Rectangular raft of any rigidity on the layer of limited thickness

Radier rectangulaire de rigidité finie sur une couche d'épaisseur limitée

D. M. Milovic – *Institute IMS, Belgrade, Yugoslavia*
M. B. Djogo – *Faculty of Technical Sciences, Novi Sad, Yugoslavia*

ABSTRACT: The behaviour of rectangular raft of any rigidity resting on a homogeneous elastic layer of finite thickness is analyzed. The raft foundation is assumed to be uniformly loaded over the entire surface and, alternatively, over only a part of the foundation. The theoretical solution for determination of settlements w, bending moments M_x and M_y, torque M_{xy}, shearing forces N_x and N_y and reactive stresses q in any nodal point of a rectangular foundation of any rigidity is obtained by finite difference method.

RESUME: Cet article présente l'étude du comportement de la fondation rectangulaire de rigidité finie reposant sur un sol homogène et élastique d'épaisseur limitée. On a supposé que la fondation est sollicitée par une charge uniformément répartie ou, alternativement, par une charge partielle. La solution théorique pour la détermination des tassements w, moments de flexion M_x et M_y, moments de torsion M_{xy}, forces de cisaillement N_x et N_y et pressions de contact q pour chaque point nodal de la fondation a été obtenu par la méthode de difference finie.

1 INTRODUCTION

The contact problem of a rectangular raft foundation has been studied by several authors. Ueshita and Meyerhof (1968) and Milovic and Tournier (1971) have presented solutions for stresses and settlements produced by a uniformly loaded rectangular area. Brown and Gibson (1979) have given the solution for surface settlement of a rectangular foundation, resting on an elastic material, whose Young's modulus increases linearly with depth. For an elastic layer of finite thickness overlying a rough incompressible base Milovic and Tournier (1973), Fraser and Wardle (1976), Dempsey and Li (1989) have presented some results for contact stresses and settlements produced by symmetric loading over a rectangular rigid raft foundation.

This Paper shows solutions for determination of settlements w, bending moments M_x and M_y, torque M_{xy}, shearing forces N_x and N_y and reactive stresses q in any point of a square or rectangular raft foundation. For soil model an elastic and isotropic half-space was assumed, whereas the raft foundation can be of any rigidity. The solutions have been obtained by the finite difference method.

2 THEORETICAL ANALYSIS

The square or rectangular plate was divided by parallel lines in x and y directions into a grid system with a size of λ_x and λ_y, as shown in Figure 1.

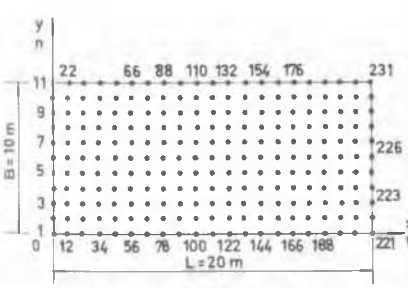

Figure 1. Finite difference grid system

The deflection of the foundation plate loaded by p(x,y) is defined by the partial differential equation:

$$\frac{\partial^4 w}{\partial x^4} + 2 \cdot \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p(x,y) - q(x,y)}{D} \tag{1}$$

where w = w(x,y) unknown function of the vertical displacement of the central part of the slab, q = unknown function of the reactive stresses. The flexural rigidity of the plate is given by:

$$D = \frac{E \cdot h^3}{12 \cdot (1 - \mu^2)} \tag{2}$$

where h is the thickness of the plate.

The system of linear algebraic equations is given in the following matrix form:

$$[A] \cdot \{w\} = \frac{\lambda_x^4}{r^2 \cdot D} \cdot [\{p\} - \{q\}] \tag{3}$$

where [A] = matrix of coefficients in the system of linear algebraic equations, {w} = one dimensional matrix for vertical displacements, {p} = one dimensional matrix for the applied load which acts in nodal points, {q} = one dimensional matrix for the reactive stress which acts in nodal points. The relationship between the unknown vectors {w} and {q} can be expressed in the following matrix form:

$$\{w\} = [F] \cdot \{q\} \tag{4}$$

where [F] = square matrix for the settlement of a point of the plate.

Using the expression (4) it is possible to determine the reactive stresses q and settlements w of all points on the plate, and after that the shear forces N_x and N_y, the bending moments M_x and M_y and the torque M_{xy}. Shearing forces and moments can be calculated by the expressions:

$$M_x = -D \cdot \left(\frac{\partial^2 w}{\partial x^2} + \mu \cdot \frac{\partial^2 w}{\partial y^2} \right) \tag{5}$$

$$M_y = -D \cdot \left(\frac{\partial^2 w}{\partial y^2} + \mu \cdot \frac{\partial^2 w}{\partial x^2} \right) \tag{6}$$

$$M_{xy} = -D \cdot (1-\mu) \cdot \frac{\partial^2 w}{\partial x \partial y} \tag{7}$$

$$N_x = -D \cdot \left(\frac{\partial^3 w}{\partial x^3} + \mu \cdot \frac{\partial^3 w}{\partial x \partial y^2} \right) \tag{8}$$

$$N_y = -D \cdot \left(\frac{\partial^3 w}{\partial y^3} + \mu \cdot \frac{\partial^3 w}{\partial y \partial x^2} \right) \tag{9}$$

3 RESULTS

On the basis of the obtained theoretical solution the values of w, q, M_x, M_y, M_{xy}, N_x and N_y have been calculated for several values of the L / B ratio (where L is the length and B is the width of the foundation), for various relative stiffnesses K of the foundation raft and for a few values of the Poisson's ratio of soil μ.

The relative stiffness of the plate is defined by the expression (Fraser and Wardle 1976):

$$K = \frac{4 \cdot E_b \cdot (1-\mu_s^2) \cdot h^3}{3 \cdot E_s \cdot (1-\mu_b^2) \cdot B^3} \tag{10}$$

where E_b, E_s = modulus of elasticity of the raft and the soil, respectively; μ_b, μ_s = Poisson's ratio of the raft and the soil, respectively; h = thickness of the raft; B = width of the raft.

Some of the obtained results are shown in terms of dimensionless coefficients. So, the coefficients I_w for the settlement calculation of the corner and the center of a raft are shown in Figure 2. These coefficients are given for various values of H / B (where H is the thickness of a compressible layer), and for several values of K. The coefficients I_w for the partly loaded raft foundation are shown in Figure 3.

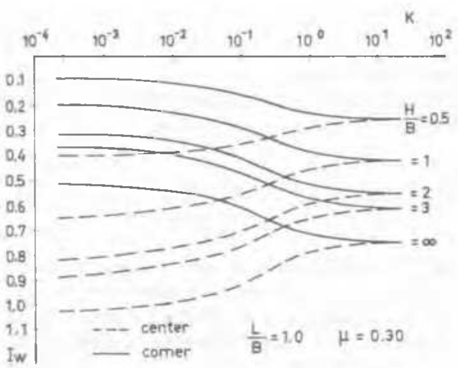

Figure 2. Coefficients I_w; uniform load

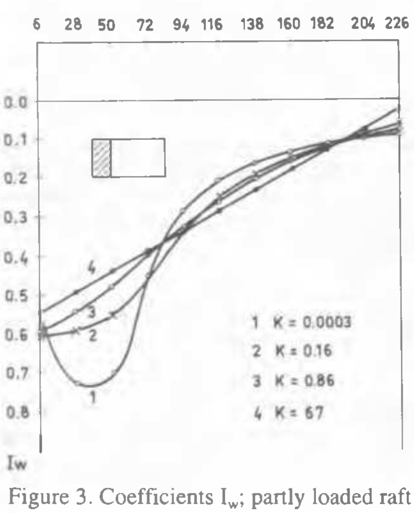

Figure 3. Coefficients I_w; partly loaded raft

The coefficients I_{Mx} for the calculation of bending moments M_x, produced by partly loaded raft are shown in Figure 4.

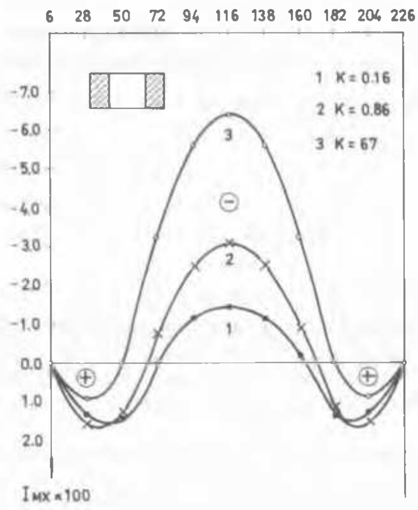

Figure 4. Coefficients I_{Mx}; partly loaded raft

The curves shown in the above Figures represent only a few examples of the obtained results. As a matter of fact, on the basis of the obtained solution it is possible to obtain for any nodal point of the raft the values of w, q, M_x, M_y, M_{xy}, N_x and N_y, for any value of the ratio H / B and for any stiffness K of the raft.

4 CONCLUSIONS

1. Relative stiffness of the slab foundation has considerable influence on values of settlements, reactive stresses, moments and shearing forces and must be involved in design of a raft foundation.
2. The thickness of the compressible layer has a significant influence on the values of settlements and moments.
3. The degree of precision of the results obtained by finite difference method becomes higher with the finer grid.

REFERENCES

Brown, P.T. 1972. *Analysis of rafts on clay.* Ph D Thesis, University Sidney, Australia.

Brown, P.T. and Gibson, R.E. 1979. Surface settlement of a finite elastic layer whose modulus increases linearly with depth. *Int. Journal for numerical and analytical methods in geomechanics,* 3:37-47.

Butterfield, R. and Banerjee, P.K. 1971. A rigid disk embedded in an elastic half-space. *Geotech. Engng 2,* 1:35-52.

Cheung, Y.K. and Zienkiewicz, O.C. 1965. Plates and tanks on elastic foundations - an application of finite element method. *Intern. Journal Solids Structures,* 1(4):451-461.

Dempsey, J.P. and Li, H. 1989. A rigid rectangular footing on an elastic layer. *Géotechnique* 39:147-157.

Fraser, R.A. and Wardle, L.J. 1976. Numerical analysis of rectangular rafts on layered foundations. *Géotechnique,* 26:613-630

Giroud, J.P. 1972. Settlement of rectangular foundation on soil layer. *Journal of the Soil Mechanics and Foundation Division,* A.S.C.E.98,SM1:149-154.

Gorbunov Possadov, M.I. and Serebrjanyi, R.V. 1961. Design of structures on elastic foundations. *Proc. 5th Int. Conf. Soil Mech. Found. Engng.,* 1:643-648, Paris.

Milovic, D. and Tournier, J.P. 1971. Stresses and displacements due to rectangular load on a layer of finite thickness. *Soils and Foundations Tokyo,* 11(1):1-27.

Milovic, D. and Tournier, J.P. 1971. Stresses and displacements due to rectangular foundation on a layer of finite thickness. *Soils and Foundations Tokyo,* 13(4):24-43.

Ueshita, K. and Meyerhof. G.G. 1971. Surface displacements of an elastic layer under uniformly distributed loads. *Highway Research Record,* 228: 1-10.