Soil-structure interaction of embedded foundations under seismic loading

Interaction sol-structure des fondations partiellement enterrées sous sollicitation séismique

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ABSTRACT: The problem of soil-structure interaction of rigid foundations embedded in an elastic half space excited by incident seismic waves is analysed. The impedance matrix and the input motion, which are necessary to determine the dynamic response of the foundation, are obtained via BEM. Results are shown for the case of strip foundations.

RESUME: On analyse le problème de l'interaction sol-structure des fondations rigides partiellement enterrées dans le semi-espace élastique soumise à la propagation des ondes sismique. La matrice d'impédance et le "input motion", que sont nécessaries pour déterminer la réponse dynamique de la fondation, sont obtenues par le BEM. Le résultats sont présentés pour le cas de la fondation filante.

1 INTRODUCTION

During the last few decades a significant amount of research activity has been devoted to the area of soil-structure interaction, and as a result a number of methods are available today for solving a variety of problems (Gazetas 1983; Wolf 1985; Manolis & Beskos 1988). Rigorous analytical solutions (Trifunac 1972; Wong & Trifunac 1974) as well as approximate solutions (Iguchi 1984; Todorovska 1993) and numerical techniques, based on FDM FEM or BEM, have been proposed to study the dynamic response of foundations. In particular, BEM has become widely diffused because of its ability to model infinite compliant media and irregular soil profiles, without the use of fictitious adsorbing boundaries at the far field being necessary. Moreover, BEM is in general less costly from a computational point of view, compared to the other numerical techniques, since discretization of only the surface of the domain is required. This numerical technique has been applied to the field of soil-structure interaction by many authors, both in the frequency domain (Dominguez 1978; Sato et al. 1983; Kobayashi & Mori 1986) and in the time domain (Karabalis & Beskos 1984; Spyrakos & Beskos 1986; Antes & von Estorff 1989).

In this paper, a frequency domain analysis is presented to study the dynamic response of embedded rigid foundations of arbitrary shape subjected to the action of both external forces and obliquely incident seismic waves. Following Luco et al. (1975), the dynamic response of the foundation is obtained once the impedance matrix and the input motion for the foundation are known. These latter are calculated numerically by BEM, with relatively low computational costs. The foundation is assumed to be rigid and completely bonded to an elastic half space along the contact surface. It should be noted that the rigid foundation assumption appears to be reasonable in many actual situations involving massive structures and block foundations. For the case of strip foundations, a parametric study is conducted to examine the effect of the embedment depth and the incidence angle of impinging seismic waves on the dynamic response of the foundation. Some results concerning the contact stresses between the foundation and the soil medium are also shown.

2 THE METHOD OF ANALYSIS

Following Luco et al. (1975), the displacement $\mathbf{u}_T$ of a rigid foundation may be represented as the sum of two contributes:

$$\mathbf{u}_T = \mathbf{u}_o + \mathbf{u}_s \quad (1)$$

where $\mathbf{u}_o$ denotes the displacement vector of the foundation under the action of external loads and in absence of seismic excitation, and $\mathbf{u}_s$ indicates the displacement vector of the foundation under the action of the seismic waves and in absence of external forces. These two contributes have been termed as *relative displacement* and *foundation input motion*, respectively. The external forces correspond to the forces and moments that the superstructure and the foundation exert on the soil, in a complete soil-structure interaction problem. Time variation is obtained multiplying the displacements by the factor $e^{i\omega t}$, with ω= frequency of vibration. In Eq.(1) and in those which follow, however, this time factor has been omitted for brevity.

The vector $\mathbf{u}_T$ has, in general, six components corresponding to three translations (U_x, U_y, U_z), two rocking rotations about the horizontal axes (ψ_x, ψ_y), and torsion about the vertical axis (ψ_z), i.e.

$$\mathbf{u}_T = \left[U_x, U_y, U_z, a\psi_x, a\psi_y, a\psi_z\right]^T \quad (2)$$

where a indicates the semi-width of the foundation and the superscript T denotes matrix transposition. In the particular case of a rigid strip foundation subjected to in-plane excitation (incident P, SV or R waves)

$$\mathbf{u}_T = \left[U_x, U_z, a\psi_y\right]^T \quad (3)$$

and in that of a strip foundation undergoing anti-plane motion (SH waves),

$$\mathbf{u}_T = \left[U_y, a\psi_x, a\psi_z\right]^T \quad (4)$$

The displacement vector $\mathbf{u}_T$ can be obtained by solving the equation of motion governing the steady-state response of the foundation that is expressed in the matrix form as

$$\mathbf{F}_o = \omega^2 \mathbf{M}\,\mathbf{u}_T + \mathbf{F} \quad (5)$$

where $\mathbf{M}$=mass matrix of the foundation; $\mathbf{F}_o$=soil-foundation interaction forces, and $\mathbf{F}$=forces that the superstructure exerts on the foundation. For seismic excitation, this latter term can be expressed in terms of the total motion of the foundation through the relation

$$\mathbf{F}=\omega^2 \mathbf{M}_b\, \mathbf{u}_T \quad (6)$$

where $\mathbf{M}_b$ is a frequency-dependent mass matrix which involves the geometry, the inertial and elastic properties of the superstructure (Luco & Wong 1982). The interaction forces $\mathbf{F}$, on the contrary, are expressed in terms of the relative displacement $\mathbf{u}_o$, i.e.

$$\mathbf{F} = \mathbf{K}\mathbf{u}_o = \mathbf{K}\mathbf{u}_T - \mathbf{K}\mathbf{u}_s \tag{7}$$

in which $\mathbf{K}$ denotes the impedance matrix of the foundation that depends on the geometry of the foundation, the elastic properties of the soil and the excitation frequency. In Eq.(5), the effect of the gravity forces is neglected. Finally, substitution of Eqs. (6) and (7) into Eq. (5) leads to the following equation

$$\left[\mathbf{K} - \omega^2\left(\mathbf{M} + \mathbf{M}_b\right)\right]\mathbf{u}_T = \mathbf{K}\mathbf{u}_s \tag{8}$$

As can be noted from Eq. (8), the dynamic response of a rigid foundation can be readily determined once the foundation impedance matrix $\mathbf{K}$ and the foundation input motion vector $\mathbf{u}_s$ are known.

3 IMPEDANCE MATRIX AND FOUNDATION INPUT MOTION

In this study, BEM is employed to calculate both $\mathbf{K}$ and $\mathbf{u}_s$ for a rigid massless foundation embedded in a homogeneous isotropic linearly elastic half space. The matrix form of the frequency domain integral equation (Rizzo et al. 1985) can be expressed as

$$\begin{bmatrix} \mathbf{H}_{ii} & \mathbf{H}_{ih} \\ \mathbf{H}_{hi} & \mathbf{H}_{hh} \end{bmatrix} \begin{bmatrix} \mathbf{u}_i \\ \mathbf{u}_h \end{bmatrix} - \begin{bmatrix} \mathbf{G}_{ii} & \mathbf{G}_{ih} \\ \mathbf{G}_{hi} & \mathbf{G}_{hh} \end{bmatrix} \begin{bmatrix} \mathbf{p}_i \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{w}_i \\ \mathbf{w}_h \end{bmatrix} \tag{9}$$

where $\mathbf{p}$ and $\mathbf{u}$ indicate, respectively, the traction and displacement amplitude vectors at the discretization nodes; $\mathbf{w}$ is the vector containing the nodal displacement amplitudes of the prescribed incident field; $\mathbf{H}$ and $\mathbf{G}$ are influence matrices constituted by integrals over the boundary elements with integrants, respectively, the components of the displacement and traction Green's tensors multiplied by the interpolation functions chosen to represent the variation of $\mathbf{u}$ and $\mathbf{p}$ inside every element. In this study, simple constant elements are used although the use of higher-order boundary elements would be better. It should be noted that, in general, the surface of the domain to be discretized only consists of the soil-foundation contact area and a finite part of the free surface surrounding the foundation. Therefore, in Eq.(9) the subscript i indicates the nodes located on the soil-foundation interface, and the subscript h denotes the nodes on the free surface of the half-space. After some manipulations, Eq.(9) can be put in the form:

$$\overline{\mathbf{H}}_{ii}\,\mathbf{u}_i - \overline{\mathbf{G}}_{ii}\,\mathbf{p}_i = \overline{\mathbf{w}}_i \tag{10}$$

where

$$\begin{aligned} \overline{\mathbf{H}}_{ii} &= \mathbf{H}_{ii} - \mathbf{H}_{ih}\mathbf{H}_{hh}^{-1}\mathbf{H}_{hi} \\ \overline{\mathbf{G}}_{ii} &= \mathbf{G}_{ii} - \mathbf{H}_{ih}\mathbf{H}_{hh}^{-1}\mathbf{G}_{hi} \\ \overline{\mathbf{w}}_i &= \mathbf{w}_i - \mathbf{H}_{ih}\mathbf{H}_{hh}^{-1}\mathbf{w}_h \end{aligned} \tag{11}$$

It is important to point out that when fundamental solutions which automatically satisfy the boundary condition on the free surface of the half space are used, the terms with the subscript h vanish in Eq.(11), and $\mathbf{w}_i$ is the displacement due to incident and reflected waves.

Furthermore, compatibility of displacements at the soil-foundation interface requires that $\mathbf{u}_i$ is related to the rigid motion of the foundation $\mathbf{U}$, by the equation

$$\mathbf{u}_i = \mathbf{T}\,\mathbf{U} \tag{12}$$

where $\mathbf{T}$ represents a rigid-body motion influence matrix. By solving Eq.(10) in terms of $\mathbf{p}_i$, and considering Eq.(12), one has

$$\mathbf{p}_i = \overline{\mathbf{G}}_{ii}^{-1}\overline{\mathbf{H}}_{ii}\mathbf{T}\mathbf{U} - \overline{\mathbf{G}}_{ii}^{-1}\overline{\mathbf{w}}_i \tag{13}$$

Consequently, the external force vector $\mathbf{P}$ is given by the equation

$$\mathbf{P} = \mathbf{T}^T\mathbf{A}\,\mathbf{p}_i \tag{14}$$

where $\mathbf{P}$ contains the resultant forces and moments acting on the soil, and $\mathbf{A}$ is a diagonal matrix whose terms are the areas of the elements.

Finally, multiplying both sides of Eq.(13) by $\mathbf{T}^T\mathbf{A}$ and considering Eq.(14), the following relationship is obtained

$$\mathbf{P} = \mathbf{T}^T\mathbf{A}\overline{\mathbf{G}}_{ii}^{-1}\left[\overline{\mathbf{H}}_{ii}\,\mathbf{T}\mathbf{U} - \overline{\mathbf{w}}_i\right] \tag{15}$$

As suggested by Sato et al. (1983), by setting $\overline{\mathbf{w}}_i = \mathbf{0}$ from this equation, the impedance matrix $\mathbf{K}$ for a rigid massless foundation is derived, i.e.

$$\mathbf{K} = \mathbf{T}^T\,\mathbf{A}\,\overline{\mathbf{G}}_{ii}^{-1}\,\overline{\mathbf{H}}_{ii}\,\mathbf{T} \tag{16}$$

whereas, by setting $\mathbf{P}=\mathbf{0}$ the foundation input motion can be obtained

$$\mathbf{u}_s = \mathbf{K}^{-1}\mathbf{T}^T\,\mathbf{A}\overline{\mathbf{G}}_{ii}^{-1}\,\overline{\mathbf{w}}_i \tag{17}$$

The accuracy of the BEM procedure just expounded has been verified by comparison with other existing solutions. However, due to space limitations, only the results of Figs.1 and 2 are shown. Figure 1 shows the vertical and horizontal impedances of strip foundations versus the dimensionless frequency a_o, as reported by Dobry & Gazetas (1986). Figure 2, on the contrary, shows the imaginary part of the longitudinal driving forces $F_s^*=Ku_s$ for a semi-elliptical foundation, generated by obliquely incident *SH* waves. These latter graphs are drawn from the above mentioned paper of Luco et al. (1975), who used an analytical solution previously derived by Wong and Trifunac (1974). The symbols in the figures represent the results obtained by means of Eqs.(16) and (17) and, as can be seen, they are in close agreement with those obtained by the other authors.

4 APPLICATIONS

The foundation model depicted in Fig.3 is considered to illustrate the effect of the incidence angle of seismic waves on the dynamic response of an embedded foundation. The foundation is two-dimensional, rigid and free from loads exerted by the superstructure. The embedment ratio $\delta=D/a$ is assumed equal to 0.2, 0.5 and 1.0, where D and a are, respectively, the embedment depth and the semi-width of the foundation. Furthermore, Poisson's ratio is equal to 0.2 and the mass density ratio $\rho=\rho_F/\rho_S=1.25$, being ρ_F=mass density of the foundation and

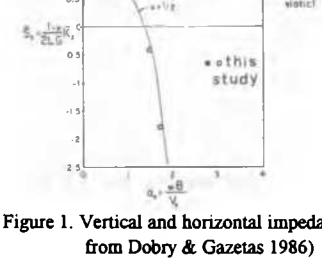

Figure 1. Vertical and horizontal impedances of strip foundations (Adapted from Dobry & Gazetas 1986)

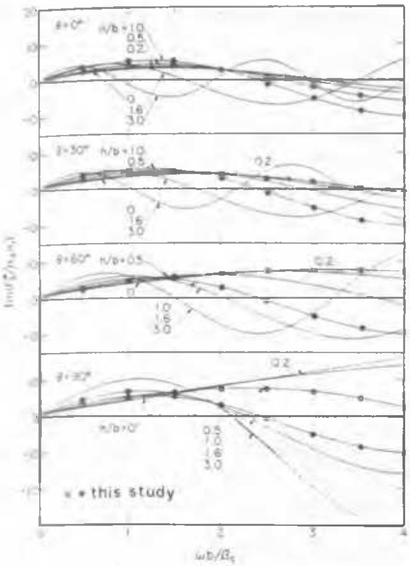

Figure 2. Imaginary part of the longitudinal driving force (Adapted from Luco et al. 1975)

ρ_S=mass density of the soil. The dimensionless frequency $a_o=\omega a/\beta$, which in practice represents the ratio of the foundation width over the wavelength of the shear waves in the soil, ranges between 0 to 6, β is the shear wave velocity. The excitation consists of plane incident waves with unit amplitude and incidence angle γ=0°, 30° and 60°, respectively. The displacement vector $\mathbf{u}_T$ is defined with respect to the barycentre of the foundation, and its components are normalized by the amplitude of the incident waves.

The foundation motion caused by vertically incident *SH* waves is first analyzed. Amplitudes of displacement U_y and rotation ψ_x of a rigid massless foundation for different values of the embedment ratio δ are presented in Figs.4 and 5, respectively. Torsion is, of course, zero in this case. The results show that, due to wave scattering from the foundation, the input motion for a embedded foundation may be very different from the free-field motion. In particular, for the shallower foundations, values of U_y greater than 2, which is the amplification at the free surface of the half space, occur in a wide range of frequency. Moreover, a significant rotation about the *x*-axis is generated. The results of Fig.6 are relative to the same case except for the mass foundation that on the contrary is assumed to be non-zero. As shown in the figure, due to the inertia forces of the foundation, the dynamic response is markedly modified with respect to case of the massless foundation (Fig.4). As can be seen, the displacement exhibits an amplification peak in the frequency band $0<a_o<2.5$, with values of U_y that are much larger than 2. The frequency at which this amplification occurs as well as the magnitude of U_y decrease with the increase of δ. Thus, when the contact area between the foundation and the soil is larger, the loss of energy due to wave radiation outgoing from the foundation is more readily allowed, and as a consequence the foundation motion is more attenuated. For higher frequencies, displacement amplitude rapidly decreases, and for $a_o>3$, i.e. for wavelengths of the order of the foundation width, it reaches a very low value that is practically independent of the embedment depth.

Figures 7 and 8 consider the case of a rigid foundation with δ=0.5 subjected to non-vertically incident *SH* waves. These results indicate that, as the incidence angle increases, the peak value of U_y markedly decreases (Fig.7), whereas the torsional component ψ_z becomes more noticeable (Fig.8). It is interesting to point out that similar trends have been also observed in the case of the dynamic response of rigid foundations undergoing incident *P* and *SV* waves.

Figure 9 shows the distribution of the normal stress along the base of the foundation caused by *P* waves impinging with different incidence angle values. The stress is normalized by the shear modulus of the soil. As can be expected, when a *P* wave vertically propagates, the contact stresses are symmetrical with respect to the vertical axis through the centre of the foundation, and the higher contact stresses are concentrated beneath the edges of the rigid foundation. For obliquely incident waves, the stresses are, of course, non-symmetrical and for γ=60° they exhibit a higher value in correspondence to the edge on the opposite side from which the waves come. Finally, in Figs.10 and 11, the stress distribution is shown for some values of δ, and for γ=0° and 30°, respectively. In these cases, the mass of the foundation is assumed to be the same, so a different portion of the foundations surfaces out of the soil. The results indicate that when γ=0° (Fig.10), apart from a more pronounced concentration at the edge, the stresses at the soil-foundation interface does not in practice depend on the δ value. On the contrary, due to the amount of rocking occurring when waves obliquely propagate, the stress distribution along the contact area may be significantly affected by the embedment depth of the foundation (Fig.11). Obviously, since the foundation model considered here is two-dimensional, the results presented are only significant for foundations with length much larger than width.

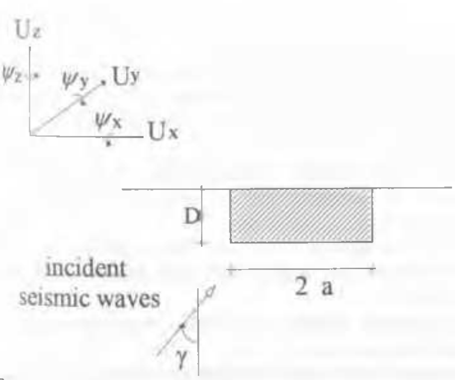

Figure 3. The foundation model

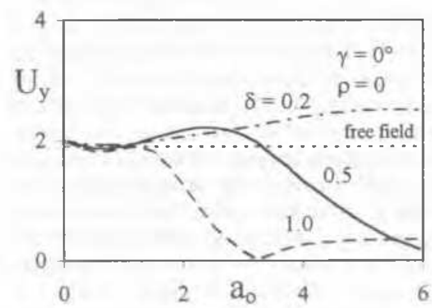

Figure 4. Effect of δ on U_y (ρ = 0)

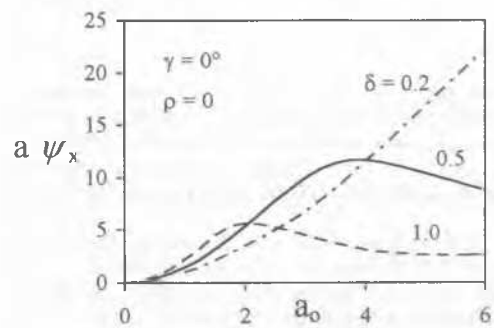

Figure 5. Effect of δ on ψ_x (ρ = 0)

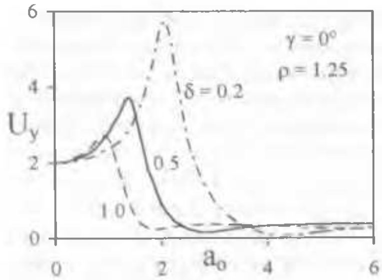

Figure 6. Effect of δ on U_y (ρ = 1.25)

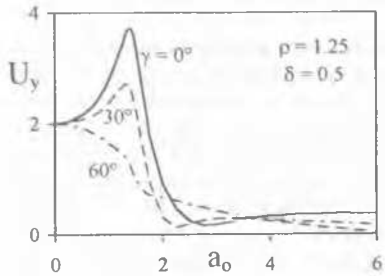

Figure 7. Effect of γ on U_y

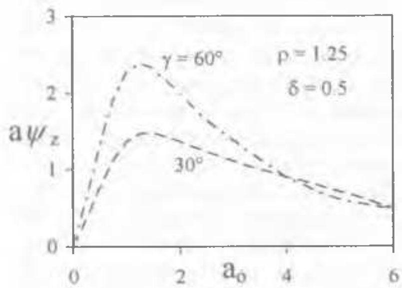

Figure 8 Effect of γ on ψ_z

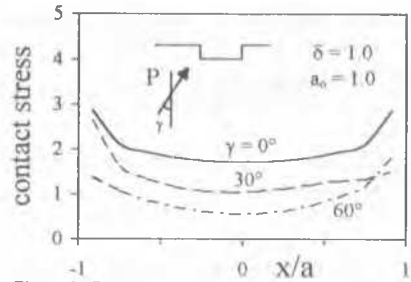

Figure 9. Contact stress distribution due to incident P waves.

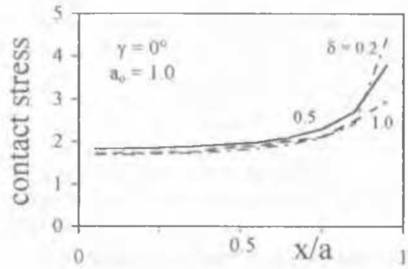

Figure 10. Effect of δ on contact stress (γ = 0°)

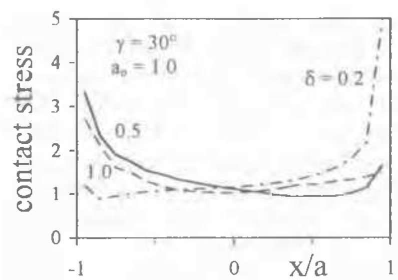

Figure 11 Effect of δ on contact stress (γ =30°)

5 CONCLUDING REMARKS

The problem of soil-structure interaction for a rigid foundation subjected to the action of external forces and incident seismic waves has been analysed. According to Luco et al. (1975), the solution can be readily found once the impedance matrix and the foundation input motion are known. To calculate these latter, a numerical procedure based on BEM has been set up. The results obtained show that the translational response of a rigid foundation to vertically incident waves increases with frequency from the free-field amplitude to a maximum value and then rapidly decreases. The maximum value is notable when the foundation is shallower. Obliquely incident seismic waves cause a significant decrease in the translation response, but may generate a considerable amount of torsion, when incident *SH* waves are considered, or rocking in the case of incident *P* or *SV* waves. Moreover, the angle of incidence of the seismic waves may significantly affect both magnitude and distribution of the stresses at the soil-foundation interface.

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