A rotational hardening elasto-plastic model for clays

Un modèle élasto-plastique à consolidation rotative pour argiles

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ABSTRACT: An elasto-plastic model incorporating mixed isotropic and rotational hardening is presented. The model, which is intended to represent the effects of development and erasure of fabric anisotropy during plastic straining, is based on Modified Cam Clay, with two additional soil parameters to describe the rotational component of hardening. An advantage over alternative rotational hardening models is that a unique critical state line is retained in q:p':v space. Model predictions are shown to be qualitatively consistent with experimental observations.

RESUME: Le modèle prèsenté est un modèle élasto-plastique prenant en compte les modes de consolidation isotropiques et rotatifs. Il est basé sur le modèle Cam-Clay et se doit de représenter les effets de l'augmentation et de la disparition de l'anisotropie présente dans l'état du sol lorsque celui-ci endure des déformations plastiques. Deux paramètres ont été ajoutés pour décrire la composante rotative de consolidation. L'avantage par rapport à d'autres modèles de consolidation rotative est qu'une unique ligne d'état critique est conservée dans l'espace q:p':v. Les prédictions de ce modèle se sont avérées être qualitativement en accord avec les résultats expérimentaux.

1 INTRODUCTION

The earliest elasto-plastic critical state constitutive models for clays, including Modified Cam Clay (Roscoe and Burland (1968)), incorporated a yield curve that was symmetrical about the mean effective stress axis. Subsequent experimental studies showed, however, that yield curves for many natural clays are inclined in the q:p' plane, with one axis of the curve corresponding approximately with the K_o line (see, for example, Graham, Noonan and Lew (1983)). The inclination of the yield curve is an indication of fabric anisotropy arising from the previous one-dimensional consolidation.

Several authors, including Mouratidis and Magnan (1983), presented elasto-plastic models incorporating an inclined yield surface, where the inclination was assumed to remain constant for a given soil. In reality, however, the inclination of the yield surface should change progressively with development or erasure of fabric anisotropy during plastic straining.

Authors such as Banerjee, Stipho and Yousif (1985) and Davies and Newson (1993) proposed elasto-plastic models incorporating a rotational component of hardening. In these models the rotational hardening attempts to drag the yield surface round until the axis of the surface aligns with the current stress point. A weakness of many such models is that they do not predict a unique critical state line in the v:p' plane, because the final inclination of the yield surface is dependent on the path taken to the critical state. This is in conflict with experimental observations and with the fundamental expectation that the location of the critical state line should be independent of any previous development of fabric anisotropy, because a critical state corresponds to a condition where fabric is being continuously destroyed.

The development of a more satisfactory rotational hardening elasto-plastic model has received relatively little attention of late. This is perhaps because the focus of many researchers has moved on from classical single yield surface models to models incorporating some form of plastic behaviour inside the main "yield surface". In practice, however, classical single yield surface elasto-plastic models, such as Modified Cam Clay, are still very widely used for numerical predictions. These models may give highly inaccurate predictions of ground behaviour if the inclination of the yield surface is ignored or incorrectly modelled (see, for example, Bowey (1996)). It would therefore be of great practical benefit if a simple and fully consistent rotational hardening model could be developed.

2 ROTATIONAL HARDENING AND CRITICAL STATE

If there is to be a unique critical state line in q:p':v space, the final orientation of the yield curve on arriving at a critical state must be independent of the route to the critical state. This includes stress paths which arrive at a critical state without the occurrence of plastic volumetric strains. It must therefore be the occurrence of intense plastic shear strains that brings the yield curve to its critical state orientation (consistent with the expectation that shearing at a critical state involves continuous destruction of fabric and hence erasure of any previously established anisotropy).

Smart (1996) reports experimental evidence, from computerized image analysis of clay particle orientations observed on optical micrographs, of the development and erasure of fabric anisotropy during shearing of natural clay samples. During shearing the degree of fabric anisotropy initially increases from its starting value but the degree of anisotropy then decreases to a final critical state value as shearing continues. This critical state level of fabric anisotropy is relatively low and is independent of the initial degree of anisotropy at the start of shearing.

3 PROPOSED ELASTO-PLASTIC MODEL

The model is presented for the simplified conditions of a triaxial compression test ($\sigma'_2 = \sigma'_3$). The proposed yield curve is in the form of a sheared ellipse, defined by

$$(q - \alpha p')^2 - (M^2 - \alpha^2)(p'_m - p')p' = 0 \qquad (1)$$

where q is the deviator stress, p' is the mean effective stress, M is the critical state value of stress ratio η (where $\eta = q/p'$) and p'_m and α define the size and inclination respectively of the yield curve (see Fig. 1). With $\alpha = 0$, Equation 1 corresponds to the Modified Cam Clay yield curve. The proposed yield curve has vertical tangents at the origin and at point A, where $p' = p'_m$ and $q = \alpha p'_m$ (see Fig. 1). The tangent to the yield curve is horizontal at B, the point of intersection with the critical state line ($q = Mp'$).

For simplicity, an associated flow rule is proposed

$$\frac{d\varepsilon_s^p}{d\varepsilon_v^p} = \frac{2(\eta - \alpha)}{M^2 - \eta^2} \qquad (2)$$

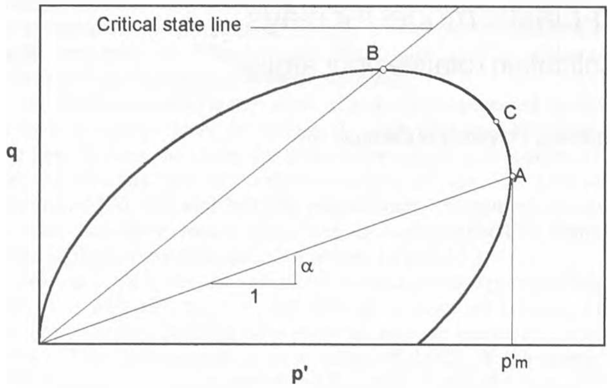

Figure 1. Inclined yield curve

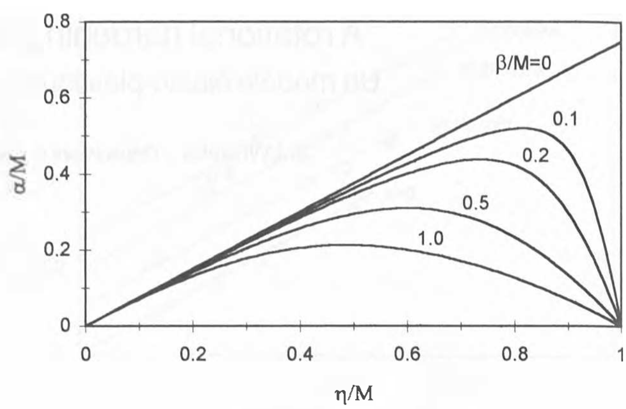

Figure 2. Equilibrium values of α / M for radial stress paths.

Change of the parameter p_m' defining the size of the yield curve, is assumed to be related solely to the plastic volumetric strain

$$dp_m' = \frac{v p_m' \, d\varepsilon_v^p}{\lambda - \kappa} \tag{3}$$

The hardening law for change of inclination of the yield curve assumes that plastic volumetric strains, which dominate during anisotropic compression, have the effect of aligning the yield curve about the current stress point, whereas plastic shear strains, which dominate as a critical state is approached, have the effect of rotating the yield curve back towards an isotropic orientation. For the former, it is important to note that the point on the yield curve that would be visually identified as defining the axis of the curve lies above the vertical tangent point A (see Fig. 1). For simplicity, it can be taken as point C, where $\eta = 4\alpha/3$. The proposed rotational hardening law is therefore

$$d\alpha = \mu\left[\left(\frac{3\eta}{4} - \alpha\right)d\varepsilon_v^p - \beta\alpha\left|d\varepsilon_s^p\right|\right] \tag{4}$$

where μ and β are two new soil constants. β controls the relative effectiveness of plastic shear strains and plastic volumetric strains in dragging α towards the respective target values of zero and $3\eta/4$, whereas μ controls the absolute rate at which α heads asymptotically towards its current target value. Note that the sign of plastic shear strain increment is assumed to be immaterial in erasing fabric anisotropy and hence dragging α towards zero.

For stress paths remaining inside the yield curve, the elastic behaviour is likely, in practice, to be anisotropic. At this stage, however, isotropic elastic behaviour is assumed

$$d\varepsilon_v^e = \frac{\kappa dp'}{vp'} \quad , \quad d\varepsilon_s^e = \frac{dq}{3G'} \tag{5}$$

A final soil constant Γ is required to define the position of the critical state line in the $v{:}p'$ plane. A total of 7 soil parameters are therefore required: 5 from the Modified Cam Clay model ($\lambda, \kappa, \Gamma, M$ and G) and two additional parameters (μ and β) related to rotational hardening. The current state of a soil sample is completely defined by the values of q, p', p_m' and α.

4 MODEL PREDICTIONS

4.1 *Radial stress paths*

For any radial stress path (η =constant), α will tend towards an equilibrium value. Setting $d\alpha = 0$ in Equation 4 (noting that plastic shear strain increments must be positive when an equilibrium value of α is reached under conditions of triaxial compression) and then combining with the flow rule of Equation 2

$$\left(\frac{3\eta}{4} - \alpha\right)\left(M^2 - \eta^2\right) = 2\beta\alpha(\eta - \alpha) \tag{6}$$

This quadratic equation can be solved to give the equilibrium value of α corresponding to any combination of η, M and β.

Equilibrium values of α/M from Equation 6 are plotted in Fig. 2. For a soil with a given value of β/M, the equilibrium value of α increases from zero at $\eta = 0$ to a maximum α_{max} at some intermediate value of η and then decreases again to zero at $\eta = M$. At low values of η/M, α is approximately $3\eta/4$, and the yield curve aligns about the radial stress path.

When yielding is occurring at a particular value of η/M, even on a non-radial stress path, the value of α/M shown in Fig. 2 represents a current target value which α/M is instantaneously heading towards. This means that, for a soil with a given value of β/M, the inclination of the yield curve can never exceed the value α_{max} corresponding to the peak of the relevant curve in Fig. 2. This suggests that realistic values of β are likely to be about $0.2M$ or less, because with higher values of β it would be impossible to achieve the highly inclined yield curves reported for natural clays.

4.2 *One-dimensional compression*

If elastic strains are much smaller than plastic strains then one-dimensional compression corresponds to

$$\frac{d\varepsilon_s^p}{d\varepsilon_v^p} = \frac{2}{3} \tag{7}$$

Combining this with Equations 2 and 6 gives a quadratic equation for the value of η corresponding to K_o compression.

Fig. 3 shows K_o values of η/M plotted against β for three typical values of M. As β tends to infinity, the yield curve inclination α tends to zero and the predicted K_o value of η is the same as that given by Modified Cam Clay, which is considerably lower than is observed experimentally. As β tends to zero, α tends to $3\eta/4$ and the predicted value of η is approximately double the corresponding Modified Cam Clay prediction. These higher values of η are much closer to experimental observations, supporting the earlier suggestion that realistic values of β are likely to be relatively low (about 0.2 or less).

4.3 *Isotropic compression*

Fig. 4a shows the variation of specific volume of a sample compressed under an isotropic stress state, having been previously compressed one-dimensionally to $\sigma_v' = 100\,\text{kPa}$ and then unloaded elastically to an isotropic stress state inside the yield curve. Results are shown for the proposed rotational hardening model and for conventional Modified Cam Clay. The following representative values have been selected for the various soil constants : $\lambda = 0.16$, $\kappa = 0.04$, $\Gamma = 2.800$, M = 1.0, G = 10 MPa, $\beta = 0.2$ and $\mu = 30$. These values are also used for the subsequent examples in Sections 4.4 and 4.5.

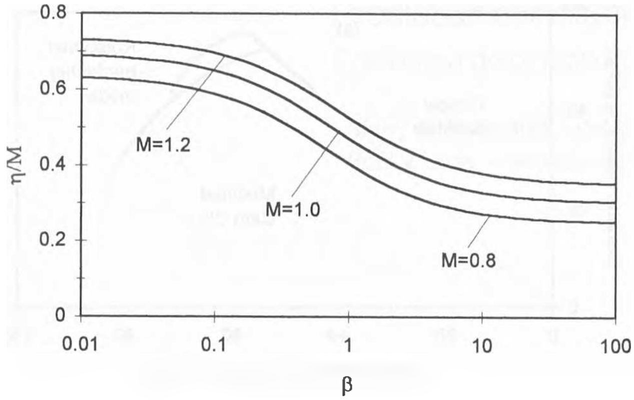

Figure 3. Values of η / M during one-dimensional compression

According to the Modified Cam Clay model, yield during isotropic compression occurs at $p' = 90.8$ kPa (see Fig. 4a) and the compression curve then follows the true isotropic normal compression line defined by

$$v = \Gamma + (\lambda - \kappa)\ln 2 - \lambda \ln p' \quad (8)$$

With the proposed rotational hardening model, one-dimensional compression to $\sigma'_v = 100$ kPa expands the yield curve to a size and inclination defined by $p'_m = 74.6$ kPa, $\alpha = 0.406$. The predicted value of v following elastic unloading is therefore significantly higher than for the Modified Cam Clay model. On subsequent isotropic loading, yield occurs at $p' = 62.3$ kPa (much lower than the yield stress predicted by Modified Cam Clay). The yield value of v lies significantly below the true isotropic normal compression line defined by Equation 8 (see Fig. 4a). As isotropic loading continues the yield curve expands and rotates clockwise towards an isotropic orientation. The compression curve therefore tends asymptotically towards the true isotropic normal compression line.

The rate at which the compression curve tends towards the true isotropic normal compression line is largely dependent on the value of the parameter μ. With $\mu = 30$, the compression curve shown in Fig. 4a is virtually indistinguishable from the true isotropic normal compression line by $p' = 200$ kPa (an isotropic stress twice the magnitude of the original one-dimensional stress). More detailed examination of the individual axial and radial strains (see Fig. 4b) shows, however, that the predicted behaviour is still significantly anisotropic at this point. At $p' = 200$ kPa the rate of axial straining is only 61% of the rate of radial straining. This ratio increases to 88% by p' = 400 kPa and 97% by $p' = 800$ kPa. These results, which are fairly sensitive to the value of μ, are consistent with experimental measurements of axial and radial strains measured directly on samples of compacted kaolin by Zakaria (1994). This suggests that $\mu = 30$ is a reasonable approximation for this material.

Inspection of Fig. 4b shows that a negative rate of axial straining is predicted immediately following yield. This is a consequence of the associated flow rule and a highly inclined yield curve. There is some experimental evidence from Zakaria (1994) that this negative axial straining under isotropic loading can occur.

The proposed rotational hardening model represents the effects of fabric anisotropy induced by plastic straining, but does not represent the effects of aging or inter-particle bonding which are important in many natural clays. It is these additional effects that produce compression curves for natural samples that lie above the intrinsic compression curve for the reconstituted material in the v:p' plane (see Burland (1990)). In contrast, the suggestion here is that fabric anisotropy induced by previous plastic straining has the effect of producing compression curves that lie below the true isotropic normal compression line (see Fig. 4a). This form of behaviour is reported by Sivakumar and Wheeler (in press) for samples of kaolin compacted one-dimensionally to different stress levels and then subsequently compressed isotropically.

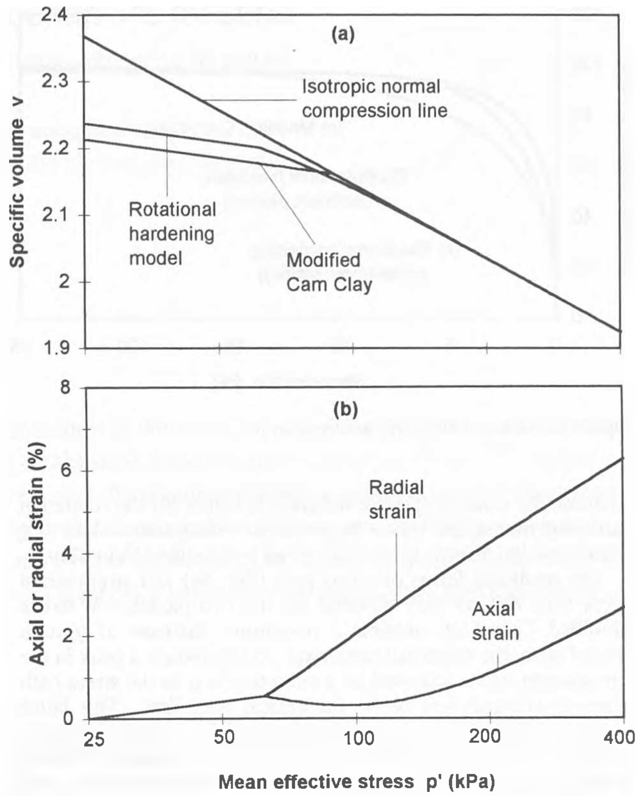

Figure 4. Isotropic compression following K_o compression

4.4 *Drained shearing*

Fig. 5 shows predicted results of drained shearing of a normally consolidated sample at constant $p' = 100$ kPa from an initial isotropic stress state. Three different predictions are shown:

(a) the rotational hardening model for a sample with an anisotropic stress history consisting of one-dimensional compression to $\sigma'_v = 100$ kPa followed by isotropic compression to $p' = 100$ kPa;

(b) the rotational hardening model for a sample with an isotropic stress history;

(c) the Modified Cam Clay model.

The inclined initial orientation of the yield curve for case (a) means that the first section of the stress path during shearing lies inside the yield curve, and the stress-strain response is therefore stiff and elastic up to a yield point at $q = 42.5$ kPa (see Fig. 5). In contrast, the other two predictions show plastic strains commencing from the start of shearing. As plastic straining progresses the rotational hardening model (cases (a) and (b)) shows higher shear stiffnesses than the Modified Cam Clay model throughout most of the shearing process. Towards the end of shearing, however, the rotational hardening model requires very high shear strains if a critical state is to be reached. This is because there is a substantial final section of the test path where the stress point is very close to the critical state line in the q:p' plane but large rotations and expansions of the yield curve (accompanied by large plastic strains) are still required to bring the yield curve to its final critical state orientation and size.

All three model predictions shown in Fig. 5 finally approach the same critical state values of q and v, but this requires shear strains of the order of 100% for the rotational hardening model.

4.5 *Undrained shearing*

Fig. 6 shows predicted results of undrained shearing of a sample previously prepared to a normally consolidated state by one-dimensional consolidation to $\sigma'_v = 100$ kPa. The initial values of q and p' for the rotational hardening model are different to those for Modified Cam Clay (see Fig. 6a), because of the different values of η during one-dimensional consolidation. In

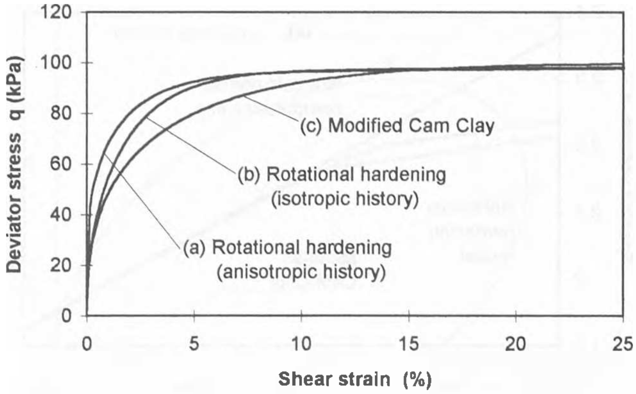

Figure 5. Drained shearing at constant p'

addition, the value of specific volume v is higher for the rotational hardening model, and hence the predicted critical state values of q and p' are significantly lower than given by Modified Cam Clay.

The predicted forms of stress path (Fig. 6a) and stress-strain curve (Fig. 6b) are very different for the two models. Whereas Modified Cam Clay predicts a monotonic increase of q to a critical state, the rotational hardening model predicts a peak in the stress-strain curve, followed by a reduction in q as the stress path slides downwards just below the critical state line. This latter form of behaviour is commonly observed in undrained triaxial compression tests on normally consolidated clay samples with an anisotropic stress history (see, for example, Whittle (1993)), and the failure of Modified Cam Clay to model this is one of its major limitations. It is interesting to note that during the post-peak reduction of deviator stress predicted by the rotational hardening model the yield curve is still expanding in size (in this sense, strain hardening is still occurring), but this is more than offset by the accompanying clockwise rotation of the yield curve.

Inspection of Fig. 6b shows that, once again, very large shear strains are required before a critical state is finally achieved with the rotational hardening model. The magnitude of shear strains required to approach a critical state during drained or undrained shearing would be reduced by employing a higher value of the parameter β. This would also have the effect of smoothing the sharp corner in the undrained stress path as the critical state line is first approached in the $q:p'$ plane (see Fig. 6a).

5 CONCLUSIONS

A simple elasto-plastic model incorporating mixed isotropic and rotational hardening has been developed. The model is intended to represent the effects of development and erasure of fabric anisotropy during plastic straining. An advantage over alternative rotational hardening models is that a unique critical state line is retained in $q:p':v$ space. Model predictions are shown to be qualitatively consistent with experimental observations, but detailed validation, and perhaps refinement, is still required. The model may not be suitable for stress paths involving yielding on the dry side of critical state.

The proposed model is based on Modified Cam Clay, with two additional soil parameters, μ and β, to describe the rotational component of hardening. A disadvantage of the model (as with other models employing rotational hardening) is that experimental measurement of the values of μ and β would typically be rather indirect. For example, the value of μ might be selected by matching the rate of erasure of anisotropy apparent from measurements of axial and radial strain during isotropic compression following one-dimensional compression. In the absence of suitable experimental data, values of 30 and 0.2 can be considered as typical for μ and β respectively. The latter is based on a compromise between the low value of β required for reasonable modelling of η values and yield curve inclinations during one-dimensional compression and the high value of β necessary to limit the magnitudes of shear strains required to approach a critical state.

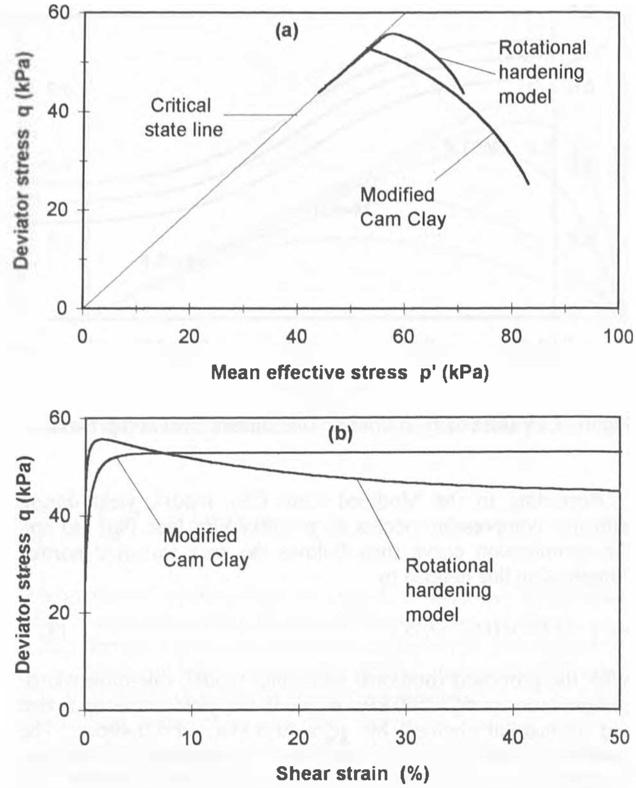

Figure 6. Undrained shearing of K_o consolidated sample

Further development of the model would include generalization to stress states other than those of the triaxial compression test and incorporation of an anisotropic form of elasticity (with the degree of elastic anisotropy linked to the level of fabric anisotropy via the inclination α of the yield curve).

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