On stability of three-dimensional isolated piles in anisotropic swelling soils

Sur la stabilité de pieux isolés tridimensionellement dans les sols anisotropes dilatables

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ABSTRACT: Depending on the kind of end effects there may appear in a pile three-dimensional stressed deformed state, not plane one. In this case one should take into account pile torsion. Thus, it is evident that to solve pile stability problems a geometrically non-linear thory of three-dimentional bars must be used. In considering contact of the pile under study with soil an allowance for anisotrogy of the soil is required. If in the plane case coefficient of rigidity in the horizontal plane is an important indicator of the soil in lateral bending of the pile it is necessary to take into account torsional rigidiry and also that in the vertical direction when studying the three-dimensional state. These two coefficients are essential in studying the swelling soil as an experiment shows that rigid cohesion is peculiar to the kind of soil under investigation. For solving the stated problem of non-linear equilibrium of a three-dimentional bar situated in the anisotropic soil it is expedient to employ of variation principles, the Reisner's prinsiple in particular. On the basis of this principle a system of determining equations is obtained in this research. Initially the variation principle for computing a three-dimensional elastic body in contact with anisotropic elastic medium is presented.

RESUME: Selon le type d'efforts de bout e etat general de contrainte et de deformation peut apparaaitre dans le pieu, pas e etat de contrainte et de deformation plan. En ce cas il est necessaire tenir compte de la torsion du pieu. Done, il est evident que pour la resolution de problemes de la stabilite du pieu il est indispensable employer une theorie non-lineaire geometriquement de barres tridimensionnels. En tenant compte de contact du pieu etant e'objet d'etude avee le sol il faut prendre en consideration e'anisotropic du sol. Si en cas d'etat plan le coefficient de rigidite dans le plan horizontale est un indice impotant du sol en flexion transversale du pieu il est necessaire tenir compte de rigidite en torsion et de rigidite dans le sens verticale aussi en etudiant e'etat general Ces deux coefficients sont essentiels en etudiant les sols dilatables car e'epreuve montre que la cohesion rigide est propre au type de sol en condideration. Pour la resolution de probleme pose d'equilibre non-lineare de la barre etant en sol anisotrope il est rationnel employer un des prineipes de variation, le principe de Reisner en particulier Sur la base de ce principe un systeme d'equations determinates est obtenu dans cette recherche. Le principe de variation pour le calcul du corps e'lastique tridimensionnel en contact avec le milieu elastique anisotrope est presente initiallement.

The distinctive feature of the swelling soils is that tensile stresses appear in them in the process of swelling. These stresses lead not only to the bulding of the soil surface but also to the occurrence of buoyant (lifting) forces. For example, if isolated piles in the swelling soil are studied one sees that in the process of swelling the lateral surface of the pile begins to be vertically affected by upward forces caused by friction between the pile and the soil (as a result of adnesion) and lift of the soil mass (as a result of swelling). The value of this force depends on a lot of factors: location of the source of swelling, physico-chemical composition of soil etc. (a variety of the factors is not studied sufficiently as yet). Among these factors we shall single out the relationship between the force of swelling and the value of compressive stress in a point of the soil under study. More-over, it is assumed that the force leading to the deformation of the pile is proportional to the difference of vectors of movement of the soil's points at the pile $\vec{W}$ and points of the contact surface of thepile and the soil $\vec{W}_k$ movement of the points of the pile as on absolutely solid body is neglected).

In this case the horizontal constituent of the force of swelling can be ignored, i.e. the horizontal force appears as the force of reaction.

From the above description of the swelling soil it follows that it is transversally isotropic medium. For the sake of generality of computations it is assumed that the soil is anisotropic. This assumption requires for restrictions on the performance of structures functioning in these media. In particular, when studying piles functioning in ordinary soil the plane of flexure is determined by parameters of the pile's section as the surrounding medium is isotropic. In the case under study the plane of flexure will depend not only on the structure itself but also on the direction of the force of swelling. From this it ensues that for the computation of the pile one must not confine himself to a model of simple bending (i.e., simple bar) but it is necessary of study a three-dimensional model (i.e., three-dimensional bar).

Because of the complexity of the pile's model it is expedient to obtain determining equations on the basis of the variational principle. As a starting principle we shall take the Reisner's variational principle of the elasticity theary in "velocities" with consideration for geometrical non-linearity. Initially we shall write a functional for the computation of a three-dimensional body contracting with soil. We shall divide all the surface of the body into the three intersecting surfaces S_u, S_r, S_c. On the surface S_u the movements $\bar{U}_i$ are specified, i.e.

$$U_i = \bar{U}_i \qquad x \in S_u \tag{1}$$

On the surface S_σ the forces $\bar{T}^i$ are specified, i.e.

$$T^k \equiv \sigma^{ij} n_j (\delta_i^k + U_{,i}^k) = \bar{T}^i \qquad x \in S_\sigma \tag{2}$$

where σ^{ij}- components of Piole's stress tensor of the second order; n_j - components of normal vector in non-deformed coordinate system (further we shall take all the values in non-deformed system); δ_i^k- Kronecher's symbols; U^k - movement vector; comma means covariant differentiation

The surface S_c is contact one - between the soil and the body. On this surface the contact forces of the type $\vec{T}_c$ are specified which are determined by the following equality

$$\vec{T}_c = G(\vec{W}-\vec{U}) \text{ or } T_c^i = G^{ij}(W_j - U_j) \qquad x \in S_c \tag{3}$$

where G - matrix of soil resistance; $\vec{W}$ - lift of swelling depending on stressed state in points of S_c.

The representation (3) allows to pass to Winkler model as in the general case ($\vec{W}=0$) so along the direction $f(\vec{W}_f=0)$. In some cases it is assumed that $\vec{T}_c=G\vec{W}$, i.e. the contact force depends on the value of the lift of swelling . In a sense the representation (3) is general because the matrix G may degenerate into a scalar, have diagonal elements distinet from each other and from.

In the general case the contact forces are determined from the solution of contact problem of the structure and the soil. That is why this force may depend on the movement $\vec{W}$, i.e.

$$\vec{T}_c=\vec{S}(\vec{W}) \quad \text{or} \quad T_c^i=S^i(W_k) \tag{4}$$

The swelling function S^i depends on some combination of the swelling vector W and it must be specified (this function is specified on the basis of the expected pattern of contact for a particular problem). Let us suppose that the body under study is elastic, i.e. $\varepsilon_{ij}=C_{ijke}\,\sigma^{ke}$; where C_{ijke} - elasticity tensor; ε_{ij} - deformation determined through the movement U_i within geometrically theory by the following equality

$$\varepsilon_{ij}=\frac{1}{2}(U_{i,j}+U_{j,i}+U_{,i}^{k}\,U_{k,j}) \tag{5}$$

The Reisner's functional for the computation of the stressed deformed state in the described body has the form:

$$J = {}_V\!\int\{\dot{\sigma}^{ij}\frac{1}{2}(U_{i,j}+U_{j,i}+U_{,i}^{k}\,U_{k,j})^{\bullet}-\frac{1}{2}C_{ijke}\dot{\sigma}^{ij}\dot{\sigma}^{ke}+\frac{1}{2}\sigma^{ij}\,\dot{U}_{,i}^{k}\,\dot{U}_{k,j}-\dot{F}^{i}\dot{U}_i\}dV-$$
$$-{}_{S\sigma}\!\int\dot{\bar{T}}^{i}\dot{U}_i dS-{}_{Su}\!\int\dot{T}^{i}(\dot{U}_i-\dot{\bar{U}}_i)dS-{}_{Sk}\!\int\frac{1}{2}G^{ij}(\dot{W}_j-\dot{U}_{j.})(\dot{W}_i-\dot{U}_i)dS+$$
$$+{}_{Sk}\!\int\frac{1}{2}\frac{\partial S^{i}}{\partial W_k}\dot{W}_k\,\dot{W}_i\,dS \tag{6}$$

By a point is implied differentiation with respect to some parameter q characterizing the process of loading. $\dot{U}_i$, $\dot{G}^{ij}$, $\dot{W}_i$ are independent varied quantities. On the basis of the variation condition it is evident that Eiler's equations of the functional (6) are:

$$\{[\sigma^{ij}(\delta_i^{\,k}+U_{,i}^{\,k})]_{,j}-F^{k}\}^{\bullet}=0;\ \{\frac{1}{2}(U_{i,j}+U_{j,i}+U_{,i}^{\,k}\,U_{k,j})-C_{ijke}\sigma^{ke}\}^{\bullet}=0 \qquad x\in V$$

$$\{(U_i-\bar{U}_i)\}^{\bullet}=0 \quad x\in S_u;\ \{[\sigma^{ij}(\delta_i^{\,k}+U_{,i}^{\,k})]n_j-\bar{T}^{k}\}^{\bullet}=0 \quad x\in S\sigma \tag{7}$$

$$\{[\sigma^{ij}(\delta_i^{\,k}+U_{,i}^{\,k})]n_j-S^{k}(W_i)\}^{\bullet}=0 \qquad x\in S_k$$

The system (7) is the system of non-linear equations of equilibrium of an elastic body contacting with soil written in differential form. If the stressed deformed state in the body when q=0 is taken as a starting condition of integration of the system (7), then after integration with respect to q the system (7) will differ, from the starting one by absence of braces. From here it follows that the stationary value of the functional (7) is obtained by solving the stated problem. The peculiarity of the system (7) consists in that it is the system of quasi-linear equations differentiated with respects to q. It is simpler to solve this problem on an electronic computer than to solve the analogous system of non-linear equations. The quantity q is selected so that the initial stressed deformed state would be trivial or would be easily determinable.

Let us employ the functional (6) for the computation of the piles. For this it is necessary to pass in (6) from integration with respect to V to that with respect to lenght of the pile, i.e. to pass from three-dimensional functional to one-dimentional one. It is supposed that the is described by S.P.Timoshenko's three-dimentional model, i.e. cross-section of the bar wich was flat before deformation remains the same after it. In this case movement of an arbitrary point of the bar has the form:

$$\widetilde{U}=U-Z\Psi_y+Y\Psi_z;\quad \widetilde{V}=V+Z\Psi_s;\quad \widetilde{W}=W-Y\Psi_s \tag{8}$$

where (y,z)- coordinates of the point of cross-section; U,V,W - movement of points of axis; Ψ_s,Ψ_y,Ψ_z - vector of rotation of the cross-section.

Then deformation of the arbitrary point with consideration only for non-linearity of deflections will become.

$$e_{ss}=\frac{dU}{dS}+\frac{1}{2}(\frac{\partial W}{\partial S})^2+\frac{1}{2}(\frac{\partial V}{\partial Z})^2+Z(-\frac{d\Psi_y}{dS})+y\frac{d\Psi_z}{dS} \tag{9}$$

$$e_{sy}=\frac{1}{2}(\Psi_z+\frac{dV}{dS}+z\frac{d\Psi_s}{dS});\ e_{sz}=\frac{1}{2}(\frac{dW}{dS}-\Psi_y-y\frac{d\Psi_s}{dS})$$

Let us substitute the relationships (8) and (9) in the functional (6) and integrate with respect to (y, z). Assuming that the body is elastic with Young's modulus E let us suppose that the axis of the bar goes through centre of symmetry of the cross-section and that contact with the soil occurs only along the lateral surface. In this case we obtain the following one-dimensional integral instead

$$J={}_0\!\int^{l}\{\dot{N}[\frac{dU}{dS}+\frac{1}{2}(\frac{dU}{dS})^2+\frac{1}{2}(\frac{dV}{dS})^2]^{\bullet}-\dot{M}_y\frac{d\dot{\Psi}_y}{dS}+\dot{M}_z\frac{d\dot{\Psi}_z}{dS}+\dot{Q}_y(\Psi_z+$$
$$+\frac{dV}{dS})^{\bullet}+\dot{Q}_z(\frac{dW}{dS}-\Psi_y)^{\bullet}+\dot{M}\frac{d\dot{\Psi}_s}{dS}+\frac{1}{2}N[(\frac{\partial\dot{W}}{\partial S})^2+(\frac{d\dot{V}}{dS})^2]+\frac{1}{2}E(\frac{\dot{N}^2}{F}+$$
$$+\frac{1}{I_y}\dot{M}_y^2+\frac{1}{I_z}\dot{M}_z^2+\frac{1}{F}\dot{Q}_z^2+\frac{1}{F}\dot{Q}_y^2+\frac{1}{I}\dot{M}^2)-P_1\dot{U}-P_2\dot{V}-P_3\dot{W}- \tag{10}$$
$$-\gamma\frac{1}{2}G^{11}(\dot{W}_1-\dot{U})^2-\gamma\frac{1}{2}G^{22}(\dot{W}_2-\dot{V})^2-\gamma\frac{1}{2}G^{33}(\dot{W}_3-\dot{W})^2+$$
$$+\frac{1}{2}\frac{\partial S^i}{\partial W_k}\dot{W}_k\dot{W}_i\gamma\}dS+J_1$$

where J_z - boundary integral with respect to end; N, M_z, M_y, M, Q_z, Q_y - forces, moments cutting the forces; F, I_y, I_z, I_s - area, moments of inertia of the section; P_i - averaged values of F_i; γ - perimeter.

When writing (10) the soil was supposed to be orthotropic. $\dot{U}$, $\dot{V}$, $\dot{W}$, $\dot{\Psi}_s$, $\dot{\Psi}_y$, $\dot{\Psi}_z$, $\dot{N}$, $\dot{M}_y$, $\dot{M}_z$, $\dot{M}$, $\dot{Q}_y$, $\dot{Q}_z$, $\dot{W}_i$ are varied quantities. It is evident that Eiler's equations of the functional (10) are non-linear equations of equilibrium of the pile contracting with the soil, and the pile is described by the S.P. Timoshenko's three-dimensional model. In case of anisotropy of the soil or asummetry of the cross-section derivation of the appropriate functional presents no difficulty (Procedure of its derivation is similar to that of derivation of (10).

Let us consider the equation of equilibrium. Variation with respect to U gives the foolowing equation:

$$[\frac{\partial N}{\partial S}+P_1+\gamma G^{11}(W_i-U)]^{\bullet}=0$$

For linear theory the derived equation together with the equation $N=FEU_{,s}$ describes the change in stresses and movements in the isolated piles carrying a vertical axial load in the swelling soils. It coincides with R.L.Litton's equation. In non-linear theory it is necessary to solve these equations jointly with those of tending.

In particular, when varying with respect to $\dot{W}$ and $\dot{\Psi}_y$ we obtain:

$$[\frac{d}{dS}(N\frac{dW}{dS})+\frac{dQ_z}{dS}+P_3+\gamma G^{33}(W^3-W)]^{\bullet}=0;\quad \frac{dM_y}{dS}-Q_z=0 \tag{11}$$

When varying with respect to $\dot{V}$ and $\dot{\Psi}_z$ similar equations are obtained. Let us note that at $P_3=0$ and $G^{33}=0$ the obtained equation is reduced to the equation of stability. Variation with respect to $\dot{\Psi}_s$ permits to obtain the equatian of torsion.

From the presented equations (11) it is seen that in most cases the flexure plane is determined not only by geometrical parameters of the cross-section but also by coefficient of soil stiffness. It points to the necessity of the study of the three-dimensional model of the pile.