Calculation of slotted foundations in spatial stress-strain state of soil base

Calcul des fondements fentes dans les conditions tridimensionnelles de l'état tensio-deformé du sol

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ABSTRACT: The necessity for investigating contact interaction of bases with rigid slotted foundations in spatial formulation is substantiated. The procedure for numerical solution of spatial contact problem for a slotted foundation on the basis of the direct boundary element method has been devised. Taking into account the foundation shape, the method proposed permits determining the foundation displacements and stresses on the contact surface as well as investigating the stress-strain state in its active zone. The questions of optimizing the shape and the loading parameters of slotted foundations are considered.

RESUME: On considère et on argumente la nécessité des recherches des interactions des contacts des fondations avec la dureté des fondements fissurés dans l'espace. On propose la méthode des calculs de la résolution numérique du problème des contacts dans l'espace pour le fondement sur la base de la méthode directe des éléments limités. La proposition de l'approche des recherches permet, en considérant les formes du fondement, de définir ses déplacements, la tension de la surface des contacts aussi bien que de rechercher l'état tensio-deformé dans sa zone active. On considère des problèmes de l'optimisation des formes et des paramètres de charge des fondements fentes.

1 INTRODUCTION

Experience of designing and building industrial and civil engineering projects in Russia shows (Sorochan, 1986) that in cohesive and slightly wet soils of natural origin (eluvial as a rule), the use of monolithic shallow foundation with a lateral work surface is justified. The notable feature of such foundations is the efficient technology of their construction that excludes backfilling. It permits one to achieve the lateral friction along walls which is impossible to obtain in conventional construction foundations in open excavations. Compared to various conventional constructions (pile, strip, columnar) of monolithic shallow foundations with lateral work surface, of most efficiency are the slotted foundations used both for industrial and civil engineering (Pavlov 1992). It should be said that slotted foundations are finding increasing use in the erection of low dwelling houses (cottages) and light industrial buildings (of a frame type) (Sorochan et al.1991, Bykov 1995) in light of modern requirements to ecological foundation engineering, lowering construction costs, high bearing capacity and simplification of production operations.

2 SLOTTED FOUNDATIONS OF VARIOUS CONSTRUCTIONAL SHAPES

Slotted foundations represent the variety of constructional shapes of foundations being constructed by the "wall-in-soil" method (Pavlov 1992). They present a narrow concrete or reinforced concrete wall in the soil base (Figure 1). The load is transmitted to the foundation by lateral planes and wall. Therefore the geometric shapes of the slot and the lateral surface are defined according to the sizes of superstructures, their contour in plan, the value and the direction of loads, geological-engineering conditions, and the freezing depth or soil weathering. In recent years using the experience of foundation construction by the "wall-in-soil" method (Smorodinov & Fyodorov 1986), Russian scientists and engineers have proposed and worked out the new designs and technologies of constructing slotted foundations (Pavlov 1992), which are profitably employed in industrial and civil structures such as foundations with cavities, two-slotted ones with a low foundation grill, wedged-slotted foundations with monolithic plates, slotted foundations with a broadened toe and some others.

Slotted foundations have the advantages of both shallow foundations and those of pile ones. In literature (Pavlov 1992, Sorochan et al.1991) it is shown that constructional shapes of slotted foundations and the method of their construction permit one to eliminate a number of disadvantages of conventional foundation types. In particular, during heir construction there is no need for digging deep trenches under the entire building, the volume of sheeting work being either missed or considerably reduced. In addition, such foundations may be constructed at a different depth without overexpenditure of materials. Thus, the application of slotted foundation in building some industrial and civil projects for different soil conditions has shown (Pavlov 1992) that as compared with foundations constructed in an excavation or made of driven piles, the former permits reduction of the earth work volume by 40-50% (and at some constructional shapes by 70-80%), concrete consumption by 5-6%, armature consumption by 15-20%, and the volume of sheeting work by 70-100%.

Despite the efficiency of foundation constructions under consideration, they are not widely applied in building. One of the main reasons for this is a low reliability of the calculation methods used in practice which are based on individual empirical data and

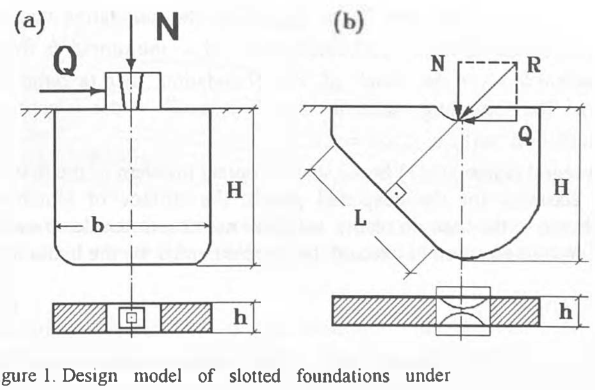

Figure 1. Design model of slotted foundations under (a) reinforced column and (b) three-hinged frame

the simplest design models (Pavlov 1992, Recommendation 1982). Up to now there are no methods for calculating slotted foundations which would take into account their constructional features, friction resistance on the developed lateral surface and various combinations of effective forces and moments. The simplified Winkler model is used. As a rule slotted foundations transmit to the base a spatial force system presented in general case by vertical, horizontal and momentary loads. In this case, according to the Russian standards of designing, stress-strain state of slotted foundation base should be investigated separately: on a vertical load using the model of linearly deformable half-space, and on horizontal and momentary loads using the Winkler model. The dependence of the foundation design model and the base model on the direction of an external loads effect is also a reason for a low reliability of the calculation results for determining displacements and inclinations of slotted foundations constructions on the operation stage.

Thus, though there is an effective and sufficiently elaborated technique of construction under different soil conditions, the methods of calculating settlements and inclinations of slotted foundations need further refinements on the basis of theoretical study, taking into account the spatial formulations and the detailed analysis of the processes of the contact foundation interaction with soil.

3 SPATIAL CONTACT PROBLEM FOR SLOTTED FOUNDATIONS

Slotted foundations are characterized by a great rigidity, so we accept the design model in the form of a volumetric deepened punch. The soil base is considered as a linearly deformable half-space weakened by a cavity, the boundaries of which are identical to the contact surface of the foundation construction being calculated. The integral equations of the spatial contact problem for the deepened punch of a given shape may be presented as follows (Aleynikov & Ikonin 1989):

$$\left|\begin{aligned} &\Delta_i = \iint\limits_{\Gamma} p_j(N)\,U_{ij}(K,N)\,d\Gamma + \xi_k\,\psi_j\,\varepsilon_{ijk}\,; \\ &\mathrm{P}_i = \iint\limits_{\Gamma} p_j(N)\,\delta_{ij}\,d\Gamma; \quad \mathrm{M}_i = \iint\limits_{\Gamma} p_j(N)\,\zeta_k\,\varepsilon_{ijk}\,d\Gamma, \end{aligned}\right. \tag{1}$$

where Δ_i - the foundation displacement in the cut level along the i - coordinate axis; ψ_j - the angle of the foundation rotation around the j - coordinate axis; $p_j(N)$ - the projection of the contact stress vector at a point N onto the j - coordinate axis; $U_{ij}(K,N)$ - the Mindlin's fundamental solution for half-space (Mindlin 1936) giving the displacement at a point $N(\zeta_k)$ of the half-space in the j - direction from a single concentrated force applied at a point $K(\xi_k)$ of the half-space along the i - direction; δ_{ij} - the Kroneker's symbol; ε_{ijk} - the Levi-Civita's tensor; P_i - the external forces applied to the foundation face in the direction of the i - coordinate axis; M_i - the moments from external loads in the level of the foundation face tending to turn the structure around the i - axis; Γ - the contact foundation surface; $i,j,k=\overline{1,3}$

Integral equations (1) fit the spatial contact problem of the theory of elasticity for the deepened punch, the surface of which is adhered to the base. In reality, soil does not absorb tensile stresses. Therefore equation (1) should be supplemented by the limitation

$$p^{(n)}(N) \geq 0\ , \tag{2}$$

where $p^{(n)}(N)$ - the projection of the contact stress vector on the outer normal to the contact surface at a point N

Equation system (1) with limitation (2) leads to a range of contact problems with the nonlinearity of a structural type (Aleynikov et al. 1993).

4 NUMERICAL ALGORITHM OF SPATIAL CONTACT PROBLEM SOLUTION BY THE BOUNDARY ELEMENTS METHOD

Up to the present time the exact solutions of the formulated spatial contact problem are unknown. Because of this, we solve it numerically using the boundary element method (Benerjee & Butterfield 1981) simultaneously with piecewise-constant approximation of the contact stress function. As a result of the algebraization of integral equation system (1) and inequalities (2) we obtain the following solving system of linear algebraic equations:

$$\left|\begin{aligned} &\Delta_i = \sum_{t=1}^{c} p_j(N_t)\iint\limits_{\Gamma_t} U_{ij}(K_f,N_t)\,d\Gamma + \xi_k\,\psi_j\,\varepsilon_{ijk}\,; \\ &\mathrm{P}_i = \sum_{t=1}^{c} p_j(N_t)\iint\limits_{\Gamma_t} d\Gamma\,\delta_{ij}\ ; \quad \mathrm{M}_i = \sum_{t=1}^{c} p_j(N_t)\iint\limits_{\Gamma_t} \zeta_k\,d\Gamma\,\varepsilon_{ijk}\ ; \\ &p^{(n)}(N_t) \geq 0, \quad i,j,k=\overline{1,3}, \quad f=1\ldots c\ , \end{aligned}\right. \tag{3}$$

where c -the number of boundary elements at the discretization of the foundation contact surface; $p_j(N_t)$ -the averaged values of the contact stresses in the direction of the j -coordinate axis within the limits of the t - boundary element; $N_t(\zeta_k) \in \Gamma_t$; Γ_t -the surface of the t- boundary element; $K_f(\xi_k)$ -the collocation points coinciding with the gravity center of the boundary elements.

According to the algorithm developed the surface of the slotted foundation contact with soil is sequentially subjected to discretization; matrix coefficients with unknowns are calculated; equation system (3) is formulated and solved for different values of external force effects. The study of the algebraic systems obtained has shown their good conditionality associated with the diagonal dominance that permits the use of conventional solution methods of the Gauss type without resorting to the regularization methods. Discretization of the contact surface is performed automatically with the use of flat boundary elements of a triangular or quadrangular shape (Figure 2). In order to increase the efficiency of the computing algorithm on quick-operation and exactness, we apply a specially developed numeric-and-analytic technique for calculating surface integrals, both singular and regular, according to the triangular and quadrangular boundary elements arbitrarily oriented in space (Aleynikov 1995).

At a perfect contact of the slotted foundation with the base, system (3) represents a closed system ($3c+6$) of the linear algebraic equations. It there is no preliminary information about the contact domain, the solution of this system is found iteratively.

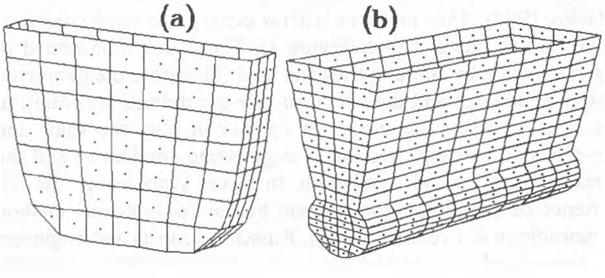

Figure 2. A general view of slotted foundations with the mesh of boundary elements on the contact surface of interaction with soil:
(a) columnar; (b) wedged-slotted foundation with broadening

Values $p^{(n)}(N_t)$ are analyzed at each step in order to define the boundary elements on whose surface tensile normal stresses arise. Rows and columns of the matrix of equation system (3) fitting such elements are zeroed on the subsequent iterations. Iterations are completed when condition (2) is satisfied for all the boundary elements. After solving the boundary problem by the predetermined values Δ_i and $p_j(N_t)$, it is easy to calculate the stress-strain state of the ground mass enclosing the slotted foundation. For this purpose it is sufficient to perform a direct integration on the basis of the stress functions obtained from solving the contact problem.

Test calculations and substantiation of the computing efficiency of the approach proposed have been performed on the examples of the well-known spatial contact problems of deepened punches in the axisymmetrical formulation (Shishov 1971).

5 CODE FOR CALCULATING DISPLACEMENTS OF SLOTTED FOUNDATIONS AND STRESS-STRAIN STATE IN GROUND MASS

The outlined technique of solving the spatial contact problem for the slotted foundations with a variously shaped lateral surface has been implemented in the computer system on FORTRAN-77. Discretization of the surface foundation contact with the base has been brought to 500 boundary elements. For the sake of the software compactness and for performing effective calculations, the Mindin fundamental solution is presented in a compact form (Aleynikov 1995). The amounts of time for calculating one variant on PC IBM/AT-486 is about 10-15 minutes.

The contact problem being solved, the calculation of the stress-strain state in the active zone of the slotted foundation is performed. For constructing the contour levels of the fields of stresses and displacements in any plane section, a specially elaborated program of forming the triangular mesh is used taking into account the concentration of stresses on the contact surface of the foundation and soil.

The detailed analysis of the calculated relationships characterizing the influence of the type of loading and sizes of the slotted foundations on their displacements and stress distribution in soil and on the contact surface of the slotted foundations is carried out in an interactive regime with a wide application of modern computer graphics. So the optimization of the shape and loading parameters is conveniently performed by the use of the software developed for slotted foundations. The software proposed also permits one to consider the problems of improving the geometric shape of slotted foundations in order to widen the fields of their application for various geological-engineering conditions as well as for effectively perceiving the loads of different directions. Calculations have been done for new effective kinds of monolithic slotted foundations in the form of vertical carriers with cross-shaped, rectangular doubled, tee, box-like and some other types of cross-sections (Pavlov 1992, Sorochan et al.1991). Some typical examples of the results of computational modelling are given in the next section.

6 ILLUSTRATIVE EXAMPLES OF COMPUTATIONAL MODELLING

The application package developed allows us to perform computational experiments for estimating the influence of different factors on the displacements of slotted foundations and the distribution of the contact stresses along the lateral and front surfaces and the toe. It is possible primarily to estimate the influence of the geometric foundation shape on its displacements and the stress-strain state of the base. Slotted foundations with variously shaped lateral surfaces have been considered as examples (Figure 3). At an equal average pressure on the foundation, the influence of the contour curvature of the longitudinal section on the parameters of the contact interaction of the foundation with the soil base at power and momentary loadings has been estimated for each coordinate direction. In case of a curviliniar longitudinal section the shape of the lateral surface has been given in accordance with

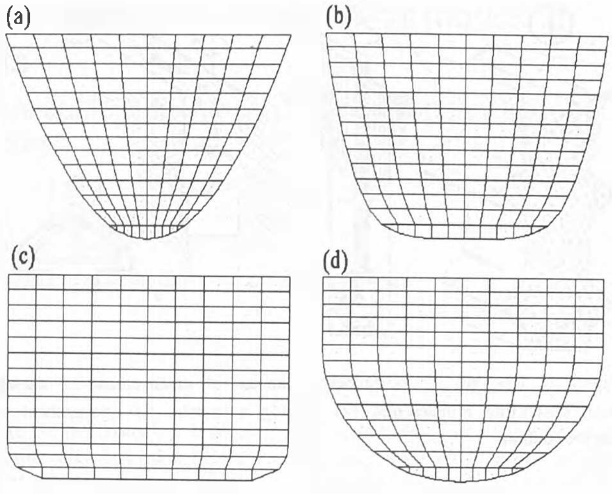

Figure 3. Discretization of lateral surface of slotted foundations with different longitudinal sections: (a) curvilinear, α = 2; (b) curvilinear, α = 5; (c) rectangular ; (d) rectangular with curvilinear shape of toe

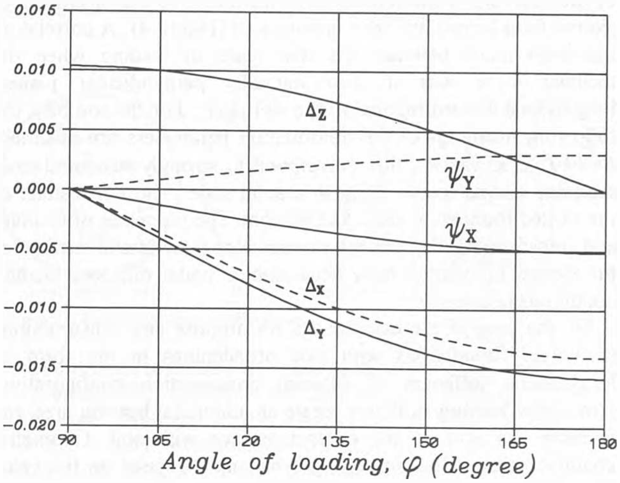

Figure 4. Displacements Δ , m and inclinations ψ (in radians) of slotted foundation at longitudinal (continuous lines) and transverse (dotted lines) loadings by inclined force

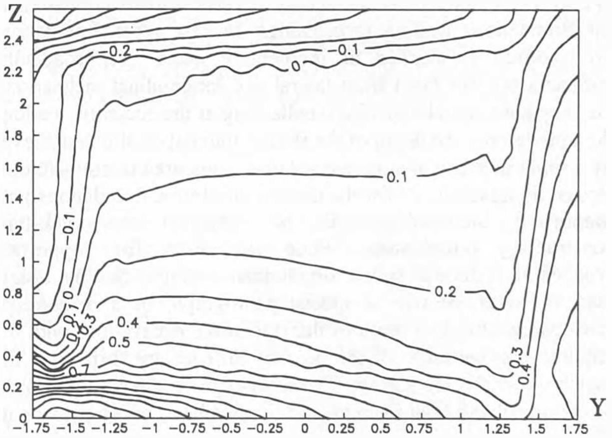

Figure 5. Contact normal stresses σ/σ_{av} on lateral surface of slotted foundation at the action of inclined load R=1000 kN in longitudinal direction (φ = 135^0)

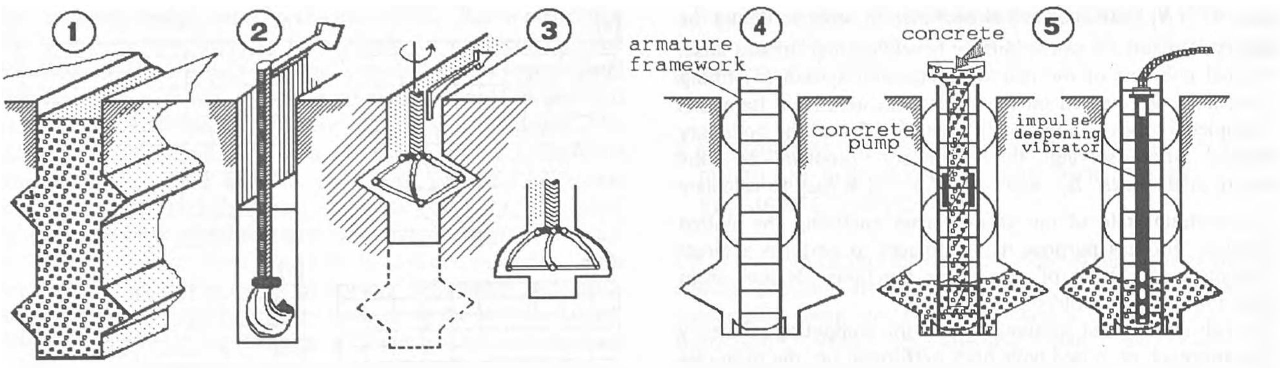

Figure 6. Diagram of technological process of arrangement of slotted foundation with supporting broadenings: (1) ready-made slotted foundation with supporting broadenings; (2) digging a trench; (3) arrangement of supporting broadening; (4) reinforcing by volume frameworks; (5) concreting the foundation

the power law $z(y) = H[1-(2|y|/b)^{\alpha}]$, where H is the foundation depth, b - is its breadth. In this case the increase of the α parameter leads to approaching the longitudinal shape of the section to the quadrangular kind. The influence of the slope angle of the resultant external load on inclinations and displacements of slotted foundations has been investigated (Figure 4). A correlation has been made between the two types of loading when the inclined force acts in two mutually perpendicular planes: longitudinal and orthogonal to the slot plane. For the soil base the following meanings of the deformation parameters are assumed: $E=2\cdot10^4$ kPa; $\nu=0.4$, that correspond to strongly structural low-moisture eluvial loams being in a solid state. The dimensions of the slotted foundation are 3.5x2.6x0.6m. The processes of forming and transforming the contact stresses along the lateral surface of the slotted foundation have been studied under different loading conditions (Figure 5).

On the basis of the calculations we propose new constructions of slotted foundations with side broadenings in the form of longitudinal stiffeners of different cross-section configuration. Horizontal bearing stiffeners create an additional bearing area and increase the area of the contact surface with soil. Geometric characteristics of the supporting broadening depend on the value and the direction of the external load applied to the foundation as well as on the physicomechanical properties of the soil. The availability of the supporting broadenings in the construction of slotted foundations, being the concentrators of stresses in the foundation active zone, permits one to decrease substantially (12%) the settlements at the action of vertical loads compared to the foundations without broadenings. In case when foundations are exposed to inclined or momentary loads, the broadening stiffeners prevent them from lateral and longitudinal inclinations, i.e. they keep foundations from collapsing at the eccentric loading. At a predetermined depth of the slotted foundation this is achieved by a slight increase of concrete volumes compared to conventional slotted foundations. As for the design, the slotted foundations with supporting broadenings can be classified as quick-type constructing foundations. When excavation for supporting broadening is done in soil, a longitudinal cutting is dug by a basic machine equipped with a special pantograph, or a petal-shape broadening unit. As a result of this operation, excavations with the required configuration of the bearing stiffener are formed in the trench under the foundation. The rotor-type broadening unit must be equipped with high strength cutting members, which provide its efficient work in a ground mass with regard to the speed of its translational displacement. In cases when the depth of the foundation placing in soil is more than 3.5 m, it is possible to repeat the machine cutting in order to form the second broadening stiffener through the foundation (Figure 6). Additional operations for profiling the cuts practically neither increase the overall labour consumption nor complicate the working cycle nor slow down the rate of work.

The analysis of the calculations conducted show that the settlements and inclinations of slotted foundations with side broadenings are greatly influenced by such factors as relative depth of the position of the bearing stiffener and its shape and sizes in the cross section. Changing these parameters makes it possible to effectively regulate the main calculated "load-settlement" characteristics of foundations.

7 CONCLUSIONS

The developed calculation algorithm and the software for PC of an average power permits us to perform effectively computational experiments with the well-known constructions of slotted foundations at various types of loading and deformation properties of the soil base. The proposed calculation technique of slotted foundations is rather universal, and it is based on the spatial design model. At present it is most adequate (among all the known techniques) for the actual processes of the foundation interaction with soil and permits us to provide a substantial reduction of terms and costs of designing with the advantage of eliminating labour consuming and expensive full-scale tests.

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