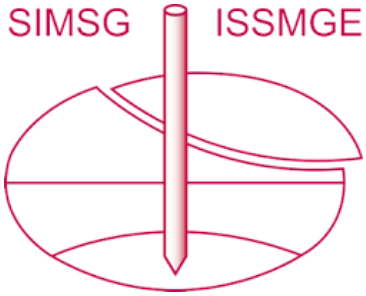
INTERNATIONAL SOCIETY FOR
SOIL MECHANICS AND
GEOTECHNICAL ENGINEERING



This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:

https://www.issmge.org/publications/online-library

This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.

A serviceability limit state design approach for footings

Limites de service pour selles de fondation

B.M. Lehane & E.R. Farrell – *Department of Civil Engineering, Trinity College Dublin, Ireland*

ABSTRACT: The paper examines the possibility of predicting the drained settlement of shallow foundations using an approach which models the stress and strain level dependent vertical stiffness of the ground and assumes that stresses imposed by the footing can be approximated from elastic theory. This approach, which is investigated for the case of a trial footing test on Dublin Boulder Clay, enables conclusions to be made regarding the sensitivity of settlement predictions to non-linear soil stiffness parameters.

RESUMÉ: Le papier explore la possibilité de prédire les déplacements à long terme des fondations de grande largeur en utilisant une approache qui modèle la raideur verticale de la terre qui depends sur des niveaux de contraintes et de déformations et qui suppose que des contraintes imposé par la surface carreé peuvent être estimé avec de theorie élastique. Cette approache, qui est examiné pour un essai avec une surface carrée sur l'argile de Dublin, permets des conclusions d'être fait en ce qui concerne la sensibilité des prédictions de déplacement sur des paramètres non-linéaires de la raideur.

1. INTRODUCTION

Eurocode 7 introduces the concept of limit state design for shallow foundations and, as such, requires separate calculations to ensure that (a) there is an adequate factor of safety against a bearing capacity failure and (b) the foundation will perform satisfactorily in service i.e. its long term settlement is acceptable. The traditional approach for foundations on clay has been to derive an allowable bearing pressure by applying a suitable load factor to the ultimate undrained bearing capacity of the footing. While Eurocode will accept this approach for simple footings, it is important to explore alternative approaches which are more in keeping with the limit state concept. These should ideally be as simple as possible but should also incorporate recent improvements in our understanding of soil stiffness.

The impact of the advances made in soil stiffness measurement is examined in this paper by comparing the settlements observed in a footing test conducted on Dublin boulder clay with settlement predictions made on the basis of state-of-the-art laboratory stiffness measurements. This comparison allows some conclusions to be made regarding the sensitivity of settlement predictions to non-linear soil stiffness parameters.

2. TRIAL FOOTING TEST AND SITE PROPERTIES

The footing test took place at a site in Tallaght on the outskirts of Dublin in Ireland. The complete site had been lowered by 3m one year prior to the test. This excavation left unweathered Dublin black boulder clay at the surface with the water table just below this surface. Four 1.5m deep 50mm diameter holes at 0.7m centres were initially drilled at the location of the footing to accelerate consolidation during the test.

The clay beneath the footing was typical of that underlying most of the Dublin area. It is a hard, heavily overconsolidated, well graded, low plasticity stoney clay (lodgement) till. Typical index parameters measured at the test site were as follows: liquid and plastic limits of ≈25% and ≈12% respectively, clay fraction ≈15%, silt fraction ≈30% and mean Standard Penetration Test (SPT) N value to 3m depth ≈45 (excluding 10% of tests which gave N>100).

The footing was a 1.5m square (relatively rigid) reinforced concrete pad founded on a 50mm thick sand blinding layer at 0.1m depth. Loading to a maximum bearing pressure of 1020kPa was achieved over a period of 3 months. Each increment of load was added after the rate of settlement had reduced to insignificant levels (< 0.01mm/day). Settlements were recorded at the four corners of the footing using dial gauges attached to reference beams; these were verified by level readings taken on graduated scales also located at the footing's corners. Unfortunately, site constraints dictated that settlement readings could only obtained after an initial dead load corresponding to a bearing pressure of 90kPa had been applied to the footing.

The best estimate bearing pressure vs. long term (drained) settlement curve derived from the test data is shown on Figure 1. The equivalent linear elastic modulus (E') of the ground approximating this curve is 65MPa; this modulus is in good agreement with the correlation proposed by Stroud (1988) of E'(MPa) =1.5 times the mean SPT N value for low plasticity clays.

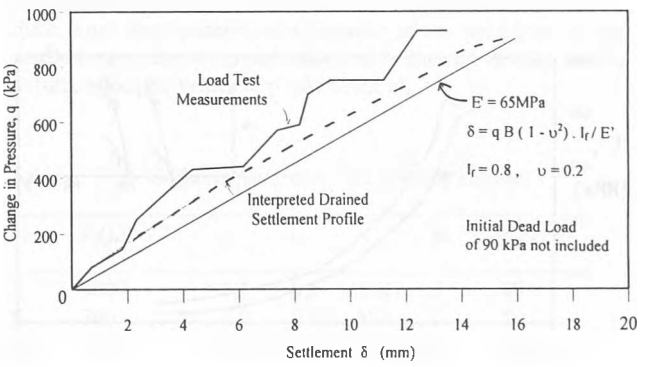


Figure 1. Applied pressure vs. settlement data from footing test

3. DUBLIN BOULDER CLAY STIFFNESS

3.1 Drained vertical stiffness characteristics

Triaxial stress path tests, incorporating local axial and radial strain instrumentation, were carried out on block and rotary cored samples of Dublin black boulder clay (DBC). While these samples were not retrieved from near the footing test location, their particle size distributions, mineralogy and Atterberg limits were virtually identical to those measured on samples from beneath the footing.

Two of the samples were tested in conventional drained compression after initial anisotropic consolidation followed by swelling to an isotropic stress, p'_0; see Figure 2. The variation of the vertical drained secant Young's modulus (E'_z) with (local) vertical strain (ε_z) measured in the drained compression stages are shown on Figure 2. Both variations highlight the strongly non-linear behaviour of DBC e.g. E'_z values at 0.01% strain are between four and five times higher than those at 0.1% strain. These tests also indicated that the ratio of the radial to vertical strain (i.e. Poissons ratio if the material is assumed elastic) was typically about 0.2 at axial strains of less than 0.1% and increased to a value of 0.4 at an axial strain of 1%.

The E'_z values measured for DBC at axial strains of 0.01% and 0.1% are compared on Figure 3 with a set of triaxial E'_z data compiled by Lehane *et al.* 1996 for medium dense and dense reconstituted quartz sands and with E'_z data reported by Lehane & Smilie (1994) for London Clay (which followed a similar stress history). The E'_z values on Figure 3, which are plotted against the samples' initial mean effective stresses (p'_0), indicate that:

- DBC, at a given stress and strain level, is appreciably stiffer than London clay and is comparable to that of a dense sand.

- Stiffness increases as the mean stress level (p') increases. Lehane et al (1996) show that, at strains up to 0.1%, the stiffness of the sand samples varied with $(p')^x$, where x is 0.7 ±0.3 while Lehane & Smilie (1994) indicate that x is approximately unity in the London Clay at strains between 0.01% and 0.5%.

In summary, it is evident from Figures 2 and 3 that the dependence of stiffness on both stress level and strain level needs to be taken into consideration in a settlement analysis which attempts to model the soil behaviour.

3.2 Expression for E'_z

The stiffness data on Figure 2 have been re-plotted using double

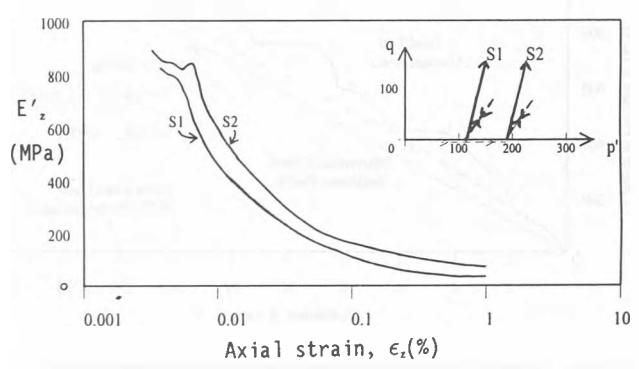


Figure 2. Vertical drained secant Young's modulus (E'_z) variation with vertical strain (ε_z) in triaxial tests on Dublin Boulder Clay.

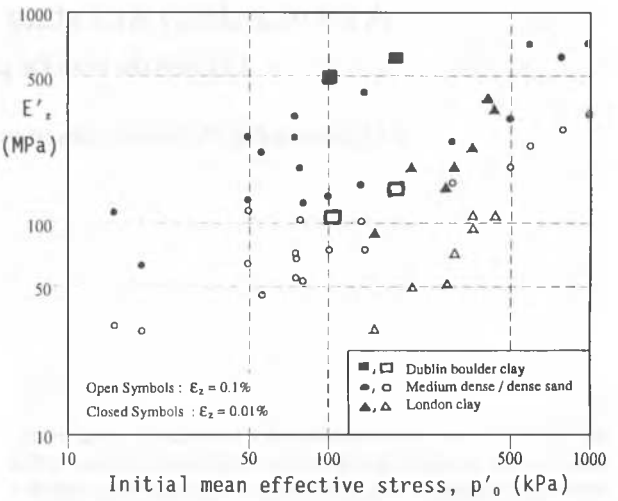


Figure 3. E'_z variation with stress and strain level in 3 soil types.

logarithmic axes on Figure 4 in the form $E'_z/\sqrt{p'}$ versus ε_z. Both curves on Figures 4 almost follow the same straight line suggesting that a suitable approximate expression for E'_z in DBC is:

$$E'_z = E_f (p')^x (10^{-3}/\varepsilon_z)^n \qquad (1)$$

where x= 0.5, n= 0.6 (≡the slope of the line shown on Figure 4) and E_f is a constant and equal to 9000 $kN^{0.5}/m$ when E'_z and p' are expressed in kPa. If, as is often assumed, stiffness is taken to be directly proportional to p' (i.e. x=1), a best fit E_f value of 700 with n=0.65 is derived for the data on Figure 2.

As the value of p' increases during drained loading of a footing, the tangent vertical drained stiffness $(E'_z)_t$ is more conveniently used in analytical methods. The corresponding tangential stiffness form of Eqn (1) is given as:

$$(E'_z)_t = (1-n)\, E_f (p')^x\, 10^{-3n}\, \varepsilon_z^{-n}$$

$$= (1-n)\, E'_z \qquad (2)$$

E'_z and $(E'_z)_t$ are assumed in the following to be constant at strains less than 0.001% i.e. stiffness is independent of strain at ε_z <1 $x10^{-5}$ and $E'_z = (E'_z)_t = E_f (p')^x\, 100^n$.

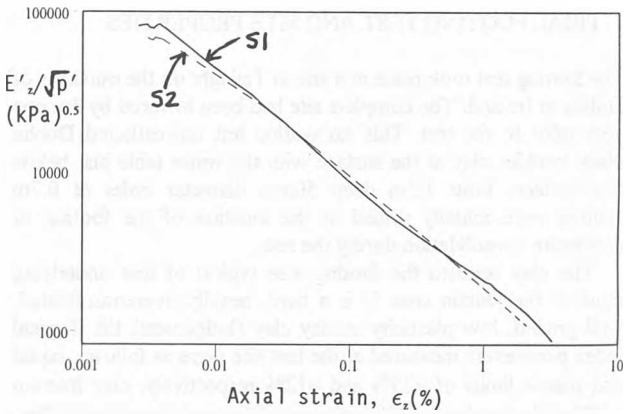


Figure 4. $E'_z/\sqrt{p'}$ vs. ε_z curves derived from E'_z data on Figure 2.

4 SETTLEMENT PREDICTION

4.1 *Simplified settlement prediction approach*

Rather than resorting to relatively complex non-linear finite element computations to incorporate expressions such as Eqn. (2) in a standard settlement prediction, designers may wish to use a less complex and faster approach based on the assumption that the imposed stresses on the ground can be approximated from elastic theory. Many settlement prediction computer programs (e.g. VDISP by OASYS 1992) adopt this assumption.

The vertical and horizontal stress increments ($\Delta\sigma'_z$ and $\Delta\sigma'_x$) beneath an area with a uniform pressure distribution (i.e. under a flexible footing) may be estimated by integration of Boussinesq's equations. Incremental elastic strains and hence settlements can then be calculated by combining these stress increments with the tangential stiffness given in Eqn. (2).

Burland et al (1977) report on factors affecting the predictions of stress distributions in elastic soils and examine effects such as stiffness non-linearity, soil non-homogeneity, Poisson ratio variability and stiffness anisotropy. They conclude that Boussinesq's equations are sufficiently accurate for the estimation of vertical stress changes ($\Delta\sigma'_z$) in a large range of soil conditions but that estimates of horizontal stress changes ($\Delta\sigma'_x$) and hence changes in mean effective stress ($\Delta p'$) are extremely sensitive to the given set of assumptions. The effect that this sensitivity has on settlements incorporating a stiffness relationship of the form given in Eqn. (2) is investigated in the following. To extend the limits of this investigation, values of $\Delta\sigma'_z$ and $\Delta\sigma'_x$ beneath the centre of a flexible footing on a cross-anisotropic linear elastic medium were calculated separately using the SAFE finite element program (Oasys 1992); these calculations assumed a ratio of the shear modulus in the vertical plane to the vertical Young's modulus of 2/3.

4.2 *Prediction technique*

In an isotropic or cross-anisotropic elastic soil, the increase in vertical strain ($\Delta\varepsilon_z$) in the soil near the centre of a footing due to an increase in pressure on the footing may be calculated from the following expression, if the stress increments of $\Delta\sigma'_z$ and $\Delta\sigma'_x$ associated with this increase in pressure are known:

$$\Delta\varepsilon_z = [\Delta\sigma'_z - (2\nu/m)\,\Delta\sigma'_x]/\,(E'_z)_t \qquad (3)$$

where 'm' is the ratio of horizontal to the vertical stiffness and ν is the appropriate average Poisson's ratio. The stress increments $\Delta\sigma'_z$ and $\Delta\sigma'_x$ also increase p' by $(\Delta\sigma'_z + 2\Delta\sigma'_x)/3$ and therefore also increase E'_t.

If increases in the applied stress increments are small, the strain in the soil after application of one of the increments is given as:

$$\varepsilon_z + \Delta\varepsilon_z = \varepsilon_z + \frac{[\Delta\sigma'_z - (2\nu/m)\,\Delta\sigma'_x]\,\varepsilon_z^{n}}{(1-n)\,E_f\,(p')^{x}\,10^{-3n}} \qquad (4)$$

where p' is the average value of the mean effective stress before and after application of the increment.

The settlement beneath the centre of a flexible footing at a given applied pressure may then be computed by integrating the vertical strains that have accumulated at this pressure. A correction factor (I_f) of 0.8 to account for the rigidity of the foundation is then applied to this settlement value to obtain an estimate of the mean footing settlement.

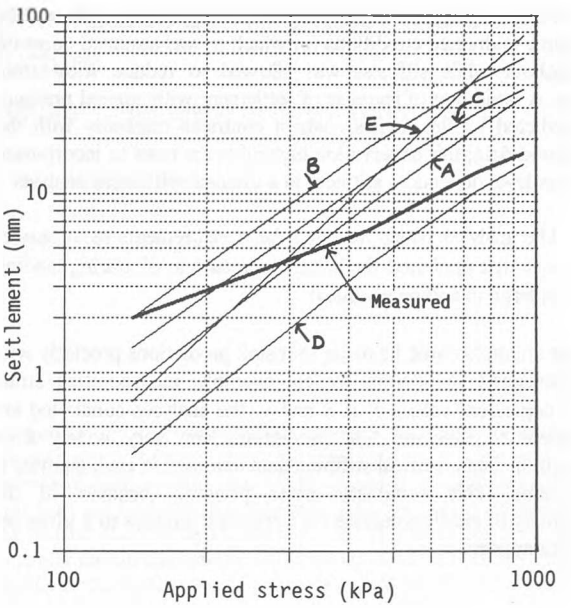


Figure 5. Predicted and measured settlements

4.3 *Predicted settlements for trial footing*

The results of a selection of predictions made using the technique described are compared on Figure 5 with the measured trial footing settlement variation with applied pressure. The stiffness parameters employed for each analysis are summarised in Table 1. All predictions assumed that the vertical stiffness of DBC was given by Eqn. (2) and that the initial mean effective stress (p'_0) varied with depth beneath the footing (z) in accordance with the equation: $p'_0 = p'_s + 20z$ (kPa). p'_s might be expected to be in the range 50kPa to 80kPa given that a bearing pressure of 90kPa had been applied to the footing prior to settlement monitoring.

Inspection of Figure 5 in conjunction with Table 1 indicates that:

(i) Despite the relatively large E'_z values employed in the analyses (by comparison with other soils; see Figure 2), predictions are in most cases larger than the measured settlements.

(ii) The differences between the settlements predicted in each analysis are appreciable and very large at high levels of applied stress. For example, input parameters for case B are identical to those of case C except for the assumed value of v. Nevertheless settlements for B are typically 50% higher than those of Case C. Such large discrepancies arise because of the sensitivity of the predictions to the computed values of horizontal stresses which, in turn, affect the values of p' and hence E'_z.

Table 1 Stiffness parameters for selected analyses

No.	$E_f(kPa)^{1-x}$	x	n	v	m	p'_s(kPa)
A	9000	0.5	0.6	0.2	2	80
B	700	1.0	0.65	0.2	1	80
C	700	1.0	0.65	0.4	1	80
D	700	1.0	0.65	0.2	2	150
E	11250 p'_0	0.0	0.6	0.5	1	80

(iii) Curve E is the pressure vs. settlement curve predicted for isotropic undrained conditions (in which p' was assumed constant throughout while stiffness was allowed to reduce with strain level). A rapid rate of increase of settlement with applied pressure is predicted by this analysis which contrasts markedly with the measured data; this observation highlights the need to incorporate a stress level dependent stiffness in a drained settlement analysis.

(iv) The analyses which matched the measurements most closely are those that predicted the largest increases in σ'_x during loading (e.g. at large m and/or ν values).

While attempts could be made to match predictions precisely with the measured settlements by, for example, incorporating strain level dependent values of n, x and ν, the analyses conducted are sufficient to illustrate that predictions can vary widely even though the basic vertical stiffness data assumed in each analysis is the same. This variability arises primarily because of the sensitivity of elastic solutions for horizontal stresses to a given set of assumptions.

5 CONCLUSIONS

1. Laboratory tests on Dublin Boulder Clay have verified the strong influence of both stress level and strain level on its stiffness characteristics.

2. The analytical procedure described for predicting settlement of shallow foundations has shown that, despite its limitations, the employment of a stress and strain level dependent modulus leads to relatively realistic variations of applied foundation pressure with drained settlement.

3. Settlement predictions, however, are too sensitive to the choice of input parameters for such a stiffness model. Even if a consistent variation of vertical Young's modulus with strain level and stress level is obtained from a set of high quality laboratory tests, predictions can vary greatly depending on the assumptions made with regard to the value of Poisson's ratio, the degree of anisotropy, the profile of initial mean effective stress and the stress history.

4. Sophisticated non-linear elastic predictions for the drained settlement of shallow foundations are unlikely to lead to more accurate predictions than those based on experience using an equivalent stress and strain level independent modulus.

REFERENCES

Burland J.B., Broms B.B. & De Mello V.F.B. (1977). Behaviour of foundations and structures. *Proc. 9th ICSMFE:* 495-538, Tokyo

Lehane B.M. & Smilie C. (1994). The stiffness of London Clay. *Internal report CE/BL(7)*, Dept. of Civil Engng., Trinity College, Ireland.

Lehane B.M., Keogh D.K. & O'Brien E.J. (1996). Soil structure interaction analysis for integral bridges. *Proc. 3rd Int. Conf. on Computational Structures Technology.* Budapest, August 1996 (in press)

OASYS (1992). Computer manuals for GEO suite. Ove Arup & Partners, London W1P 6BQ

Stroud M.A. (1988). The standard penetration test- its application and the engineering properties of glacial materials. *Proc. Conf. on Penetration Testing in the UK*, Thomas Telford publ.