Soil improvement by multi-stage preloading – A method of analysis

Amélioration des sols par pré-chargement en étapes – Une méthode d'analyse

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ABSTRACT :. Asaoka's observational procedure is the best available method for predicting settlement evolution over time once previous readings are known. In this paper, a new method is proposed, which is useful for more general conditions of loading. Load is applied by increments, in steps. The application of the present method provides an estimation of the settlement components and of the coefficient of consolidation for each step, allowing for assessment of the improvement of soil with applied load. Furthermore, the method permits us to identify the initial movements of the instrumentation, which are not related to soil deformation, and which are due to impacts produced by the machinery used in the application of load. An application of the method to a practical case is presented.

RESUME : La méthode graphique proposée par Asaoka permet la prediction des tassements et leur évolution dans le temps. Cet article analyse une méthode pour conditions de chargement plus générales que celles analysées par Asaoka. La méthode ici developpée permet d'obtenir les components du tassement pour chaque palier de charge dans un chargement par étapes et d'évaluer la variation du coefficient de consolidation dans chaque étape. L'exploitation de la méthode permet aussi de déterminer les tassements initiels produits par d'autres causes comme l'action des équipements pendant l'application de la charge. On presénte une application pratique de la méthode.

1 SETTLEMENT ANALYSIS. ONE-STEP LOADING

Let us suppose that the process of loading is done in only one step. The load is increased linearly with time until it reaches its maximun value q_f, at time t_c. (Figure 1)
Afterwards, the applied load remains constant. At time t, the settlement would be:

$$s(t) = s_o + s_e\, t/t_c + s_c\, U_1(t,t_c) \quad (t<t_c) \tag{1a}$$

$$s(t) = s_o + s_e + s_c\, U_2(t,t_c) \quad (t>t_c) \tag{1b}$$

where: s_o is some kind of initial or previous settlement due to causes other than a change in the effective stresses such as induced movement by machinery. S_e is an "elastic" settlement which is proportional to the maximum applied load q_f. S_c is a "consolidation" settlement which is produced by dissipation of pore pressure. $U_{1,2}(t,t_c)$ are time functions that describe the settlement evolution over time at different periods, and t_c is the construction time.

For edometric conditions, both functions $U_{1,2}$, can be obtained by integration of the non-homogeneous equation of consolidation. This approach will be considered in this paper. An analytical expression for $U_{1,2}$ is provided by Olson (1977).(eq.2)

$$U(T,T_c) = \frac{T}{T_c}\left[1 - \frac{2}{T_c}\sum_{m=0}^{m=\infty}\frac{1}{M^4}\left[1 - e^{-M^2T}\right]\right] \quad T \le T_c$$
$$U(T,T_c) = 1 - \frac{2}{T_c}\sum_{m=0}^{m=\infty}\frac{1}{M^4}\left[e^{M^2T_c} - 1\right]e^{-M^2T} \quad T \ge T_c \tag{2}$$

Where $M=(2m+1)\pi/2$; $T=Ct$; $T_c=C\,t_c$; $C=c_v/H^2$.
c_v= Coefficient of consolidation; H= drainage length.

The Olson solution, which is expressed by series, can be managed easily by taking only the first term of the series. In this case, for $t>t_c$, the settlement can be obtained through the expression:

$$s(t) = S_F - s_c\ \beta(t_c)\, e^{-\lambda t} \tag{3}$$

where $S_F = s_o + s_e + s_c$ and $\lambda = \pi^2 C/4$

$$\beta(t_c) = \left[e^{\lambda t_c} - 1\right]\frac{8}{\pi^2 t_c \lambda} \tag{4}$$

For $t_c=0$, $\beta(0) = 8/\pi^2$, and the well-known solution of Terzaghi is obtained. S(t) is solution of the first order differential equation: (eq.5)

$$\frac{ds}{dt} = \lambda(\, S_F - s(t)\,) \tag{5}$$

Equation (5) is similar to the one used by Asaoka in his method. The next first order difference equations are obtained by discretizing in time equations (3) and (5).
From the differential equation (eq.3):

$$s_t = \frac{s_{t-1}}{1+\lambda\,\Delta t} + S_F\,\frac{\lambda\,\Delta t}{1+\lambda\,\Delta t} \tag{6}$$

From the solution s(t) (eq.5):

$$s_t = \frac{1}{\mu}\, s_{t-1} + S_F\,\frac{\mu - 1}{\mu} \tag{7}$$

where s_t=s(t), s_{t-1}=s(t-Δt), μ= exp (λ Δt)

Both expressions imply a linear relationship between s_t and s_{t-1} and this fact was used by Asaoka in order to obtain λ and S_F for the case t_c=0. Equations (6) and (7) show that the method is also useful for a single step in loading, as was pointed out by Asaoka. Magnan & Deroy (1980) analyzed the graphic method of Asaoka and concluded its great practical value.

Once S_F is obtained, the solution s(t) provides a linear relationship between $Ln(S_F-s(t))$ and t according to equation (3), which allows us to determine s_c and, afterwards, s_e.

The representation of s_t versus s_{t-1} shows an ultimate zone of the graph with a different slope, which has been atributted to creep. The expression for creep settlement is:

$$s(t) = S_T - A_L\, e^{-\lambda_f t} \tag{8a}$$

which can be expressed as:

$$s(t) - S_F = s_L\left[1 - e^{-\lambda_l(t-t_{of})}\right] \quad (8b)$$

S_T being the total or final settlement . $s_L=S_T-S_F$ is the creep settlement and t_{of} is a reference time. Obviously:

$$A_L = S_L e^{\lambda_l t_{of}} \quad (8c)$$

2 MULTI-STAGE LOADING:

Let us assume a process of loading consisting of N steps, each one with a duration $t_{c,k}$ and beginning at times $t_{o,k}$ ($t_{o,1}=0$).(Figure 2)

For the period n, and for $t_{on} + t_{cn} < t < t_{o,n+1}$, the settlement $s^n(t)$ can be obtained by adding the contributions of the previous periods. The linear character of the solution allows us to express the settlement as:(eq.9)

$$s^n(t) = s_o + \sum_{k=1}^{k=n} s_{e,k} + \sum_{k=1}^{k=n} s_{c,k}\left(1 - \beta_k e^{-\lambda_k(t-t_{o,k})}\right) \quad (9)$$

which can be expressed as:

$$s^n(t) = s_{F,n} - \sum_{k=1}^{k=n} A_k e^{-\lambda_k t} \quad (10)$$

$$A_k = s_{c,k}\beta_k e^{\lambda_k t_{o,k}} \quad (11)$$

$$s_{F,n} = s_o + \sum_{k=1}^{k=n} s_{e,k} + s_{c,k} \quad (12)$$

It has been assumed that the parameter λ is different in each period, which supposes that the coefficient of consolidation can change for each period due to the soil improvement caused by loading.

For each period n, $s^n(t)$ is the solution of the first order differential equation:

$$s^n(t) + \frac{1}{\Omega_n}\frac{ds^n(t)}{dt} = s_{F,n} + \frac{1}{\Omega_n}F^n(t) \quad (12)$$

$$F^n(t) = \sum_{k=1}^{k=n} A_k(\Omega_n - \lambda_k)e^{-\lambda_k t} \qquad \Omega_n = \sum_{k=1}^{k=n} \lambda_k \quad (13)$$

Equation (12) is similar to the equation obtained for the one-step case and analyzed in the previous paragraph, but with an independent term which is function of time. To guarantee similar behaviour, the independent term should be smaller than a fixed value ε:

$$\left|\sum_{k=1}^{k=n} \frac{A_k(\Omega - \lambda_k)}{\Omega} e^{-\lambda_k t}\right| < \sum_{k=1}^{k=n} A_k e^{-\lambda_k t} \le$$
$$\le S_F - s(t) \le n\overline{A}\ e^{-\Omega T} \le \varepsilon \quad (14)$$

where $A = \max(A_k)$ and T is an upper bound of time until which the previous method can be used. For large values of t, s(t) tends to S_F and the linear relation between s_t and s_{t-1} is guaranteed.If λ_k is constant, the solution to the problem is (Celma, 1994)

$$s^n(t) = S_{F,n} - \sum_{k=1}^{k=n} A_k e^{-\lambda t} = S_{F,n} - B_n e^{-\lambda t} \quad (15)$$

with $B_n = \sum_{k=1}^{k=n} A_k$ and $A_k = B_k - B_{k-1}$.

In this case, the equation is similar to that obtained for one-step loading (eq. 8) and ,for each period, values of B_n, $S_{F,n}$, λ, and A_n can be obtained in a recursive process.

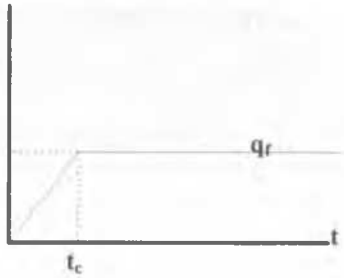

Figure 1. Single-step loading

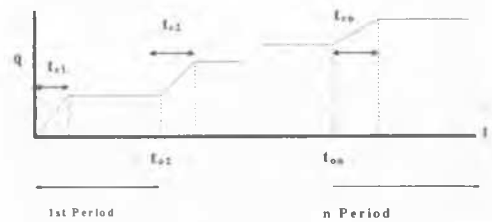

Figure 2.- Multi-step loading

3 INTERPRETATION OF RESULTS:

The interpretation of a set of measurements is difficult for the general case and an approximate method is proposed.

For the first period, n=1, the simple relation between s(t) and t allows us to obtain $S_{F,1}$, A_1 and λ_1 through the plots of s_t versus s_{t-1} and $Ln[S_F-s(t)]$ versus t.

For the second period the expression of settlement would be:

$$s(t) = S_{F,2} - A_1 e^{-\lambda_1 t} - A_2 e^{-\lambda_2 t} \quad (16)$$

where A_2 and λ_2 are unknowns. The previous expression can be transformed into:

$$\rho_2(t) = S_{F,2} - A_2 e^{-\lambda_2 t} \quad (17)$$

where $\rho_2(t) = s(t) + A_1 e^{-\lambda_1 t}$.

This new expression allows us to use the general procedure to obtain $S_{F,2}$, λ_2 and A_2. The process is repeated until period n

For each period, $s_{c,n} + s_{e,n} = S_{F,n} - S_{F,n-1}$, with $S_{F,0}=s_o$ and:

$$s_{c,n} = \frac{A_n}{\beta_n} e^{-\lambda_n t_{o,n}} \quad (18)$$

The available data between steps is limited in most cases. The capabilities of the method can be reduced for lack of data. Only the last period has enough information to obtain reliable results An alternative method for this case is proposed in the following steps:

1. Assume a constant value of λ, coincident to the nth period, and represent s_t versus s_{t-1}, which allows us to obtain the values of $S_{F,n}$.
2. Represent $S_{F,n}$-s(t) versus t, in a semilogarithmic plot, to obtain λ_n and B_n. The values of A_n and $s_{c,n}$ can be obtained in a recursive way.
3. Once λ_1 and A_1 are obtained, $S_{F,2}$ - $\rho_2(t)$ is represented versus t in a semilogarithmic plot, which provides new values of A_2 and λ_2. The process is continued until period n.
4. With the new values of λ_k obtained for each period, and assuming a linear relationship between s_t and s_{t-1}, recalculate the values of $S_{F,n}$ and repeat the process until convergence is assured.

4 APPLICATION.

A circular trial embankment ,4.20 m high and 23 m in diameter,

TABLE I. Calculations for cells 1-5

	Cell1	(*)	Cell2	(*)	Cell3	(*)	Cell4	(*)	Cell5	(*)
SF1		82,88	137,35	139,55	165,2	166,43	100,06	102,59	192,13	192,81
SF2	112,48	111,74	167,66	167,03	199,9	199,47	124,28	123,22	210,78	210,64
SF3	171,44		227,54		249,8		181,41		244,98	
A1	35,16		45,27		21,77		44,26		18,74	
A2	161,2	173,78	194,57	198,34	117,23	118,17	115,23	116,74	222,01	224
A3	1474	1550	2203	2378	638,4	644,45	720,57	730,18	1146	1155
Sc1	32,42	32,42	41,14		20,95		41,07		14,79	
Sc2	33,18	32,12	35,85	33,84	30,64	29,59	30,21	27,76	25,36	24,74
Sc3	57,23	54,53	61,24	58,58	46,49	48,17	64,22	66,15	23,31	24,01
So	39,94	41,55	83,58	85,91	140,24	140,99	49,33	50,87	169	170,42
λ1	5,30E-02	5,30E-02	5,55E-02		4,55E-02		4,54E-02		6,96E-02	
λ2	5,90E-02	6,25E-02	6,76E-02	6,51E-02	5,12E-02	5,26E-02	5,11E-02	5,43E-02	7,81E-02	7,92E-02
λ3	6,36E-02	6,54E-02	6,97E-02	7,19E-02	5,21E-02	5,24E-02	4,84E-02	4,81E-02	7,54E-02	7,50E-02
λ	4,47E-03		4,29E-03		3,30E-03		2,87E-03		8,47E-03	
AL	58,73		59,14		60,34		59,62		57,22	
ST	210,38		269,22		293,85		223,92		268,87	
	(*)	2nd.Iteration								

Symbols used in Table I

SF 1,2,3 : S_{fk} : Estimated final settlement for stages 1,2,3 (mm)

Sc1,2,3 : S_{ck} : Estimated consolidation settlement dor stages 1,2,3 (mm)

A1,2,3 A_k Coefficients defined in eq. 15

λ 1,2,3 : λ_k Coefficients related with the coefficient of consolidation for each stage (eq 3)

λ ,AL : $=\lambda_f, A_L$. Creep coefficients (eq. 8)

ST : Estimated total settlement (mm)

So : "Initial" settlement (mm)

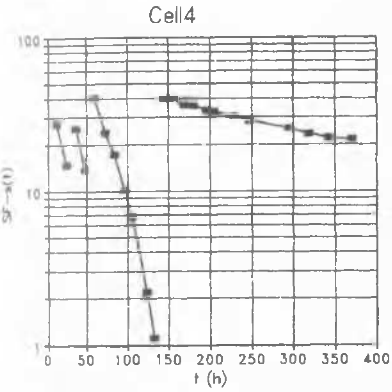

Figure 3. Proposed construction for cell 4.

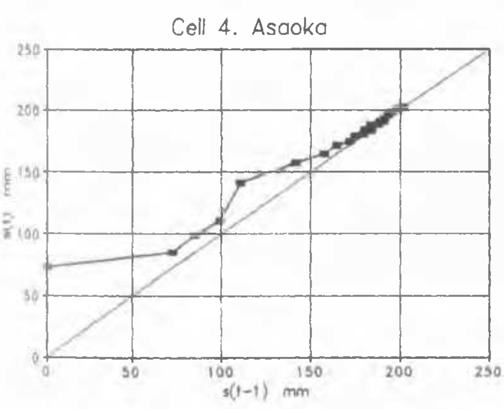

Figure 4 : Asaoka's graphical construction for cell 4

was constructed at an area gained to the sea through a hydraulic fill. The fill has a depth of 14 m and overlies old alluvial sediments. The objective of the trial embankment was to obtain an estimation of the soil response from in situ observations in order to evaluate the expected settlements produced by the construction of storage tanks. Four hydraulic overflow settlement gauges (H.O.S.G) were located at the base of the embankment, one at the center and three at the edge of the loaded area Another cell was placed 2m bellow ground surface, under the center of embankment and it was attached to the cap of a multiple rod extensometer with 4 rod anchors.

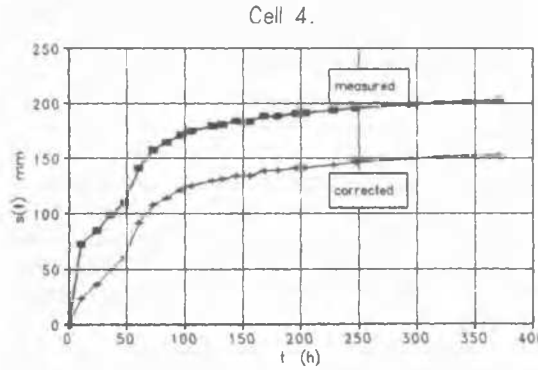

Figure 5 : Measured and corrected settlements

Loading was done in three steps, each one lasting 24 hours. Each step had a construction period (t_c) of 12 hours except for the first step which took 10.5 hours.

Readings were taken every 12 h except for the first period. Only two readings where taken during periods 1 and 2.

At the first reading, an anomalous excessive settlement was observed in all cells, which is attributable to the effect of concentrated loads transmitted by compaction machinery to the shallow cells . Because of this effect, an initial settlement s_o ,of unknown value, should be determined in order to correct the readings.

The following relations can be evaluated for each period:

Period 1: $s_1=\mu\, s_o + S_{F,1}\,(1-\mu)$; $s_2=\mu\, s_1 + S_{F,1}\,(1-\mu)$

Period 2: $s_4=\mu\, s_3 + S_{F,2}\,(1-\mu)$

Period 3: $s_k=\mu\, s_{k-1} + S_{F,3}\,(1-\mu)$: k>6; $\mu = e^{-\lambda \Delta t}$

For each cell, the previously stated methodology is applied and the results are shown in Table I. The initial and the first iteration are presented. As can be observed, the process converges quickly and the error assumed in taking only the first iteration is small, which is due to the similar values of λ in each period.

Figures 3 to 5 show the plots used in the method and the original and corrected measurements.

5 CONCLUSIONS

An analysis of the evolution of settlement over time under loaded areas is presented which improves Asaoka's observational method. The method presented permits the consideration of multi-step loading, the determination of the settlement components in every loading period and the corresponding coefficient of consolidation, which varies owing to the soil improvement caused by loading. Asaoka's method appears as a particular case of the current method.

The method has been applied to a practical case, a circular trial embankment over a hydraulic fill, in order to determine the nature and magnitude of an anomalous component of settlement, which is due to the action of machinery on the shallow settlement gauges during the process of application of load.

The settlement of a point under a loaded area can be calculated by different methods. Each method uses a model of soil behaviour characterized by soil properties that should be known in advance. The objective of calculations is to predict both the evolution of settlement over time as well as its magnitude. In situ measurement of settlements allows us to contrast the quality of the model and to predict the future evolution of settlement on the basis of previous readings.

Asaoka's method is based in the analysis of the consolidation equation under edometric conditions and assumes a uniform load which has been applied instantaneously. The application of this method provides the magnitude of final settlement, the creep settlement and the coefficient of consolidation Settlement is considered to be composed of an elastic settlement, which is proportional to actual load, a consolidation settlement and a creep settlement. Asaoka's method appears as a particular application of the proposed method. It should be pointed out that, in spite of these restricted previous hipotheses, Asaoka recommends the use of his method for more general conditions, although it was not analyzed in his paper

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