Rational prediction of lateral behaviour of concrete piles incorporating pile (concrete) non-linearity

Prédiction rationnelle du comportement latéral des pieux en béton incorporant non-linéaire du pieu en béton

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ABSTRACT: Conventional recommended methods for predicting the lateral response of pile foundations are primarily based on the assumption of linear elastic pile and soil behaviours. Concrete behaves non-linearly at very low tensile stress levels due to the cracking as recognised in structural design practice. In the current study, a simplified non-linear analysis, based on the subgrade-reaction approach, has been developed to incorporate the non-linearities of both pile and surrounding soil. The pile-soil system is simulated as a frame analogy with beam-column pile elements and soil springs. It is shown that the non-linearity of concrete piles significantly increases the pile deflection and also redistributes the flexural moments & subgrade reactions. The proposed method is feasible for routine design office applications.

RESUME: Les methodes conventionnelles et recommendees pour la prediction de la reponse laterale des pieux de fondation sont basees, basiquement, sur l'hypothese de lois de comportement lineaire du sol et du pieu. Le beton ne se comporte pas de maniere lineaire aux faibles contraintes tensorielles, `a cause des fractures, ainsi qu'il est accepte, en pratique, dans les calculs des efforts structurels. Pour la presente etude, on a developpe une methode d'analyse simple et non-lineaire, basee sur une approche de reaction de sous-couche. Cette methode inclut un comportement non-lineaire du pieu et du sol. Le systeme sol-pieu est represente dans le cadre de l'analogie avec des elements poutre-colonne pour le pieu et des "ressorts" pour le sol. La non-linearite des pieux en beton augmente de maniere significante la deflection des pieux, et favorise aussi une redistribution des moments de flexion et des reactions des sous-couches. La methode preconisee est applicable dans tout bureau d'etude.

1 INTRODUCTION

Concrete piles are used in many structural and geotechnical applications and can be either reinforced or prestressed. Pile foundations are generally used to reduce the settlements and lateral deflections and also to enhance the bearing capacity by transferring the applied surface loadings to deeper strata. Piles are subjected to simultaneous axial force (either tensile or compressive), flexural moments, and lateral forces in resisting the applied loadings on the superstructure. In addition, they can be subjected to deformation-induced loadings generated by the swelling & shrinkage of the surrounding soil and by the soil movements caused by bridge abutments, slopes, etc.

In the past, the performance of deep foundations was mainly evaluated through adequate bearing capacity with an acceptable factor of safety against the working (service) loads. Deflections under service loads were treated rather cursorily neglecting the soil-pile interaction in particular. However, in recent modern Standards (eg, AS 2159) and design practices advocating limit state design philosophies, the rational prediction of the pile behaviour under different limit states is required, particularly the serviceability limit state of limited deflections. The accurate analysis of pile foundations is complicated due to the soil-pile interaction involving relative stiffnesses, local yielding of soil, etc. In the case of concrete piles, the design of the piles should be in accordance with the relevant structural Standards (AS 3600) in regard to serviceability as well as ultimate limit states while the analysis of the soil-pile system is in accordance with the piling Standard (AS 2159). Accordingly, an effective interaction between structural and geotechnical disciplines is very important in the design of pile foundations.

Conventional methods for predicting the lateral load response of piles have been based primarily on the linear elastic behaviours of the soil and the pile. A few researchers (Poulos 1980, Sogge 1982, Gabr 1994) have studied the pile behaviour assuming the soil to be non-linear, but the pile to be linear. However, in contrary, the flexural behaviour of concrete piles would be non-linear at very low load levels as concrete cracks due to relatively smaller tensile strength values. Consequently, the stiffness of the pile varies along the pile length depending on the distribution of the flexural moments as well as the pile sectional parameters.

Concrete is a material with a very low tensile strength, and accordingly it is ignored in the estimation of the structural capacity of concrete sections (AS 3600). Cracking of concrete results in a reduction of the pile stiffness with a non-linear behaviour when the tensile strength is exceeded. The degree of non-linearity of a reinforced concrete section depends on the load level and therefore the mere use of a constant reduced stiffness value is invalid. This non-linear behaviour will increase the deflections and redistribute the flexural moments and soil reactions along the pile.

In the current study, a simplified analysis methodology has been developed to incorporate the non-linearly of concrete as well as the soil. The pile-soil system is idealised as a plane frame (frame analogy) with pile beam elements supported on discrete Winkler soil springs (struts) with respective non-linear behaviours. The analysis is carried out with the direct stiffness method in an incremental fashion. It is shown that the cracking of concrete modifies the pile behaviour significantly even at low load level.

2. GENERAL METHODS PREDICTING PILE RESPONSE

The flexural behaviour of a linear elastic pile can be expressed by the general equation:

$$(EI)\frac{d^4 w}{dx^4} + kd_p w = p \qquad (1)$$

where E = Young's modulus of elasticity of the pile material, I = pile flexural stiffness inertia (lenght^4), w = lateral deflection, k = modulus of subgrade reaction (force/length^3), d_p = pile diameter, and p = applied transverse loading (force/length) at point x. The effect of the simultaneous axial force on the pile deflection has been ignored in Eq.1 due to its insignificance being a second-order effect. Standard analytical solutions based on a beam-on-elastic foundation (BEF) system, are available (Hentenyi 1946) for special cases. The numerical solutions to Eq. 1 have been obtained by finite-difference discretisations (Poulos 1980) and they are used widely in design practice. In order to incorporate the non-linear soil behaviour, subgrade reaction-deflection relationships (usually known as p-y curves) are used in an iterative approach.

The subgrade-reaction models for soil behaviour originally proposed by Winkler, in which the subgrade reaction at a given point is only related to the deflection at that point, explicitly ignore the continuous nature of the soil medium. Subsequently, elastic methods incorporating the continuous nature of the soil medium have been developed (Poulos 1980) by using the Mindlin formulations for calculating the displacements within an elastic-half-space (EHS). They have the obvious disadvantage of being unable to include the non-linear soil behaviour and the soil behaviour is therefore assumed to be linear elastic. However, the local yielding of the soil has been incorporated (Poulos 1980) assuming elastic-perfectly plastic behaviour on reaching the ultimate pressure. This would be of limited validity as a non-linear analysis as the soil begins to behave non-linearly well before the ultimate pressure is reached. Therefore, it is reasonable to conclude that the adoption of a subgrade-reaction approach with p-y curves allows the soil non-linearity to be incorporated in a full-range pile analysis. Strain-softening effects (in which the strength of the soil falls below the ultimate peak value at larger strains) can also be incorporated.

In the representation of general non-linear pile behaviour, the finite-difference discretisation of Eq. 1 is of limited validity for special cases only (Kramer 1988). Furthermore, the responses of concrete piles have unique characteristics compared with those for steel piles in which the elastic-perfectly plastic moment-curvature relationships exist.

3 PROPOSED ANALYSIS METHODOLOGY

In Section 2, the advantages and disadvantages of the available methods for evaluating the lateral pile response were discussed. The main objective of the current study is to develop a simplified analysis method for the incorporation of the non-linearities of both concrete and soil. The subgrade-reaction method is used to represent the soil behaviour with non-linear p-y curves. If necessary, elastic half space theory (Poulos 1980) for modeling the elastic soil response, can be included easily.

Figure 1 shows the simulation of the pile-soil system referred to as the frame analogy simulation. The pile is represented by rigidly-connected beam-column members while the Winkler subgrade reaction is represented by discrete springs or pin-ended struts. This simulation can be carried out in its simplest form in a standard structural engineering software for plane frame analysis. The pin-ended struts are assigned with the respective subgrade modulus over their contributory area as axial stiffness. The beam members are assigned with the respective flexural stiffness (EI) of the pile, and the defective pile sections occurring during pile installations can be easily modelled.

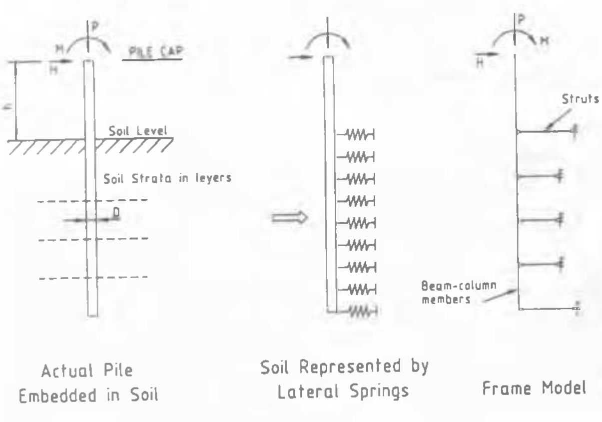

Fig 1. Frame-analogy simulation of the pile-soil system.

3.1 *Concrete (pile) behaviour*

Concrete cracks when the tensile strength f_t is exceeded. A lower value for f_t can be expected in practice compared with those given in concrete design Standards due to less favourable field conditions. Concrete pile elements are modelled as beam members with the composite action of the concrete and reinforcing steel, ie, macro-level analysis. A micro-level analysis, in which the behaviours of both concrete and steel are considered individually, will be permitted only in a sophisticated finite element analysis procedure having the capability to model the discrete cracks.

A simple beam-column element has been developed (Vitharana 1991) representing the non-linear behaviour of reinforced concrete by an effective flexural stiffness. These have been verified experimentally on proto-type concrete elements. In the design of reinforced concrete piles, the required amount of reinforcing steel is governed either by the strength criteria or more frequently by the durability requirements (Grayson 1995). In order to avoid the corrosion of reinforcing steel under aggressive exposure conditions, the crack widths should be limited to allowable values, eg, about 0.1 mm corresponding with a reinforcing steel stress of about 110 MPa generally.

The stiffness of the cracked concrete section, in which there is no tensile resistance of concrete, is much lower than that of the uncracked gross section. Tension stiffening effect provided by the bond between concrete & steel in between the cracks enhances the average member stiffness above the cracked section stiffness, Fig. 2. This effect should be considered in any rational evaluation of the stiffness of a reinforced concrete member, and this effect decreases with the increasing load above the cracking load. Typical characteristics of the moment-curvature response of a reinforced concrete member are schematically shown in Fig. 3. The effective secant stiffness E_cI_e, which includes the tension stiffening effect, can be expressed by (Vitharana 1991):

$$I_e = (M_{cr} / M)^3 I_{tg} + \left[1 - (M_{cr} / M)^3\right] I_{cr} \tag{2}$$

where I_e = effective secant moment of inertia (stiffness inertia), E_c = Young's modulus of elasticity of concrete, M = flexural moment at which I_e is being calculated, M_{cr} = cracking moment corresponding with f_t (Eq. 3), I_{tg} = transformed gross section moment of inertia, and I_{cr} = cracked section moment of inertia calculated from an elastic cracked section analysis (Fig. 4).

$$M_{cr} = \left[\frac{N}{A_{tg}} + f_t\right] \frac{I_{tg}}{(d_p / 2)} \tag{3}$$

where N = simultaneous axial force (compressive +ve), f_t = tensile strength of concrete, A_{tg} = transformed gross sectional area, and d_p = pile diameter. The values of A_{tg} and I_{tg} are given by:

$$A_{tg} = (1 + n\rho) A_g \tag{4}$$

$$I_{tg} = \left[1 + 2n\rho (d_r / d_p)^2\right] I_g \tag{5}$$

$$\text{with } A_g = \frac{\pi d_p^2}{4}, \quad I_g = \frac{\pi d_p^4}{64} \quad and \quad \rho = \frac{A_s}{A_g}$$

where n = modular ratio (= E_s/E_c), E_s = Young's modulus for steel, d_r = diameter of the annular reinforcing steel ring determined by the cover to reinforcing steel, and ρ = reinforcing steel ratio (= A_s/A_g).

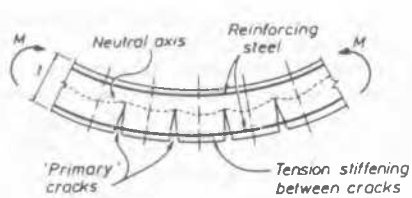

Fig 2. Tension stiffening mechanism.

The advantage of the form of Eq. 2 is the ability to express the tangential flexural stiffness (E_cI_t) mathematically in order to use in an incremental non-linear analysis:

$$I_t = \frac{I_e^2}{[4I_e - 3I_{cr}]} \qquad (6)$$

Once the reinforcing steel yields corresponding with the yield moment M_y, the yield plateau can be assumed to occur with a flat moment-curvature response until the ultimate moment M_u occurs with a compressive failure of concrete (AS 3600). However, this stage does not exist under service loading conditions.

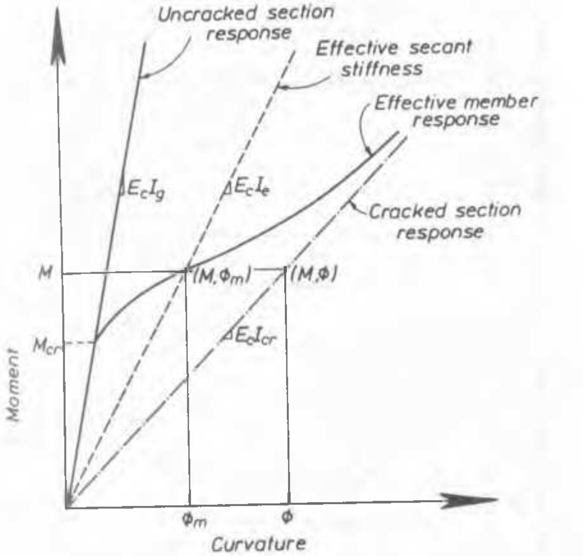

Fig 3. Schematic representation of moment-curvature relationship

The calculation of the moment of inertia of the cracked transformed section I_{cr} shall be based on a cracked section analysis which satisfies the strain compatibility between concrete & steel and equilibrium of applied loadings & resisting actions, Fig. 4. The general procedure is detailed elsewhere (Vitharana 1996). Once the neutral axis depth (at which the strain is zero) is found, the cracked section properties can be calculated. The value of the subtending angle (2β) defining the neutral axis depth is given by:

$$\left(\beta - \frac{\sin 2\beta}{2}\right)\left(\frac{2\sin^3\beta}{3(\beta - \sin\beta\cos\beta)} - \cos\beta\right) - \pi(n\rho)\cos\beta = 0.0 \qquad (7)$$

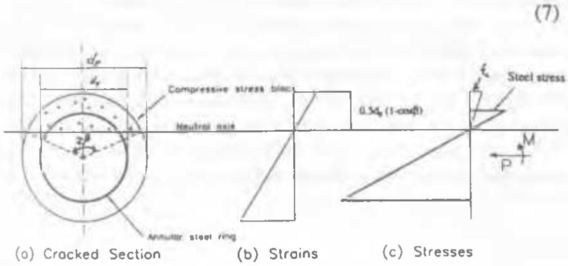

Fig 4. Details of the cracked section for a circular pile.

A particular feature to note in Eq. 7 is that the value of β is dependent only of the product $n\rho$, but not of the steel ratio ρ. Consequently, the cracked section properties can be expressed in terms of the steel parameter $n\rho$.

The available formulations for crack width calculations would give widely differing results for the same parameters, and consequently these not favoured by the design profession. This can be indirectly achieved by limiting the cracked section steel stresses f_s under service loading conditions: usually around 100-140 MPa. The values of the cracked section moment of inertia ratio r_{Icr} (= I_{cr}/I_g) and steel stress ratio f_s (= $M/d^3{}_pf_s$) are given in Fig. 5 for the practical ranges of pile sectional parameters. The effective moment-curvature relationship obtained from the above equations are given in Fig. 6 for a typical reinforced concrete pile having the parameters: d_p = 1000 mm, d_r = 800 mm, p = .01, E_c = 27500 MPa, and f_t = 1.0 MPa. As can be seen, the stiffness degradation with the onset of cracking is significant and rapidly varies with the flexural moment. In this particular case, the non-linear behaviour occurred well before the yield moment.

As shown, the design aids derived herein can be used for the rapid determination of the reinforced concrete pile behaviour particularly where the serviceability limit state performance is critical.

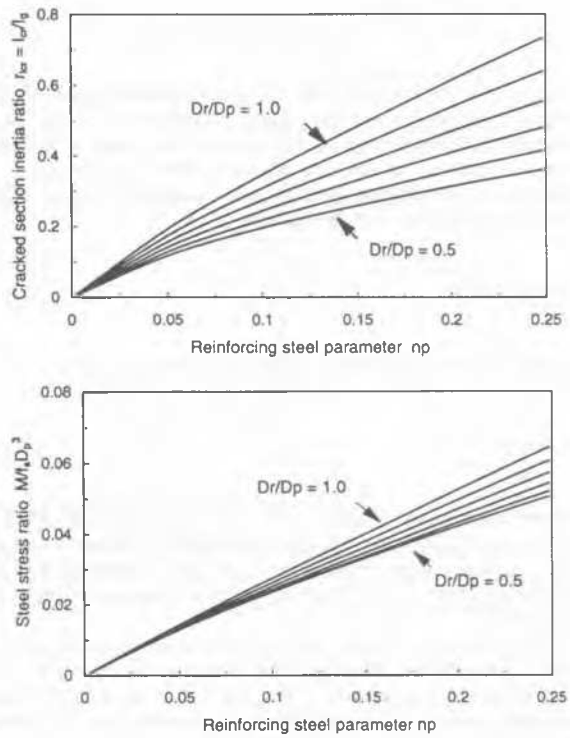

Fig 5. Cracked section properties for typical sections.

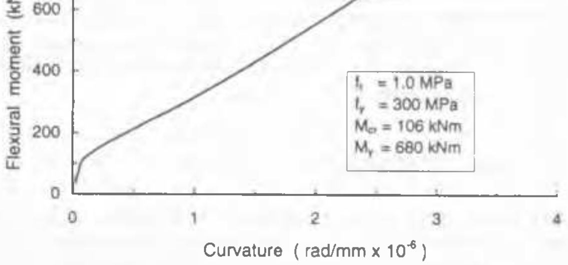

Fig 6. Moment-curvature response of a typical pile.

3.2 *Soil behaviour*

The non-linearity of the subgrade-reaction modulus k occurs in two ways: (a) with respect to the depth along the pile, and (b) with respect to subgrade reaction or lateral displacement values at a given depth as defined by p-y curves. These can be incorporated in the proposed frame analogy simulation. The former can be modelled in an elastic analysis while the latter should be modelled in a non-linear analysis.

The variation of the subgrade-reaction modulus with pile depth has been investigated by several researchers, and a generalised equation for determining initial (low strain) k_i for various soil types has been proposed by Sogge (1982):

$$k_i = k_{max}\left(\frac{y}{l}\right)^m \tag{8}$$

where y = depth at which k_i is being calculated, k_{max} = maximum value of k_i occurring at pile length l, and m = empirical index the typical values of which are 0.0 for *clay* and 1.0 for *sand*.

$$\begin{aligned} k_{max,Clay} &= \frac{(1700-15000)}{d_p} \quad (kN/m^3) \\ k_{max,Sand} &= \frac{(300-5000)}{d_p} l \quad (kN/m^3) \end{aligned} \tag{9}$$

where d_p and l are in meters. The non-linear relationship between the subgrade reaction p and the lateral displacement w has been expressed in different forms by various researchers relating to initial subgrade modulus k_i and the ultimate lateral pressure p_u. A hyperbolic relationship has been proposed recently by (Pender 1993) based on verifications with the field data:

$$w = \frac{p}{k_i}\left(\frac{p_u}{p_u - p}\right)^n \tag{10}$$

and the secant subgrade modulus k_s (= p/w) is then given by:

$$k_s = k_i\left(\frac{p_u - p}{p_u}\right)^n \tag{11}$$

Typical values for the empirical index n are: n = 1 for sand, and n = 0.2 for clay. When n $\rightarrow$ 0.0, the elastic-perfectly plastic behaviour with p_u is given. Typical values for p_u are: sand - 3 times the passive pressure, and clay - $2C_u$ at the ground surface to constant $9C_u$ below a depth of $3.5d_p$.

In a non-linear analysis, the tangential subgrade modulus k_t (= dp/dw) value obtained from the differentiation of Eq. 10 can be used in an incremental procedure. The value of k_t is updated before proceeding to the next load increment, and the procedure is repeated until the given final applied load is reached.

$$k_t = \frac{dp}{dw} = k_i \frac{(p_u - p)^{n+1}}{p_u^n\{np + p_u - p\}} \tag{12}$$

The variations of the subgrade modulus ratios k_s/k_i and k_t/k_i with the subgrade reaction ratio (p/p_u) is shown in Fig. 7 for various values of the index n. Therefore, the subgrade reaction-displacement (p-y curves) relationships can be easily established in an analysis procedure for a given set of soil parameters.

4 TYPICAL EXAMPLE CALCULATIONS

In this section, typical example calculations are presented to show the versatility in the use of the frame analogy simulation to investigate the lateral response of concrete piles, particularly the effect of the cracking of concrete. It is somewhat difficult to find test results or field data with a comprehensive description of the sectional & strength parameters for concrete piles in the literature. The analyses were carried out based on the following unless stated otherwise:

(a) Pile dimensions and details are: pile dia (d_p) = 1000 mm, annular steel ring dia (d_r) = 800 mm, pile length (l) = 15 m, E_c = 27500 MPa, and f_t = 1.0 MPa, and reinforcing steel ratio ρ = .01. The pile properties are determined from the design aids presented in Section 3.1.

(b) Soil is medium dense sand with m = 1.0, k_{max} = 40,000 kN/m^3, and n = 1.0. The ultimate pressure p_u is given by $3K_p\sigma_v'$ where K_p = passive earth pressure coefficient equal to 3, and σ_v' is the effective vertical stress at a given depth.

In order to show the effect of the non-linear behaviour of the pile, two basic types of analyses were carried out:

(i) Elastic pile behaviour as per the conventional analysis (no cracking of the pile occurs), but the soil behaviour is non-linear.
(ii) Non-linear behaviours of both the pile and the soil.

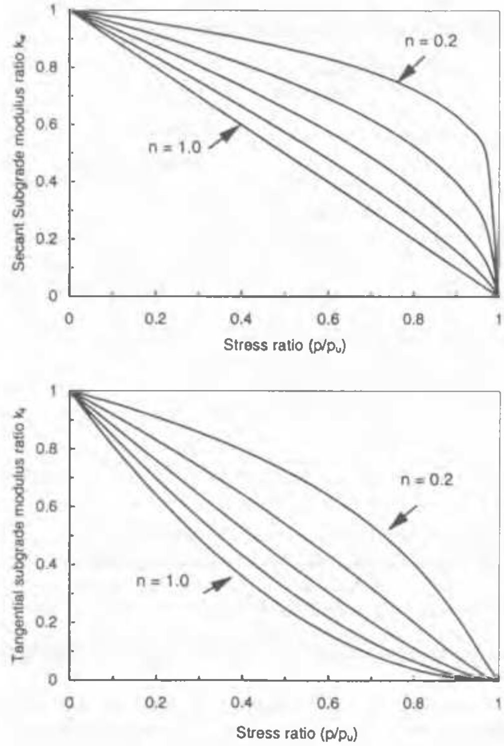

Fig 7. Characteristics of non-linear subgrade reactions.

Example No. 1

The pile is subjected to a flexural moment M of 500 kNm at the pile head. The pile is allowed to rotate at the head, ie, pinned-head.

The distributions of the flexural moment, subgrade reactions (pressures in kN/m^2), and lateral displacements are given in Fig. 8. As can be seen, the non-linearity of the pile due to the cracking of concrete has redistributed both the flexural moment and the subgrade reactions significantly. The non-linear pile behaviour has reduced the peak values by about 35% in this particular case while the lateral displacement at the pile head has been increased by about 100%. Therefore, this aspect should be considered rationally if a reinforced concrete pile has to meet stringent limits on the deflection as a serviceability criterion.

Example No. 2

In this case, the pile is subjected to a lateral (horizontal) force H of 200 kN, and the pile head is assumed to be restrained against rotation, ie, fixed-head.

It is expected that the non-linearity of the pile behaviour would significantly relax the restraining moment developed at the pile head thus relieving the moments exerted on the foundation or the pile cap. Figure 9 shows the distributions of the flexural moment, subgrade reactions (pressures in kN/m^2), and lateral pile displacements along the pile. As can be seen, the pile non-linearity has relaxed the moment induced at the pile head by about 25%, and this relaxation is quite significant for economical design of the pile. In this case, the

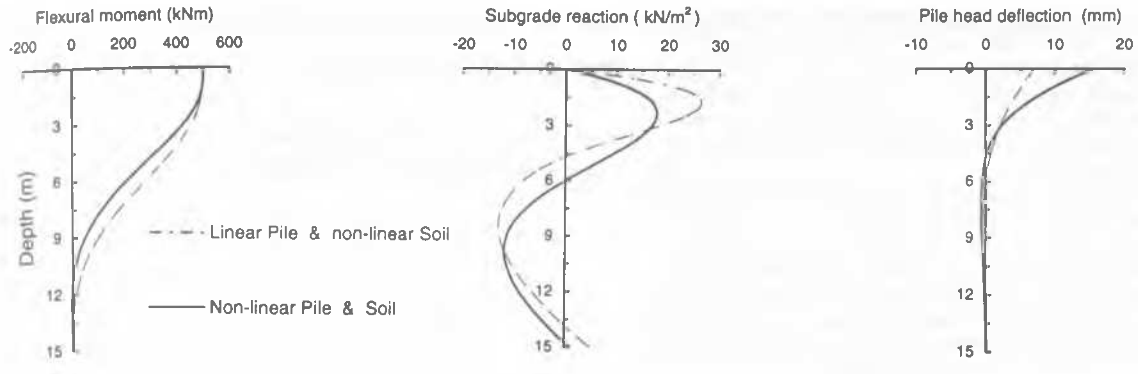

Fig 8. Lateral responses of a free-head pile subject to a flexural moment M = 500 kNm at pile head *(Example No. 1)*.

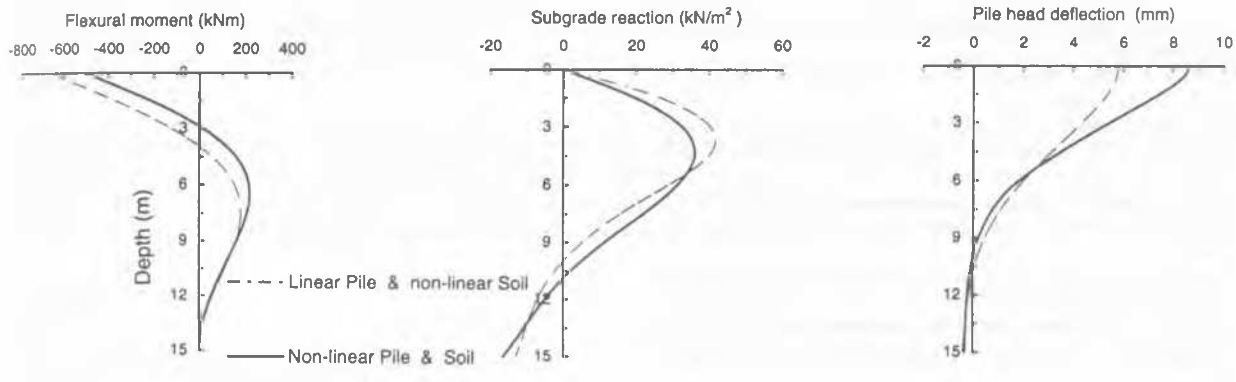

Fig 9. Lateral responses of a fixed-head pile subject to a lateral force H = 200 kN at pile head *(Example No. 2)*.

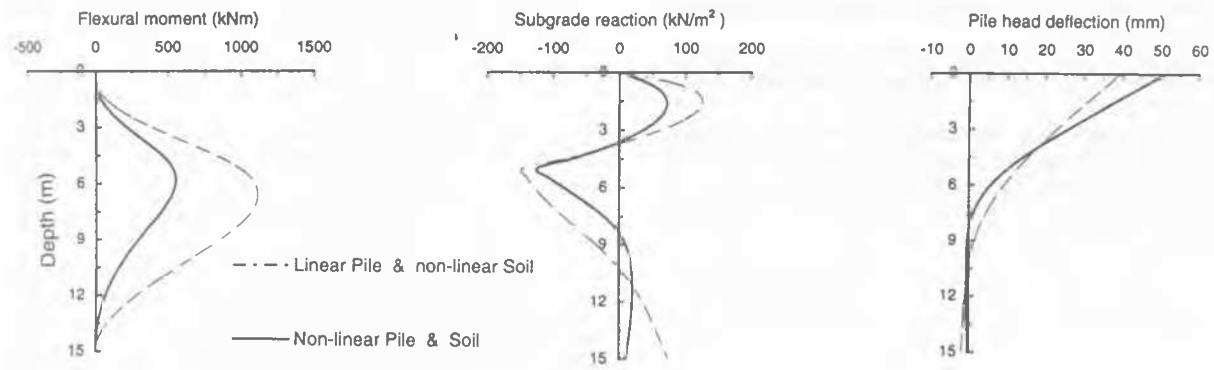

Fig 10. Loadings induced on a free-head pile subject to a triangularly-distributed free-field soil movement *(Example No. 3)*.

displacement at the pile head has been increased by about 50% by the pile non-linearity.

Example No. 3

Piles are subjected to lateral deformations imposed by various structures such as bridge abutments, slopes, and tectonic movements of the top soil strata. By nature, deformation-induced loadings are stiffness-dependent, ie, reduction in stiffness relaxes the magnitude of the induced loadings. Therefore, it can be expected that the cracking of concrete piles with a reduced stiffness would relax the deformation-induced loadings proportionately.

It was assumed that the pile is subjected to a free-field lateral soil movement of 75 mm at the pile head triangularly reducing to zero at a depth of 5 m, ie, within top 30% of the pile length. The pile head is allowed to rotate and translate, ie, free-head condition. Figure 10 shows the variation of the flexural moment and lateral displacement along the pile. As expected, the pile non-linearity has relaxed the deformation-induced loadings significantly: by about 60% in the flexural moment in this particular case. Therefore, the evaluation of deformation-induced loadings on a pile without the rational consideration of the pile non-linearity could result in a gross-overestimation of the induced loadings. Pile non-linearity has increased the lateral displacement being much more flexible compared with a (uncracked) linear elastic concrete pile.

5 CONCLUSIONS

1. Rational prediction of the behaviour of concrete pile foundations is essential in particular as required by of the modern Standards advocating limit state design philosophies.
2. Reinforced concrete piles crack at very low load levels thus resulting in a non-linear pile behaviour. This is significant even at service (working) load levels.
3. Pile-soil system can be easily simulated as a frame analogy

simulation. This simulation has much more flexibility with respect to the handling of various pile and soil details, compared with conventional methods and solutions.

4. The methodology developed in the current study is feasible for routine design office applications. The non-linear analysis is carried out in an incremental fashion incorporating the non-linear behaviours of the pile as well as the soil.

5. As shown by typical examples, the pile non-linearity (cracking of concrete) has redistributed and relaxed the forces and flexural moments significantly under applied loadings.

6. The pile non-linearity has relaxed the deformation-induced loadings caused by the movement of the surrounding soil due to the stiffness dependency of the deformation-induced loadings. This aspect should be considered rationally if the economics of the pile design is to be achieved under imposed deformations.

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