Modeling elastic volume changes of unsaturated soil

Modélisation des changements de volume élastiques dans les sols non-saturés

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ABSTRACT: A new equation to describe elastic volume changes of unsaturated soil is presented for use within elasto-plastic constitutive models. The proposed form of relationship takes account of the effect on inter-particle forces of suction within bulk water (in water-filled voids) and suction within meniscus water (at particle contacts around air-filled voids). As a consequence, the new equation gives the required forms of behaviour as suction tends to zero or infinity. It is tentatively proposed that a suction-dependent parameter, which appears in the proposed equation for elastic volume changes, also describes the influence of suction on critical state shear strength. This would have the effect of simplifying existing elasto-plastic models for unsaturated soils.

RESUME: Une nouvelle équation decrivant les changements de volume élastiques dans les sols non-saturés est presentée dans le cadre des modèles constitutifs elasto-plastiques. Celle-à prend en compte l'effet des forces de succion entre particules dans l'eau libre (à l'interieur des vides remplis d'eau) et dans l'eau des menisques (aux contacts des particules autour des vides remplis d'air). Par conséquent, la nouvelle équation donne de bous résultats pour des succion variant de zéro à l'infini. On propose qu'un paramètre dépendant des succion, qui apparait dans l'equation proposées pour les changements de volume élastiques, decuit également l'influence des succion sur la force de cisaillement d'état critique. Ceci pourrait à l'avenir simplifier les modèles elasto-plastiques existants pour les sols non-saturès.

1 INTRODUCTION

The mechanical behaviour of unsaturated soil cannot be modelled in terms of a single effective stress tensor (see Jennings and Burland (1962) and Wheeler and Karube (1996)). The approach most commonly employed is therefore to use as stress state variables the net stress tensor (where net stress is the excess of total stress over pore air pressure) and the matric suction (the difference between pore air pressure and pore water pressure). This means that, for the conditions of the triaxial test ($\sigma_2 = \sigma_3$), three independent stress parameters are required: mean net stress p, deviatoric net stress q and suction s, defined as follows

$$p = \frac{\sigma_a + 2\sigma_r}{3} - u_a \quad (1)$$

$$q = (\sigma_a - u_a) - (\sigma_r - u_a) = \sigma_a - \sigma_r \quad (2)$$

$$s = u_a - u_w \quad (3)$$

where σ_a and σ_r are the axial and radial total stresses respectively and u_a and u_w are the pore air pressure and pore water pressure respectively.

An elasto-plastic critical state framework for unsaturated soil, expressed in terms of the stress state variables p, q and s, was first presented in qualitative form by Alonso, Gens and Hight (1987). Alonso, Gens and Josa (1990) developed the framework to a full mathematical model, which tends to saturated Modified Cam Clay at zero suction. Modifications to the model were subsequently proposed by Wheeler and Sivakumar (1995), in the light of experimental data from controlled-suction triaxial tests on compacted kaolin, and other authors have suggested further modifications. The models assume elastic (reversible) behaviour if the stress state remains inside a yield surface defined in q:p:s space, with plastic (irreversible) volumetric and shear strains commencing once the yield surface is reached.

Alonso, Gens and Josa (1990) suggested the following expression for the elastic variation of specific volume v

$$dv^e = \frac{-\kappa dp}{p} - \frac{\kappa_s\, ds}{s + p_{at}} \quad (4)$$

where κ and κ_s are two elastic indices and p_{at} is atmospheric pressure. Equation 4 predicts elastic compression during an increase of p (loading) or an increase of s (drying), and elastic swelling during a decrease of p (unloading) or a decrease of s (wetting). The collapse compression that is sometimes observed on wetting is not included in Equation 4, because this is associated with plastic behaviour (see Alonso, Gens and Hight (1987)). Isotropic elasticity is assumed, and therefore Equation 4 includes no influence of deviator stress q.

The form of Equation 4 was selected by Alonso, Gens and Josa (1990) as a simple first approximation, given the limited experimental data available at the time. The inclusion of atmospheric pressure in the denominator of the last term allowed the expression to be used down to zero suction, but was essentially arbitrary.

The form of elastic behaviour predicted by Equation 4 is unsatisfactory in several respects. In particular, it is not entirely consistent with the widely accepted explanation of the physical mechanism responsible for elastic strains in soils. It is also unable to represent certain features observed in laboratory tests. Before describing these shortcomings more explicitly, it is helpful to consider the physical mechanisms responsible for elastic volume changes in unsaturated soils.

2 MECHANICS OF ELASTIC VOLUME CHANGES

Within a particulate material, such as soil, elastic strains at a continuum level are generally considered to be caused at a particle level by elastic deformation of individual soil particles or aggregations of particles, caused by a change in the external forces acting on each particle or aggregation (in particular, the inter-particle or inter-aggregate forces transmitted through contact points). In contrast, plastic strains at a continuum level are caused at a particle level by slippage or rolling at inter-particle or inter-aggregate contacts, or by yielding and damage of individual particles or aggregations.

In considering, for an unsaturated soil, the influence of suction on the inter-particle contact forces, and hence on the occurrence of elastic strains, it is necessary to consider the manner in which water is distributed within the voids. As a soil is progressively dried from a saturated state, individual voids successively empty of water, with the largest voids generally emptying first (more correctly, it is the voids with the widest entry routes, corresponding to the lowest air-entry values of suction). Small lenses of water remain, however, around the inter-particle contacts surrounding each void that has emptied of water. Neglecting any adsorbed water (which is tightly bound to the soil particles and acts as part of the soil skeleton), the free water within the soil voids therefore takes two different forms: bulk water within those voids that are still water-filled, and meniscus water at inter-particle contacts around air-filled voids (see Wheeler and Karube (1996)). At high degrees of saturation (corresponding to low values of suction), the majority of soil particles are surrounded by bulk water, whereas at low degrees of saturation (corresponding to high values of suction) most particles are acted upon by meniscus water.

The suction within bulk water acts on the soil particles in the same fashion as the pore water pressure in a saturated soil, except, of course, that this suction now acts on only those particles that are adjacent to water-filled voids. An increase in suction within the bulk water therefore results in an increase in the inter-particle forces between those particles affected by bulk water (just as would be produced by reduction of pore water pressure in a saturated soil). This produces elastic deformation of the affected particles and hence elastic compression of the soil at a continuum level. At zero suction, with the soil in a saturated state and bulk water filling all the voids, a change of suction s should have the same effect as a change of mean net stress p of the same magnitude, because both contribute equally to the mean effective stress p' in a saturated soil.

The effect of suction within meniscus water is to produce an additional component of inter-particle contact force at those contacts where meniscus water exists. For idealized spherical particles this additional component of inter-particle force acts normal to the plane of contact and an analytical expression for the variation of the force with suction is given by Fisher (1926). The additional force is non-zero at zero suction (which corresponds to a condition where the two principal radii of curvature of the meniscus are of the same magnitude, but opposite sign). The additional force increases monotonically with increasing suction, and tends to a limiting value as suction tends to infinity (because the infinite suction acts over an infinitesimally small area of meniscus). The increase in inter-particle normal force produced by an increase of the suction within meniscus water will result in elastic deformation of the soil particles affected by meniscus water and hence elastic compression of the soil at a continuum level. There will be a limiting value of elastic volumetric strain as suction tends to infinity, because each inter-particle force tends to a limiting value.

3 SHORTCOMINGS OF EXISTING EQUATION

Four shortcomings can now be identified in the existing expression for elastic variation of v given by Equation 4.

(a) The inclusion of atmospheric pressure in the final term of Equation 4 is completely arbitrary.

(b) At zero suction, when the soil is implicitly assumed to be saturated, a small change of suction should have the same effect as a change of mean net stress of the same magnitude, and the equation for elastic changes of v should therefore become

$$dv^e = \frac{-\kappa(dp + ds)}{p} \tag{5}$$

This is not the case with Equation 4.

(c) As suction tends to infinity, the elastic change of v should tend to a limiting value, whereas Equation 4 predicts that elastic compression will continue indefinitely with increasing suction.

(d) Equation 4 predicts that the elastic compressibility with respect to changes of p is constant, at a value κ, irrespective of the magnitude of suction, whereas experimental data from Vicol (1990) and Al-Mukhtar, et al. (1993) suggest that the compressibility decreases with increasing suction.

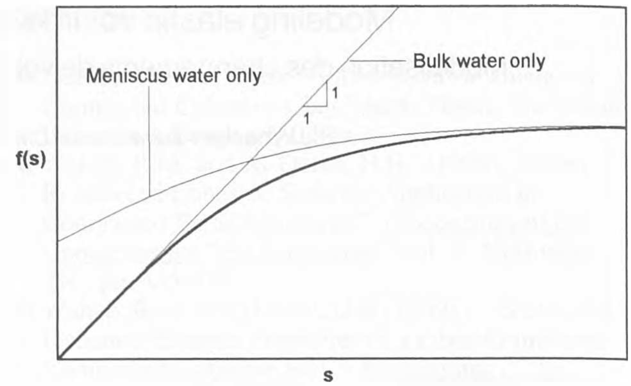

Fig. 1 Variation of f(s) with suction

4 PROPOSED EQUATION

According to the Modified Cam Clay model, the expression for elastic volume changes in a saturated soil is

$$dv^e = \frac{-\kappa dp'}{p'} \tag{6}$$

Given that elastic volume changes are caused by elastic deformations of individual particles and aggregations, which are themselves caused by changes in inter-particle forces, the corresponding equation for an unsaturated soil should be

$$dv^e = \frac{-\kappa d(p + f(s))}{p + f(s)} = \frac{-\kappa(dp + \frac{df(s)}{ds}ds)}{p + f(s)} \tag{7}$$

where the function $f(s)$ represents the contribution of suction in increasing inter-particle forces and hence causing elastic compression.

The form of $f(s)$ must take account of the gradual transition from a preponderance of bulk water at low suctions to a preponderance of meniscus water at high suctions. If all the voids were filled with bulk water, $f(s)$ would be given by the simple relationship $f(s) = s$ (see Fig. 1). With all the water in the form of meniscus water $f(s)$ would have a non-zero value at zero suction and would increase monotonically to a limiting value as suction tended to infinity. In practice, therefore, the overall form of $f(s)$ is likely to be as shown in Fig. 1, with a gradual transition from the bulk water line to the meniscus water curve as suction increases.

It is not possible to derive a rigorous theoretical expression for $f(s)$, because complexities such as non-spherical particles, a distribution of particle sizes and uncertainty in the packing arrangement make it impossible to analyse the case of a soil containing only meniscus water, before even considering the additional complexities of the gradual transition from bulk water to meniscus water. It is, however, possible to set down four conditions that should be satisfied by the function $f(s)$

$$f(s) = 0 \quad \text{at} \quad s = 0 \tag{8}$$

$$\frac{df(s)}{ds} = 1 \quad \text{at} \quad s = 0 \tag{9}$$

$$\frac{df(s)}{ds} \geq 0 \quad \text{for} \quad 0 \leq s < \infty \tag{10}$$

$$f(s) = \alpha \quad \text{at} \quad s = \infty \tag{11}$$

where α is a finite positive number. Condition 11 requires that $df(s)/ds = 0$ as suction tends to infinity. The idealized analysis of Fisher (1926) shows that the limiting value α, reached as suction tends to infinity, is inversely proportional to some representative particle size. Accurate theoretical prediction of α is, however, impossible for a real soil, because of the complexities of particle shapes, sizes and packing arrangements.

In Equations 10 and 11 it has been assumed that, as suction tends to infinity, the lenses of meniscus water surrounding each inter-particle contact become infinitesimally small but do not disappear completely. If individual inter-particle contacts were to dry out completely, *f(s)* would be expected to fall at very high suctions, with *f(s)* tending to zero for completely dry conditions. Shear strength data presented by Escario and Juca (1989) provide tentative evidence that this may occur. At this stage, however, the evidence is inconclusive. It is also likely that any fall in the value of *f(s)* would occur only at extremely high values of suction. Equations 10 and 11 are therefore considered appropriate for most situations of practical interest.

Two relatively simple expressions that fit the requirements of Equation 8 to 11 are

$$f(s) = \alpha \tanh\left(\frac{s}{\alpha}\right) \quad (12)$$

$$f(s) = \frac{2\alpha}{\pi}\tan^{-1}\left(\frac{\pi s}{2\alpha}\right) \quad (13)$$

Each of these functions involves only a single constant: the limiting value α. The function in Equation 12 tends more rapidly to the limiting value than the function in Equation 13, as shown in Fig. 2. In practice, however, a smooth analytical function, such as Equation 12 or Equation 13, may not provide a good fit to *f(s)* for a real soil. For example, a soil with a bi-modal pore size distribution, typical of many compacted fills, would be expected to show two marked discontinuities in the slope of *f(s)*, at suctions corresponding to the air entry values of the two dominant void sizes.

All four shortcomings identified in the existing expression of Equation 4 have been avoided in the proposed expression of Equation 7. Firstly, the form of Equation 7 follows naturally from a consideration of the physical processes involved and, as a result, does not require the inclusion of an arbitrary reference stress such as p_{at}. Secondly, provided that *f(s)* fulfils the conditions of Equations 8 and 9, the desired result of Equation 5 is achieved at zero suction. Thirdly, provided that *f(s)* fulfils Equation 11, the magnitude of elastic compression induced by drying reaches a limiting value, even if suction is increased indefinitely. Finally, inspection of Equation 7 shows that the elastic compressibilty with respect to changes of *p* is predicted to decrease with increasing suction. Elastic unloading-reloading lines of *v* plotted against the logarithm of *p* are now predicted to be curved, with a local slope κ_{eq} given by

$$\kappa_{eq} = \frac{p\kappa}{p + f(s)} \quad (14)$$

For unloading-reloading tests carried out over a specified range of *p*, the average compressibility is predicted to decrease with increasing suction in a manner that is qualitatively consistent with the experimental results reported by Vicol (1990).

5 POSSIBLE LINK TO SHEAR STRENGTH

The function *f(s)* in Equation 7 represents the contribution of suction to the inter-particle normal forces which cause elastic compression. It is therefore possible that other aspects of unsaturated soil behaviour which depend upon inter-particle normal forces (such as shear strength) can also be related to the stress parameter *p+f(s)*. This does not mean that all aspects of behaviour can be related to *p+f(s)*. For example, yielding of the soil depends upon the existence of tangential as well as normal

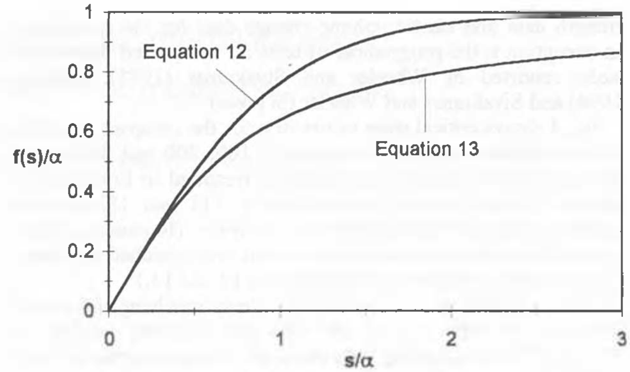

Fig. 2 Possible analytical functions for *f(s)*

forces at the inter-particle contacts. As a consequence, correct representation of the yield behaviour requires the inclusion of a further stress variable (such as suction) in addition to *p+f(s)* (see Wheeler and Karube (1996)). There is no suggestion, therefore, that *p+f(s)* could be termed an "effective stress" for unsaturated soil, in the sense defined by Terzaghi (1936).

If *p+f(s)* is the stress parameter controlling shear strength of unsaturated soil, the appropriate critical state relationship for deviator stress *q* is

$$q = M(p+f(s)) \quad (15)$$

where *M* is the value of critical state stress ratio for the soil in a saturated state.

Equation 15 can be compared with the conventional shear strength expression for unsaturated soil proposed by Fredlund, Morgenstern and Widger (1978)

$$\tau = c' + (\sigma - u_a)\tan\phi' + (u_a - u_w)\tan\phi^b \quad (16)$$

According to Equation 15, at a critical state the cohesion c' in Equation 16 will be zero (as expected), ϕ' will be a constant (related, in the normal fashion, to *M*) and ϕ^b will vary with suction (because *f(s)* is a non-linear function of suction). In particular, Equation 15 implies that ϕ^b will be equal to ϕ' at zero suction, because of the condition of Equation 9, and the tangent value of ϕ^b will decrease to zero as suction tends to infinity, because of the condition of Equation 11. This is exactly the form of shear strength variation observed in experimental tests reported by Escario and Saez (1986) and Gan, Fredlund and Rahardjo (1988).

The non-linear increase of shear strength with suction described by Equation 15 is certainly more realistic than the linear variation proposed in the model of Alonso, Gens and Josa (1990). At the other extreme, Wheeler and Sivakumar (1995) suggested that the parameter M (and hence ϕ') could also be a function of suction, but more recent experimental evidence (Sivakumar and Wheeler (in press)) suggests that this degree of flexibility is unnecessary.

Although experimental results suggest that Equation 15 will normally be an appropriate form of expression relating *q*, *p* and *s* at critical states, it remains to be seen whether the function *f(s)* in Equation 15 is the same function as used in Equation 7 to represent elastic volume changes. If this were the case, it would have the obvious advantage of reducing the number of independent soil constants required in the elasto-plastic model. This would, in turn, simplify the programme of laboratory tests required to determine the values of the model parameters for a given unsaturated soil.

6 EXPERIMENTAL EVIDENCE

Unfortunately, it is rare to find experimental results from controlled-suction tests on unsaturated samples which show shear strength data and elastic volume change data for the same soil. An exception is the programme of tests on compacted Speswhite kaolin reported by Wheeler and Sivakumar (1995), Zakaria (1994) and Sivakumar and Wheeler (in press).

Fig. 3 shows critical state values of q for the compacted kaolin for four different values of suction (0, 100, 200 and 300 kPa). The best-fit lines shown in the figure correspond to Equation 15 with M = 0.863 and $f(s)$ values of 63, 111 and 153 kPa at suctions of 100, 200 and 300 kPa respectively. (In passing, it can be noted that these values of $f(s)$ are not well-matched by either of the analytical expressions in Equations 12 and 13.)

Zakaria (1994) reports results of 5 tests involving unloading from p = 400 kPa to p = 200 kPa at a constant suction of 100 kPa. These unloading tests show an average increase of v of 0.016. This corresponds to a κ value of 0.028, if the elastic swelling behaviour is governed by Equation 7 and the value of $f(s)$ is 63 kPa at a suction of 100 kPa (as suggested by the critical state shear strength data).

It is now possible to examine whether the variation of $f(s)$ established from the critical state shear strength data and the value of κ calculated from the elastic unloading data can be used to predict correctly the elastic swelling observed in wetting tests. Wheeler and Sivakumar (1995) report results from four samples wetted at p = 50 kPa from a high initial value of suction to a final suction of 300 kPa. Eleven further samples, with the same history, were wetted at p = 50 kPa to a lower final suction of 200 kPa. According to Equation 7 (with κ = 0.028 and $f(s)$ values of 111 and 153 kPa at suctions of 200 and 300 kPa respectively), the predicted elastic increase of v on wetting from s = 300 kPa to s = 200 kPa at p = 50 kPa is 0.0065. This compares well with the measured value of about 0.006 for the average additional increase in v for those samples brought to s = 200 kPa rather than s = 300 kPa.

These experimental data for Speswhite kaolin provide an encouraging first indication that it may be possible to use the same function $f(s)$ in Equation 7 (describing elastic volume changes) and in Equation 15 (representing the shear strength at a critical state). Clearly, however, confirmation of this suggestion requires a substantially larger body of supporting experimental evidence.

7 CONCLUSIONS

A new equation to describe elastic volume changes of unsaturated soil has been presented for use within elasto-plastic constitutive models. The proposed form of relationship, given in Equation 7, is based on the assumption that elastic volume changes at a continuum level are caused at a particle level by elastic deformation of individual soil particles due to a change in the inter-particle forces acting at particle contacts. These inter-particle forces are influenced by the suction within bulk water (in water-filled voids) and by the suction within meniscus water (at particle contacts around air-filled voids).

As the proposed equation is based on consideration of the physical processes involved, the need to include an arbitrary reference pressure, such as p_{at}, is avoided. In addition, the equation gives the required forms of limiting behaviour as suction tends to zero or infinity. Finally, the equation correctly predicts that the compressibility with respect to changes of p decreases with increasing suction.

It is possible that the function $f(s)$ (which appears in the proposed equation for elastic volume changes) also describes the influence of suction on critical state shear strength, as shown in Equation 15. This suggestion requires further experimental validation, but if it were true it would have the effect of reducing the number of independent soil parameters required in the elasto-plastic models for unsaturated soils. The most robust and straightforward laboratory testing strategy, to measure the values of the soil parameters controlling elastic volume changes, would therefore be to perform an isotropic unload-reload sequence under saturated conditions, to measure κ, and then controlled-suction shear tests (in a triaxial cell or direct shear apparatus) to measure the variation of $f(s)$ with suction.

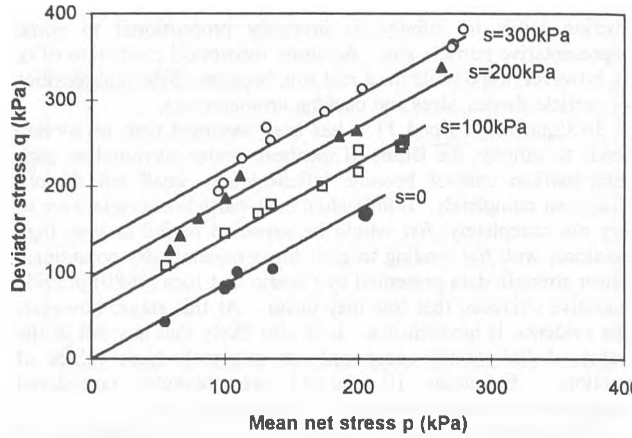

Fig. 3 Critical state data for compacted Speswhite kaolin

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