# A dual model for unidimensional consolidation

## Un modèle dualiste pour la consolidation unidimensionnelle

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SYNOPSIS: "Total Consolidations of Soils", (Primary and Secondary) may be stisfactorily explained together through a "dual soil" or "soil in soil" model, constituted by a primary clay matrix with invaded zones of less permeable coloidal soil, resulting different type consolidation curves for laboratory sample and prototype. Besides, the model helps explain the existance of a recharge and a virgin portions of the e-log p curve.

INTRODUCTION. During the First International Conference of ISSSMFE, some results of long duration settlement tests were presented, which showed a final settlement, apparently proportional to long t, of secular character: the secondary consolidation (Buisman, 1936) which, according to Buisman, must have a limit sometime.

Since 1939 (Merchant, 1939), several authors tried to explain this phenomenon through the presence of a hipothetical "structural viscosity", with only one exception to this author's knowledge a rather complex theory with nonfamiliar parameters (de Josselin de Jong, 1968) of limited Secondary Consolidation (the only acceptable type, in this author's opinion), based in a former suggestion of Buisman, showing that statistical distribution of permeabilities (a model of channels and cavities) could also explain some aspects of Secondary Consolidation, at laboratory scale.

Notwithstanding, subsequent theories followed the former tendency: "structural viscosity".

THE NEW MODEL: In an attempt to capture the very essence of the de Josselins de Jong's paper, applying more familiar Soil Mechanics parameters and concepts, the present author presents a simple model (dual soil or soil in soil model) consistent of a primary matrix invaded by coloidal soil tablets (Figure 1) toghether with some simplifying hypothesis, namely:

1.- The primary matrix (consolidation coefficient: $c_{v2}$) consolidates vertically according to Terzaghi's theory as if secondary tablets were not present.

2.- Tablets consolidate vertically according to Terzaghi's differential equation (consolidation coefficcient: $c_{v2}$), under variable charge (the surrounding primary matrix effective pressure).

Under this conditions, if T is the Time Factor of the primary matrix and the Time Factor of the tablets is expresed as $\alpha_H$ times the former ($\alpha_H$ scales up according with the $H^2$ rule), mean Tablets Consolidation Degree may be estimated (Figueroa-Vega, 1996) through time convultion and z averaging of Terzaghi's classical solution (Terzaghi,1925), as:

$$U_s(\alpha_H, T) = (1/2H\int_0^{2H} (\partial U(T')/\partial T')(U_H\ (\alpha T\ \ \alpha T'))\ dT'dz$$

and, after some forward calculations:

$$U_s(\alpha_H, T) = U(T) - 1/(1-\alpha_H)[U(\alpha_H T)] \qquad (2$$

where U(T) is the undimensional consolidation solution of Terzaghi (vertical consolidation, double drainage), which numerical value may be approximated by (Hansen, 1961):

$$U(T) = [T^3/(T^3 + 0.5)]^{1/6} \qquad (3$$

with negligible error.

Be $\rho$ the vertical fraction of primary matrix, $(1-\rho)$ the corresponding fraction of tablets, $m_{v1}$ and $m_{v2}$ their coefficients of volume decrease and $M_1 = \rho\ m_{v1}$ and $M_2 = (1-\rho)m_{v2}$ their relative compressibiilties, then, the total layer consolidation may be expressed (Figueroa-Vega 1996) as:

$$\Delta H = \Delta P\ H\ [M_1\ U_s(\alpha_H,\ T) \qquad (4$$

The $U_s\ (\alpha_H,\ T)$ family of curves is shown in Figure 2, including the Terzaghi U(T), which corresponds to the $\alpha_H = \infty$ case.

Expression (4) generates practically all the tipe curves presented in primary-secondary consolidation of soils literature (Lo, 1961), and the upper slope of the secondary portion provides the supposedly constant $C_\alpha$ slope of Mesri, showing the rest of the curve its non constancy through time, its non conservatism within the

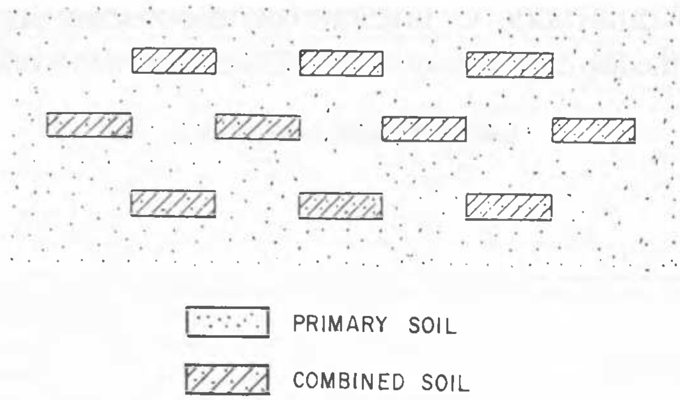


FIGURE 1 SOIL IN SOIL SQUEME

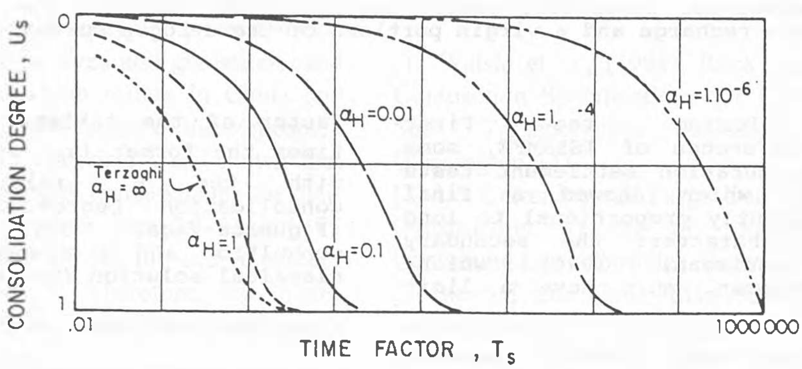


FIGURE 2 $U_S$ FAMILY OF CURVES

expected lifetime of works and the limited nature of the consolidation of soils.

When expression (4) is applied to a laboratory sample (calibration phase), $\alpha_H$ must be scaled up before building the predictive Field Consolidation Type Curve which, generally, will differ from the laboratory one.

APPLICATION EXAMPLE. In 1984, a light weight test embankment (1.2 ton/m$^2$, 140m X 280 m) was built over the 53 meters of very soft clays of the Texcoco Lake area as an aid in the design of a new platform for Aiport pavements (one to one scale) and observed during 2,866 days.

A typical laboratory consolidation curve which covers approximately the expected range of effective pressure at 14.3 m depth, is shown in Figure 3.

Through calibration, the obtained parameters were:

$$\alpha_H = 0.02; \; r = M_1/(M_1 + M_2) = 0.2$$

where r is the primary consolidation, expressed as a fraction of the total one. This way, additionally

$M_1 = 0.022566$ cm$^2$/kg; $M_2 = 0.10264$ cm$^2$/kg; $C_{vt} = 29.54$ m$^2$/yr

where $C_{vt}$ is a bulk consolidation coefficient, different of the classical laboratory $c_v$.

By scaling T and $\alpha_H$ and building the corresponding field type curve, predictions where made (a posteriori, Figueroa-Vega, 1996), with the following results:

T A B L E 1

| TIME<br>days | OBSERVED SUBSIDENCE<br>cm | COMPUTED SUBSIDENCE<br>cm |
|---|---|---|
| 191 | 9.5 | 13.9 |
| 608 | 21.3 | 24.5 |
| 973 | 32.9 | 29.4 |
| 2866 | 53.2 | 52.0 |
| FINAL | ----- | 81.6 |

FINAL COMMENT. The above proposed model helps explain, as a by-product, another important aspect of soil behaviour: the existence of a recharge on a virgin portions of the e-log p curve.

In fact, for applied pressures below the preconsolidation pressure $p_c$, the tablets surround the same number of grains of the invaded primary matrix. For applied pressures above $p_c$, which represents an extrusion pressure level, ,the colloidal soil is extruded towards the surrounding voids of the primary matrix,, with the corresponding increase of compressibility. This extrusion is irreversible before new pressure reductions.

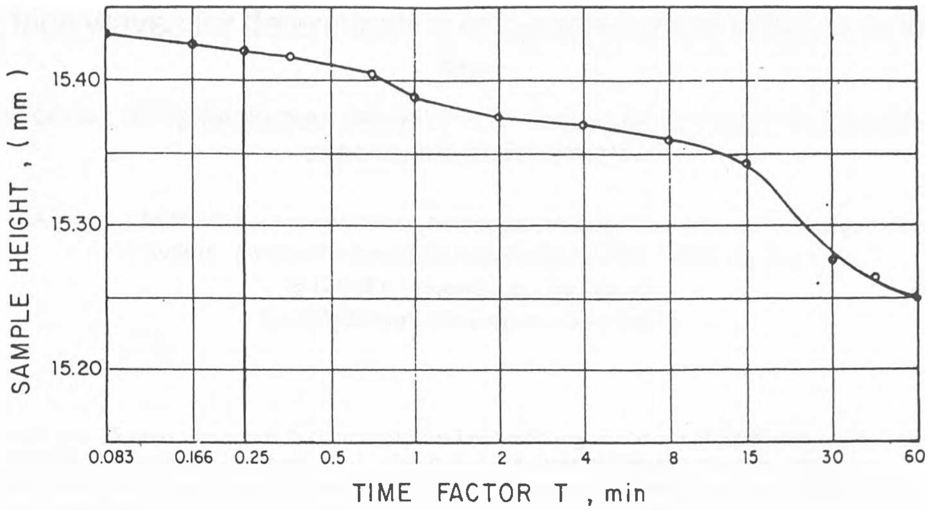


FIGURE 3 LABORATORY CURVE

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