Deformation analysis of composite ground by homogenization method

Analyse de la déformation d'un terrain complexe par méthode d'homogénisation

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ABSTRACT: Composite ground improved by deep mixing or sand compaction pile methods generally has a non-homogeneous structure with pile shaped inclusions. A new homogenization method for deformation analysis of the composite ground has been proposed, based on a two-phase mixture model. The elastic moduli for composite ground are obtained by determining stress distribution parameters for the vertical and horizontal directions. The validity of this method is confirmed through comparison between two-dimensional and three-dimensional finite element analyses and test results on composite model ground.

RESUME: La terre composite améliorée par un mélangement fond ou pieu de sable compactée généralement à une structure pas homogène avec des inclusions avec un profil d'un pieux. Une méthode nouvelle d'homogénisation à base d'un modèle de mélange, a été proposeé pour analyser la terre composite. Les modules d'élasticité pour la terre composite sont obtenus en déterminant les paramètres pour la distribution des contraintes pour les directions verticale et horizontale. La validité de cette méthode est confirmée par une comparaison entre les analyses bidirectionnelles et à trois dimensions osant la méthode des éléments finis et des résultati des essais sur une un modèle de terre composite.

1 INTRODUCTION

There are many kinds of mixtures consisting of different materials and it is therefore important to use analytical methods that are suited to individual mixtures. Composite ground with pile shaped inclusions in the form of cement improved columns or sand compaction piles is regarded as a non-homogeneous material. Although three-dimensional analysis (Kamon et al. 1988) and quasi-three-dimensional analysis (Ishizaki 1992) of the composite ground have been developed, homogenization method for two-dimensional deformation analysis is one of the problems in geotechnical engineering practice that needs to be solved.

In this study, a two-phase mixture model is proposed taking into account the stress distribution rate in the mixture. Elastic moduli of the mixtures are obtained by determining a newly developed stress distribution parameter. This homogenization method is applied to composite ground with improved piles. In order to confirm the validity of this method, results of the two-dimensional finite element analysis are compared with those of three-dimensional analysis for the composite ground for embankment and horizontal loads. Furthermore, laboratory model tests were conducted for various types of composite ground for pile shaped inclusions and the effectiveness of the proposed method is confirmed through comparisons between the model test and the calculated results.

2 TWO-PHASE MIXTURE MODEL

2.1 *Elastic moduli of two-phase mixtures*

A two phase mixture consists of a basic material and another one which are called a matrix and an inclusion respectively. It is necessary to evaluate stress and strain distribution in the mixture in order to predict the stress-strain behavior of the mixtures from the properties of the constituent materials. A new parameter for evaluating stress distribution in the mixture is introduced here as,

$$b = \bar{\sigma}_s / \bar{\sigma}^* \tag{1}$$

where $\bar{\sigma}_s$ and $\bar{\sigma}^*$ are average stresses applied to inclusion and matrix respectively. Young's modulus and Poisson's ratio of the two-phase mixtures are represented using a stress distribution parameter b as follows (Omine et al. 1993)

$$\bar{E} = \frac{(b-1)f_s + 1}{\dfrac{f_s b}{E_s} + \dfrac{(1-f_s)}{E^*}} \tag{2}$$

$$\bar{\nu} = \frac{\dfrac{\nu s}{E_s} f_s\, b + \dfrac{\nu^*}{E^*}(1-f_s)}{\dfrac{f_s\, b}{E_s} + \dfrac{(1-f_s)}{E^*}} \tag{3}$$

where, E_s , E^* and ν_s , ν^* are Young's moduli and Poisson's ratios of inclusion and matrix respectively, and f_s is volume content of inclusion.

2.2 *Determination of stress distribution parameter*

Elastic moduli of various types of mixtures are obtained by substituting the following stress distribution parameters into Eq.(2).

$b = E_s / E^* = (E_s / E^*)^1$; a horizontal laminate (4)

$b = 1 = (E_s / E^*)^0$; a vertical laminate (5)

$b = (E_s / E^*)^{1/2}$; a mixture with inclusions distributed at random (6)

$b = (E_s / E^*)^{1/6}$; a mixture with pile shaped inclusions (7)

These values are summarized in Fig.1. Constant stress and constant strain are assumed for horizontal and vertical laminates respectively. Constant strain energy is assumed for a mixture with inclusions at random in three dimensions. Eq.(7) is obtained from the results of finite element analysis for a mixture with pile shaped inclusions as an approximation value (Omine et al. 1996). These parameters are therefore given as power functions of elastic modulus ratio of inclusion and matrix.

3 NUMERICAL ANALYSIS OF COMPOSITE GROUND

3.1 *Average elastic moduli of composite ground*

As mentioned in the previous chapter, elastic moduli of mixtures are represented using stress distribution parameters. Composite

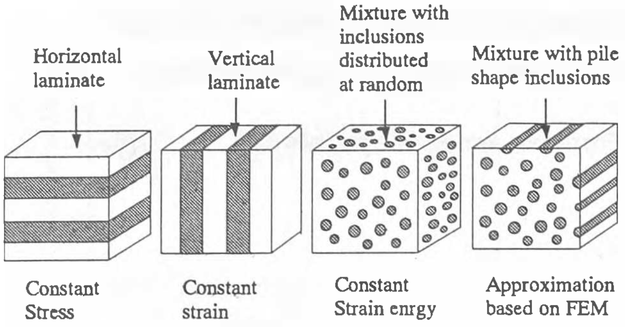


Figure 1. Evaluation of stress distribution parameter of various kinds of mixtures.

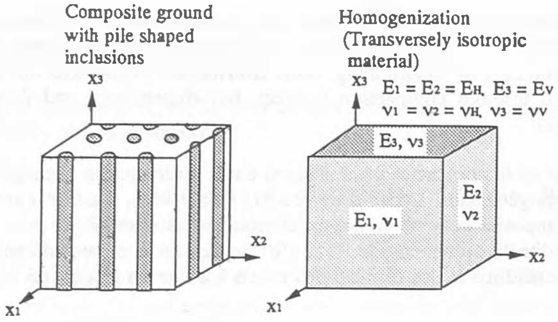


Figure 2. Idealization of composite ground with pile shaped inclusions as a transversely isotropic material.

ground with pile shaped inclusions is idealized as a transversely isotropic material (Fig.2). Eq.(5) is applied for vertical direction assuming constant strain. On the other hand, Eq.(7) is used for horizontal direction. The stress-strain relationship of composite ground for using two-dimensional FEM analysis is therefore represented as follows

$$\{\bar{\varepsilon}_i\}=[\bar{C}_{ij}]\{\bar{\sigma}_j\} \qquad (8)$$

where

$$[\bar{C}_{ij}]=\begin{bmatrix} \bar{C}_{11} & \bar{C}_{12} & \bar{C}_{13} & 0 & 0 & 0 \\ \bar{C}_{12} & \bar{C}_{11} & \bar{C}_{13} & 0 & 0 & 0 \\ \bar{C}_{13} & \bar{C}_{13} & \bar{C}_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & \bar{C}_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{C}_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \bar{C}_{66} \end{bmatrix}$$

$\bar{C}_{11}=1/\bar{E}_H$, $\bar{C}_{12}=-\bar{\nu}_H/\bar{E}_H$, $\bar{C}_{13}=-\bar{\nu}_V/\bar{E}_V$, $\bar{C}_{33}=1/\bar{E}_V$, $\bar{C}_{44}=1/\bar{G}$, $\bar{C}_{66}=1/\bar{G}_H$

$$\bar{E}_H=\frac{(b_1-1)f_s+1}{f_s b_1/E_s+(1-f_s)/E^*} \qquad \bar{\nu}_H=\frac{f_s b_1 \nu_s/E_s+(1-f_s)\nu^*/E^*}{f_s b_1/E_s+(1-f_s)/E^*}$$

$$\bar{E}_V=\frac{(b_3-1)f_s+1}{f_s b_3/E_s+(1-f_s)/E^*} \qquad \bar{\nu}_V=\frac{f_s b_3 \nu_s/E_s+(1-f_s)\nu^*/E^*}{f_s b_3/E_s+(1-f_s)/E^*}$$

$$\bar{G}=\frac{(b_4-1)f_s+1}{f_s b_4/G_s+(1-f_s)/G^*} \qquad \bar{G}_H=\frac{(b_6-1)f_s+1}{f_s b_6/G_s+(1-f_s)/G^*}$$

$b_1=b_2=(E_s/E^*)^{1/6}$, $b_3=(E_s/E^*)^1$, $b_4=b_5=(G_s/G^*)^{7/12}$, $b_6=(G_s/G^*)^{1/6}$

The value of $\bar{C}_{44}$ is a component of shear stress on both planes of vertical and horizontal directions so that an average value of b_1 and b_3 was taken for the stress distribution parameter b_4 herein.

3.2 *Comparison between two and three-dimensional analyses*

(a) *Analytical condition* In order to confirm the validity of the proposed method, two-dimensional analysis based on the homogenization method and three-dimensional analysis of composite ground with improved piles were conducted under the following two loading conditions

Case I : embankment load
Case II : horizontal load

In the analysis, improved soil and unimproved soil are regarded as linear elastic isotropic material. Poisson's ratios of improved soil and unimproved soil are $\nu_s=0.2$ and $\nu^*=0.4$ respectively and Young's modulus ratios E_s/E^* are 2, 5, 10 and 50. Improvement rate f_s which is defined by volume fraction of improved soil in the composite ground are 0, 20, 40, 60 and 80%. The mesh for three-dimensional analysis is shown in Fig.3. Element number of 5000 is used for representing composite ground with improved piles of 10 in three-dimensional analysis. On the other hand, element number of 200 is used for two-dimensional analysis by the homogenization method.

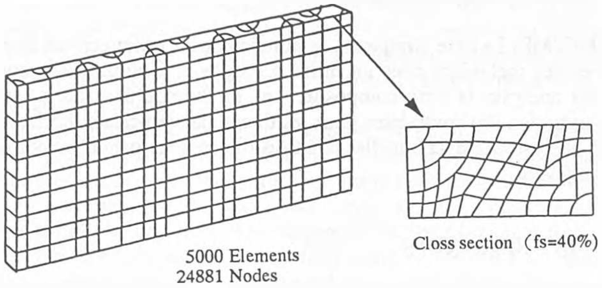


Figure 3. Mesh for three-dimensional analysis of composite ground.

(b) *Results of analyses* The relationship between S/S_0 and improvement rate f_s obtained from two and three-dimensional analyses is shown in Fig.4, where S and S_0 are settlements in the center of embankment on the ground with and without improved piles respectively. The value of S/S_0 decreases with increase in improvement rate and this tendency varies for Young's modulus ratio of improved soil and unimproved soil. As seen in the figure, S/S_0 decreases remarkable as the value of E_s/E^* increases. The results of two-dimensional analysis become almost the same as those of three-dimensional analysis. On the other hand, The relationship between δ/δ_0 and improvement rate f_s is shown in Fig.5, where δ is lateral deformation in the toe of the slope. The lateral deformation ratio δ/δ_0 also decreases with increase in improvement rate and Young's modulus ratio. The results of two-dimensional analysis using a homogenization method agree with the results of three-dimensional analysis. Figure 6 shows the comparison between two and three-dimensional analyses for the settlement shape on the ground surface. A result for isotropic material in the case of $b_1=b_2=b_3=E_s/E^*$ is also shown in the figure. The settlement around the center of embankment in this case ($b_1=b_2=b_3$) is underestimated comparing to that of three-dimensional analysis. However, the results of two-dimensional analysis using the stress distribution parameter of $b_1=b_2=(E_s/E^*)^{1/6}$ for horizontal direction agree well with the result of three-dimensional analysis. Thus, the deformation behavior of the composite ground can be expressed by evaluating stress distribution parameters in horizontal and vertical directions.

Comparison between two and three-dimensional analyses for the case applied horizontal load is shown in Fig.7. As shown in the figure, the value of L/L_0 decreases as improvement rate and Young's modulus ratio increase, where L is horizontal displacement on upper point of the load and L_0 is horizontal displacement for unimproved ground with $f_s=0\%$. The results of two-dimensional analysis using a homogenization method

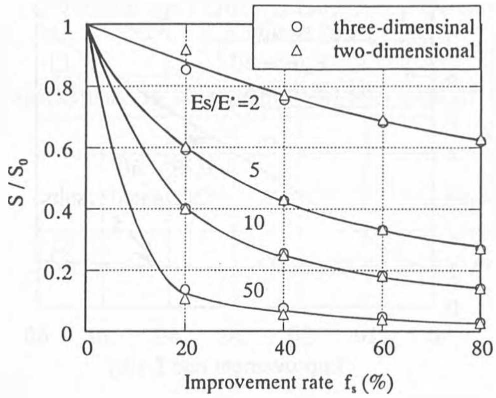


Figure 4. Relationship between S/S_0 and improvement rate f_s (Comparison between two and three-dimensional analyses).

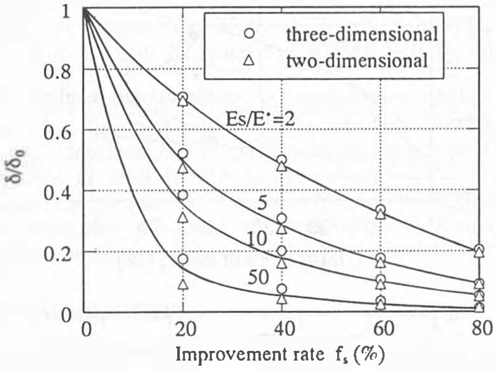


Figure 5. Relationship between δ/δ_0 and improvement rate f_s (Comparison between two and three-dimensional analyses).

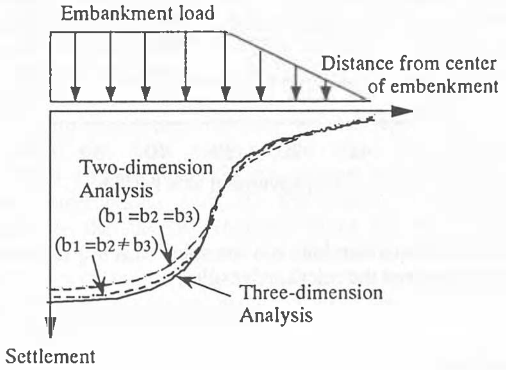


Figure 6. Comparison between two and three-dimensional analyses for the settlement shape on the ground surface.

relatively agree with the results of three-dimension analysis. Figure 8 shows the comparison between two and three-dimensional analyses for the deformation shape on the surface of applied load. The results in the cases of $b_1=b_2=E_s/E^*(=b_3)$ and $b_1=b_2=1$ being maximum and minimum values of stress distribution parameter respectively are also shown in the figure. The result of two-dimensional analysis using the stress distribution parameter of $b_1=b_2=(E_s/E^*)^{1/6}$ for horizontal direction agree with the, result of three-dimensional analysis. The results of two-dimensional analysis in the cases of $b_1=b_2=E_s/E^*$ and $b_1=b_2=1$ underestimate and overestimate a horizontal displacement respectively. It is therefore very important for two-dimensional analysis using a homogenization method to determine an appropriate stress distribution parameter.

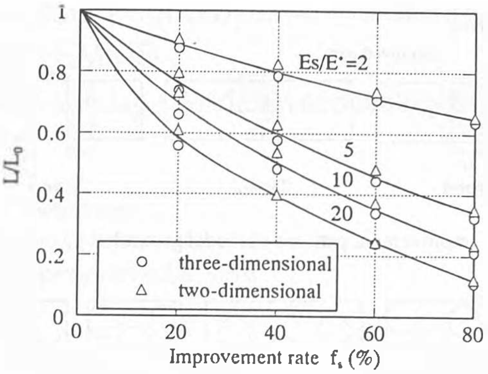


Figure 7. Relationship between L/L_0 and improvement rate f_s (Comparison between two and three-dimensional analyses).

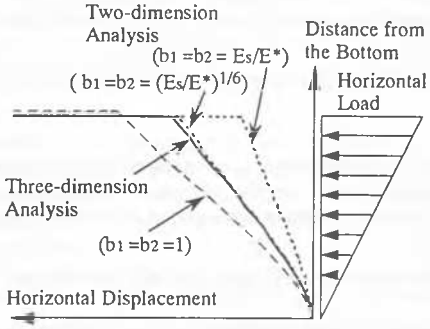


Figure 8. Comparison between two and three-dimensional analyses for the deformation shape on the surface of applied load.

Thus, average deformation behaviour of composite ground with improved piles can be evaluated based on the proposed method. Furthermore this homogenization method is effective as a simple deformation analysis of the composite ground compared with three-dimensional analysis.

4 VERIFICATION BY MODEL TEST OF COMPOSITE GROUND WITH PILE SHAPED INCLUSIONS

4.1 *Model tests*

Stiffness ratio between improved and unimproved soils is a very important factor for evaluating average elastic modulus of composite ground in this study. From this point of view, the unimproved model ground was created by adding Kaolin clay slurry to a small amount of cement without consolidation and rubber and expanded resin material of pile shape in diameter of 2.0 cm were used instead of improved soils. The unconfined compressive strength of the unimproved soil was q_u=18kPa and initial deformation modulus was E^*=3.04MPa. On the other hand, The initial deformation moduli and Poisson's ratio obtained from unconfined compression test were E_s =14.7MPa and ν_s =0.38 for the rubber and E_s =182.7MPa and ν_s =0.18 for the expanded resin material respectively. The initial deformation modulus ratio E_s/E^* was therefore about 5 in the case of the rubber and 60 in the case of the expanded resin material.

All the tests were performed under plane strain condition in a specimen box with inside dimensions of 50cm in length, 10cm in width and 15cm in depth. Improvement patterns of model ground are shown in Fig.9. The patterns are of the following three kinds;

Type-1 : Improvement of all area under loading plate
Type-2 : Improvement of upper half space under loading plate
Type-3 : Improvement on both side space of loading plate

Arrangements of the materials as improved soils are shown in

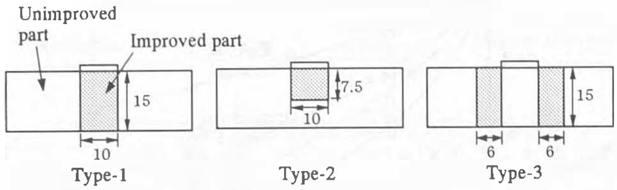
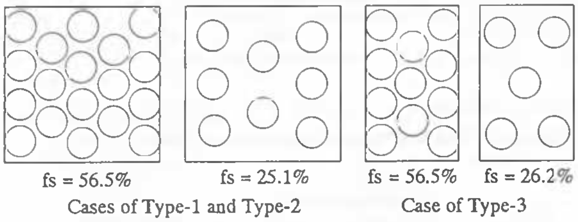


Figure 9. Improvement patterns of model ground.



Cases of Type-1 and Type-2

Case of Type-3

Figure 10. Arrangements of the materials as improved column

Fig.10. A plover arrangement is used for all types of improvement pattern. Loading tests for the composite model ground were conducted under the constant deformation rate of 0.5mm/min.

4.2 *Comparison between calculated and test results*

A coefficient of subgrade reaction K $(=p/S)$ is defined by dividing loading stress p with settlement S. A value of K/K_0, where K_0 is a coefficient of subgrade reaction of unimproved ground, is used as a parameter for representing improvement effect.

Comparison between the value of K/K_0 obtained from the tests and the calculated results is shown in Fig.11. Although the value of K/K_0 at $E_s/E^*=5$ in the case of Type-1 is almost constant, the value at $E_s/E^*=60$ increases linearly with increasing improvement rate f_s. As shown in the figure, close agreement between calculated and test results was obtained. On the other hand, the relationship between K/K_0 and f_s in the case of Type-2 is not linear. The calculated results have a relatively good agreement with the test results except for the case of f_s =25.1% at E_s/E^*=60. The improvement effect in the case of Type-3 is similar to that in the case of Type-1. However the value of K/K_0 at the same improvement rate is about half compared with that of Type-1. The calculated results agree well with such a tendency. It is therefore confirmed that the proposed method is effective for deformation analysis of the composite ground.

5 CONCLUSIONS

The main conclusions of this study are as follows:

1) Elastic moduli of various types of mixtures constituted of inclusions and matrix are represented by a formula with a new parameter for evaluating stress distribution rate in the mixture. This stress distribution parameter is given as an power function of the ratio of elastic moduli of inclusion and matrix.

2) Young's modulus and Poisson's ratio of composite ground with varying improvement rate are estimated based on the evaluation of stress distribution parameter in vertical and horizontal directions.

3) Results of deformation analysis of composite ground by the homogenization method agree well with those of three-dimension analysis.

4) The proposed method is effective for two-dimension deformation analysis of composite ground.

5) The validity of the proposed method has been confirmed by model test of composite ground.

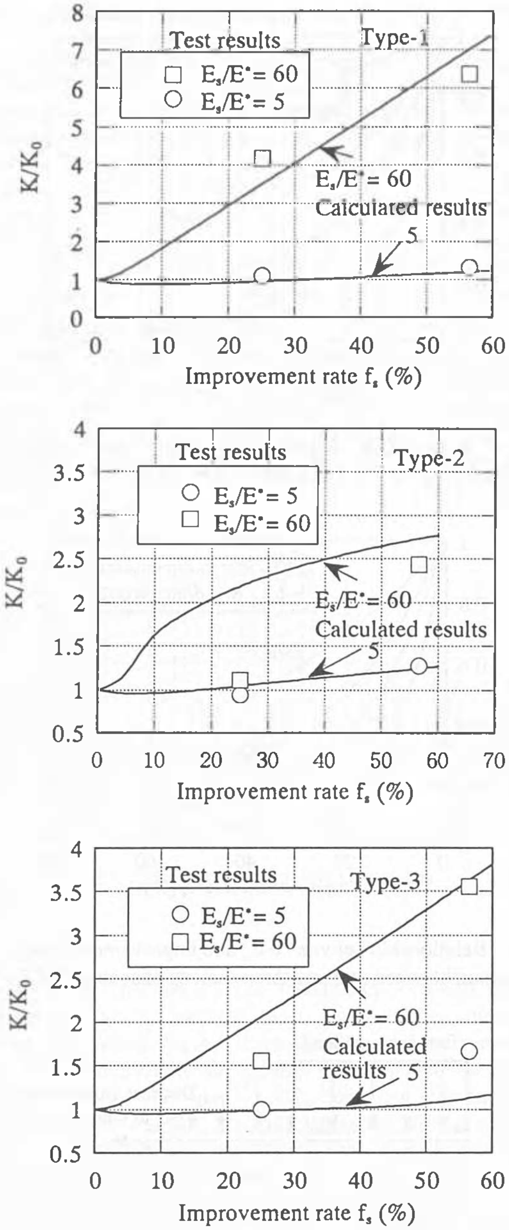


Figure 11. Comparison between the value of K/K_0 obtained from the model tests and the calculated results.

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