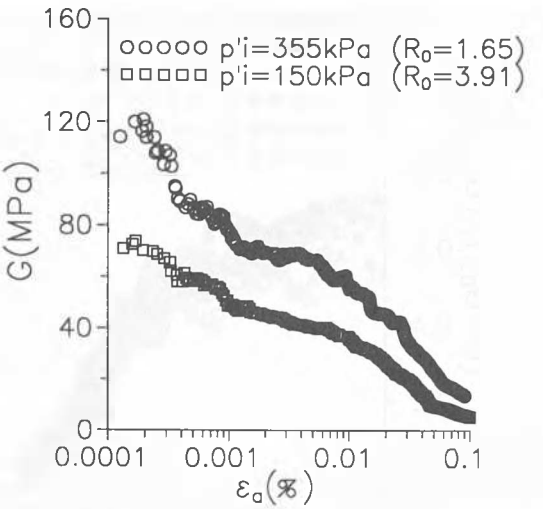

Figure 6. Stiffness:strain curves for overconsolidated kaolin

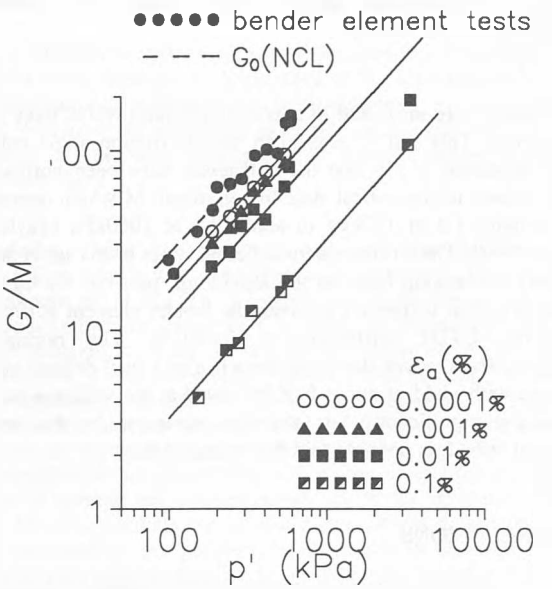

Figure 7. Bender element and LVDT stiffnesses for normally consolidated Boom clay

means was sought of normalising the stiffness:strain curves with respect to G_0 which would allow curves for different stress levels to be superimposed. Because of the convergence of the lines for different strain levels on the logG:logp' plot this comparison cannot be made simply by normalising the tangent stiffness, G with respect to the bender element stiffness, G_0 at that stress level as at different stress levels the stiffness reduces at different rates. Figure 3 shows an attempt to overcome this problem by deducting the final stiffness from the probe, at 0.1 % axial strain ($G_{0.1}$) from both the denominator and the numerator so fixing the stiffness:strain curve to pass through the point zero at 0.1% axial strain. The resulting plot then gives a unique relationship from which it is again clear that there is good agreement between the bender element stiffnesses and the LVDT tangent stiffnesses at the smallest strain levels achievable of 0.0001%. There is however little evidence of a region of linear elastic behaviour as the stiffness drops almost immediately.

Dyvick & Madshus (1985) estimated that the strains imparted to the soil by the transmitting element were around 0.001%, although dynamic finite element analyses by Jovicic (1996) showed that the geometric damping as the wave travelled through the sample resulted in a reduction in the strain level by an order of magnitude by the time it reached the receiver. The good agreement seen on Fig.3 would indicate however an average strain level of 0.0001%. This good agreement also indicates that any inherent anisotropy of the sample has not affected significantly the stiffnesses measured.

The comparison between the dynamic and continuous loading stiffnesses for the kaolin samples is given in Figs.4 and 5. All samples were normally consolidated and three bender element tests define a unique line on the logG:logp' plot. The lines for larger strain levels again converge as p' increases. For the normalisation of the stiffness:strain curves from the LVDT measurements in Fig.5 the value of G_0 used was that on the line shown on Fig.4 for that stress level. Again there is little evidence of a linear elastic region and the bender element stiffnesses agree well with the continuous loading measurements at a strain of 0.0001%. Probe tests were also carried out on two overconsolidated kaolin samples (Fig.6), but the shapes of the stiffness:strain curves which are not normalised in this case still show little sign of an elastic region.

Figures 7 and 8 show the same comparisons for normally consolidated samples of Boom clay. The bender element

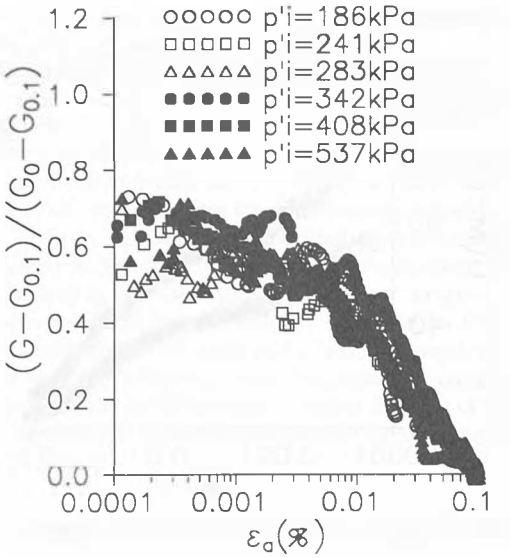

Figure 8. Normalised stiffness:strain curves for normally consolidated Boom clay

stiffnesses were measured in two different tests which gave good agreement. This soil is unusual in that its friction angle reduces with increasing p', so that the stiffnesses have been normalised with respect to the critical state line gradient, M which decreased from about 1.2 at 100kPa to about 0.8 at 1000kPa (Taylor & Coop, 1993). The stiffnesses for different strain levels again define slightly converging lines on the logG:logp' plot but for this soil there is a clear difference between the bender element stiffnesses and the LVDT stiffnesses at 0.0001%. The normalised stiffness:strain curves show that there is also a well defined elastic plateau with yield at about 0.005% but that the stiffness on this plateau is significantly less that that measured by the bender element tests. It is possible that this is a rate effect.

4. CONCLUSIONS

A new system of local axial strain measurement has been developed for the triaxial apparatus which has sufficient accuracy to allow comparison with dynamically measured stiffnesses. For a sand and a clay of low plasticity the bender element stiffnesses corresponded to the tangent stiffnesses measured under continuous loading for a strain of about 0.0001% which was the lowest strain at which a stiffness could be defined. For these two soils there was little evidence of a plateau of constant stiffness within the range of strains above 0.0001%. In contrast a well defined region of almost constant stiffness was seen for a plastic clay. This soil also gave poor agreement between the dynamic and continuous loading measurements of stiffness and it is possible that this results from a rate effect which is not seen for the low plasticity soils.

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