INTERNATIONAL SOCIETY FOR SOIL MECHANICS AND GEOTECHNICAL ENGINEERING

Some aspects of numerical analysis of piled rafts

Quelques aspects de l'analyse numériques des radiers sur pieux

G. Russo & C. Viggiani – *University of Napoli Federico II, Italy*

SUMMARY: The design of a piled foundation according to the modern settlement based approach requires the availability of methods for the analysis of the interaction among the soil, the piles and the raft. The complete BEM solutions are possible only for relatively simple problems; their accuracy as benchmarks for simplified procedures is discussed. Two recently developed simplified algorythms (Clancy, 1993; Russo, 1995) are employed to assess the significance of a proper modelling of the subsoil. One of the above methods is finally employed to predict the results of centrifuge model tests.

RESUME: Le projet des fondations sur pieux, basé sur la moderne approche qui tien en compte les tassements, exige l' utilisation d' une fiable methode d'analyse de l'interaction entre le sol, les pieux et le radier. Une solution rigoureuse avec BEM est possible seulement pour des problèmes relativement simples; on discute sa validité comme solution de référence pour méthodes de calcul simplifiée. Deux algorithmes récemment mises au point (Clancy, 1993; Russo,1995) sont utilisés pour évaluer l'importance d'une modelisation appropriée du sous-sol. Un des deux méthodes est adoptée pour enterpreter les resultats d' essais sur modèle en centrifuge.

1. INTRODUCTION

There are two reasons for taking the decision to use piles instead of a spread foundation (Burland, 1995): (i) there is the risk of a bearing capacity failure, or (ii) the settlements are deemed to be too large. In the former case (foundations for offshore platforms being an obvious example) piles are required purely from a capacity point of view, and the load-settlement behaviour is of lesser concern. In most onshore cases, on the contrary, the main purpose of the piles is to limit settlement.

Conventional methods of pile group design are essentially capacity based and do not distinguish between these two fundamentally different reasons for using piles. On the other hand, a variety of analytical and numerical techniques for the analysis of the interaction of a piled raft and the subsoil have been developed in recent years, and a number of powerful computer programs are becoming available. Extensive reviews of the latest development have been provided by Burland *et al.* (1977), Poulos (1989), Randolph (1994), Burland (1995), Mandolini & Viggiani (1996).

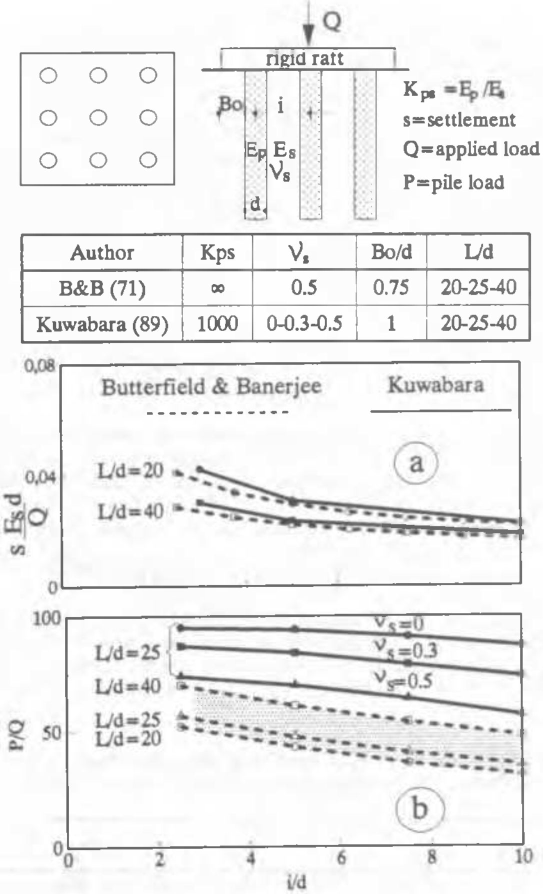
The choice of the most suited method for a given problem, the determination of the numerical values of the soil properties, the modelling of the subsoil, the assessment of the boundary conditions still require a considerable amount of judgement. Some of the factors to be considered are discussed in the present paper.

2. BENCHMARKS

The most widespread technique for the analysis of the interaction between piled rafts and soil is provided by the Boundary Element Method (BEM) (Poulos & Davis, 1980; Banerjee & Butterfield, 1981). The interface between the soil and the foundation (piles and pile cap or raft) is divided into elements and an appropriate Green function, such as that due to Mindlin, is used to relate the displacement of each element to the traction on the other elements. Corresponding equations are written for the structural response of the foundation, using either a finite difference or a finite element approach. The two sets of equations, together with those for overall equilibrium, allow the unknown tractions to be found, and hence the settlement and load distribution throughout the foundation to be evaluated.

In practice, the computational resources required to perform the ideal analysis described above become excessive for all but the simplest foundation systems. For the analysis of a practical engineering problem, and particularly for the simulation of non linearity by a stepwise linear incremental procedure, it is necessary to introduce simplifications.

The few available complete BEM solutions have been used as benchmarks to assess the accuracy of simplified methods. Unfortunately, the available computer programs are probably too rough for such a purpose. To illustrate this point, some results of a complete BEM analysis of a raft on piles, carried out by Butterfield & Banerjee (1971) and Kuwabara (1989), are reported in fig. 1.



Author	Kps	ν_s	Bo/d	L/d
B&B (71)	∞	0.5	0.75	20-25-40
Kuwabara (89)	1000	0-0.3-0.5	1	20-25-40

Fig. 1. Complete BEM solutions. a) settlement; b) load P taken by the piles as a percentage of the total load Q

Both analyses refer to an uniformly loaded infinitely rigid raft resting on 9 compressible piles of varying length and spacing, embedded in a homogeneous elastic half space. Some of the parameters (Poisson's ratio of the soil ν_s; slenderness L/d of the piles; relative stiffness $k_{ps} = E_p/E_s$; B_o/d) are given slightly different values in the two analyses.

In terms of settlement s (fig. 1a) the agreement between the two analyses is good, though not perfect. In terms of load sharing between the piles and the raft (fig. 1b), on the contrary, a substantial disagreement occurs, in spite of the differences in the values of parameters that should influence the results in the opposite direction.

An overview of the literature confirms that, despite the use of supposedly "rigorous" numerical techniques, the results in terms of load sharing between piles and raft and load distribution among piles tend to be rather more dispersed than those in terms of settlement. According to Randolph (1994), for a number of piles exceeding 100 the accuracy of available computer programs is probably not better than ± 20%. While this is probably sufficient for most engineering purposes, it seems to indicate a limit to the extent to which it is fruitful to conduct "rigorous" analyses of piled rafts.

In this paper some simplified numerical procedures will be adopted; the results obtained and the conclusions drawn are believed to be significant, at least in a semi qualitative sense.

3. SUBSOIL MODEL

3.1. Problem and method of analysis

The evaluation of the properties of the subsoil is the most uncertain step of the analysis for almost all problems in geotechnical engineering; in the case of piled foundations, however, the difficulties are increased by the significant influence of the installation of the piles.

Some Authors (e.g.: Mandolini & Viggiani, 1992, 1996) suggest to overcome such a difficulty, in the analysis of a real case, backfiguring the deformation properties of the foundation soil by the axial stiffness of a single pile, as obtained for instance in a full scale load test. It could be argued that, once a load test has been carried out, no further data are needed and the behaviour of the whole foundation may be deduced only by analysis.

Such a conclusion could induce engineers to limit the site investigation to a load test on a prototype pile. This would be a very dangerous practice, as already pointed out by Terzaghi & Peck (1948, art. 56) in their classical treatise. To further exploring the subject, an uniformly loaded square raft of finite stiffness resting on a square group of 36 piles embedded in an elastic layer of thickness H (fig. 2) has been studied by means of two simplified methods of analysis. The parameters considered are: the length L of the piles; the stiffness of the raft; the depth H of the rigid base. The input parameter is supposed to be the axial stiffness of a single pile; the elastic properties of the subsoil are backfigured from this value. Of course, given a value of the single pile stiffness, different values of E_s are obtained for different values of H; the larger H, the larger is E_s to match the same pile stiffness.

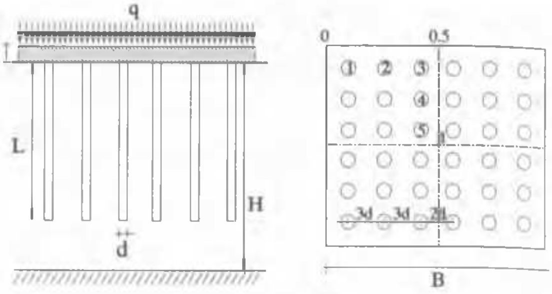
The axial stiffness of single piles of different length embedded in an elastic half space has been adopted as basic value; the values of the Young modulus of the soil E_s back calculated from this stiffness for different values of H are listed in table 1. The BEM program SINGPALO (Mandolini & Viggiani, 1996) has been used for these computations.

The analyses of the piled raft have been carried out by the following algorythms.

- HyPR (Clancy, 1993; Randolph & Clancy, 1993). This method is based on the hybrid approach proposed by Chow (1986). The subsoil is modelled as a homogeneous elastic layer of finite thickness H. A load transfer approach (Randolph & Wroth, 1978) is used to express the relation between local traction and displacement of each pile, together with an elastic continuum analysis of the additional displacements due to the tractions acting on other elements. HyPR takes into account to some extent the non linear behaviour of the piles with a load cut-off for the elements where pile-soil slip is computed to occur, using an incremental elastic analysis.

Table 1. Values of E_s (MPa) back calculated from a given stiffness of the single pile and different subsoil models

L (m)	H/L 1,5	2	∞
10	144	158	200
20	155	168	200
40	165	175	200



B=18m; d=1m; L=10-20-40m; t=1-4m; L/H=0-0,5-0,66

E_p=40000 MPa ν_p=0.15

Fig. 2. Geometry of the problem and values of the parameters

- NAPRA (Russo, 1995). In this method the interaction factors are used to represent the influence of a whole pile on the displacement of another pile or an element. The soil is modelled as an elastic horizontally stratified layer and the raft as a thin plate. As suggested by Caputo & Viggiani (1984), non linearity is concentrated in the load-settlement response of the single pile while the interactions are considered to be linear. Another form of non linearity is accounted by a no-tension algorythm at the raft-soil interface.

The main limits of HyPR are the assumption of homogeneous subsoil, and the possibility of tensile forces developing at the raft-soil interface. The former factor exerts no influence in the schematic case considered here. NAPRA has significant advantages from a computational viewpoint, thus removing most practical limits to the analysis of engineering problems.

3.2. Discussion of the results

The results obtained are reported in figs. 3 and 4. The two approaches predict substantially similar trends for both the load distribution among piles and raft and the absolute and differential settlement. The agreement is better for settlement than for load distribution and for relatively rigid than for relatively flexible rafts. Largest discrepancies occur for the cases in which HyPR analysis results in a significant portion of the raft-soil interface being subject to tensile stress. In these cases NAPRA iterates the solution till such stress disappears; this is the main reason of the differences between the two sets of results, especially in terms of load distribution. Further discussion will be based essentially on NAPRA results.

The influence of the subsoil model is very significant in the prediction of the absolute settlement of the raft. Fig. 3a reports the values of the group settlement ratio $R_s = s_g/s_s$ (s_g = average settlement of the group; s_s = settlement of the single pile at the same average load). The data refer to the case of t = 1 m (relatively flexible raft); the corresponding results for a relatively rigid raft (t = 4 m) are practically equal. It can be seen that the amplification of the settlement of the single pile by group action can vary by a factor as high as 3 when the subsoil model changes from a half space (L/H=0) to a relatively thin layer (L/H=0,66).

The differential settlement, espressed as a percentage of the average settlement (fig. 3b), is obviously influenced primarily by the stiffness of the raft. For a given stiffness, however, the influence of the subsoil model does not seem to be very significant.

The load sharing between the raft and the piles (fig. 3c) depends essentially on the pile length; the longer the piles, the higher being the load they take. For a given pile length, however, the influence of the subsoil model is very small. Also in this case the influence of the stiffness of the raft is negligible; the data shown refer to t = 1 m.

The load distribution among the piles (fig. 4) is obviously influenced by the raft stiffness, in the sense that the load concentration on peripheral piles is higher the stiffer the raft.

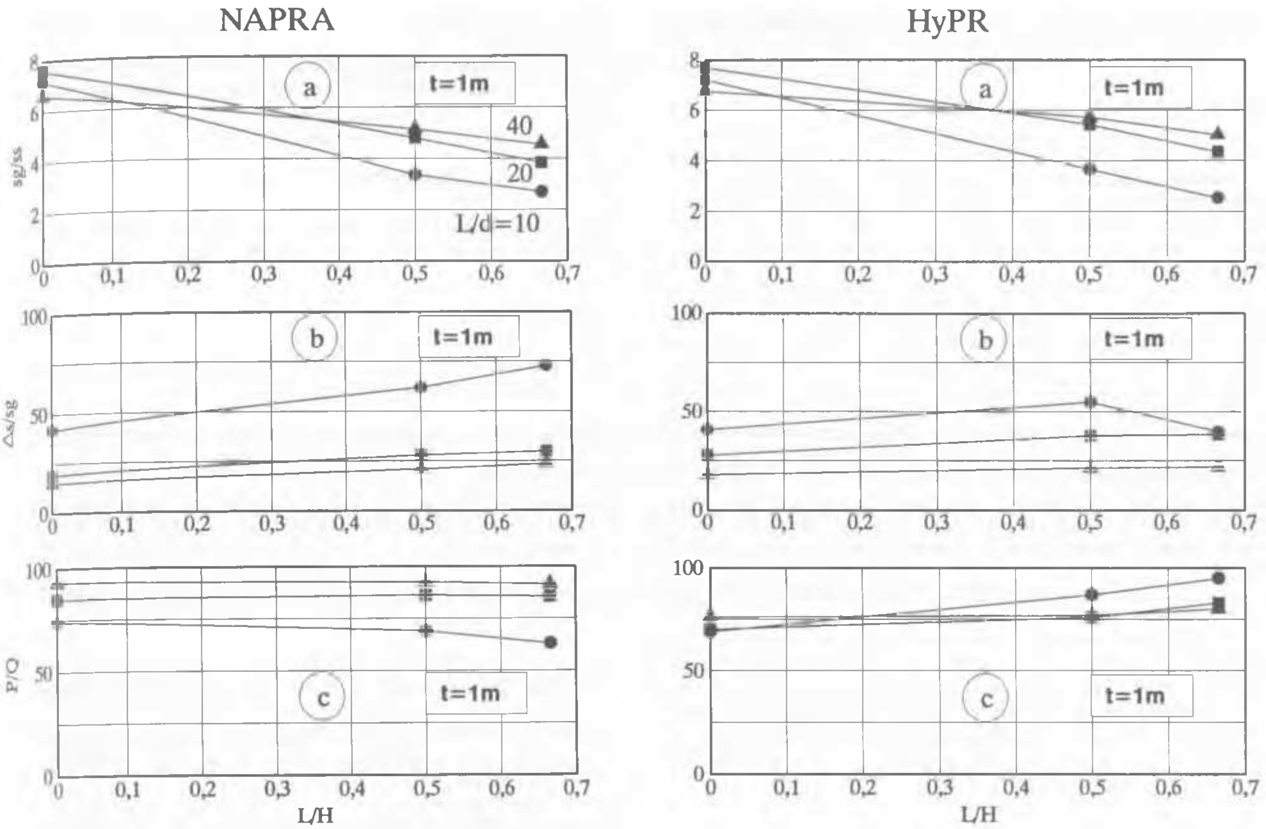


Fig. 3. Comparison of the results of the analyses carried out by NAPRA and HyPR. a) group settlement ratio; b) differential settlement Δs as a percentage of the average settlement s_g; c) load P taken by the piles as a percentage of the total load Q

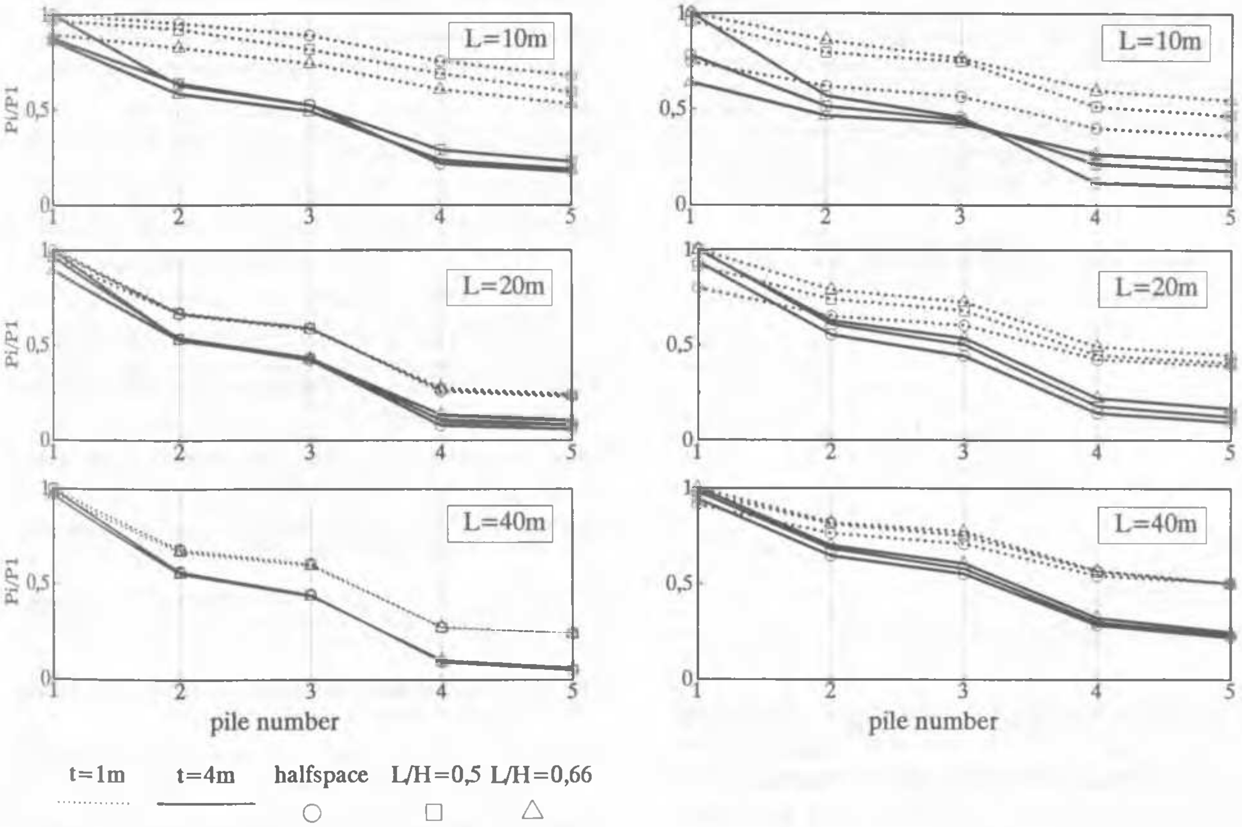


Fig. 4. Comparison of the results of the analyses carried out by NAPRA and HyPR. Distribution of the load among the piles. (Pi = load on the pile number i)

There is a slight influence of the subsoil model only for the shortest piles (L=10 m) and for the relatively flexible raft (t=1 m).

4. ANALYSIS OF A CENTRIFUGE TEST

Thaher & Jessberger (1991) carried out a series of centrifuge tests on a square raft, both with and without piles, resting on a saturated reconstituted kaolin with an undrained shear strength s_u = 200 kPa. They measured settlement, pile loads and contact pressure, and explored the influence of pile number, pile length and pile diameter.

For the scope of the present paper, NAPRA has been used to back analyse some of the results, namely those concerning a raft on eight piles of three different lengths. The relevant data are reported in fig.5. The Young's modulus of the clay has been obtained by fitting the settlement of the unpiled raft to the solution given by NAPRA. A value of E_s = 9,900 kPa was thus obtained. A lower bound of the limiting shaft friction may be evaluated in 180 kPa; the safety factor of the single piles, taking into account the base resistance, is as low as 1.1 ÷ 1.7. The back analysis by NAPRA has been hence carried out by the incremental non linear procedure, assuming that the load-settlement response of the single pile can be represented by a hyperbola.

For the sake of comparison, a linearly elastic analysis has been also carried out.

The results obtained are compared to those of the experiments in fig. 5. It may be seen that the linear analysis (L) underpredicts the settlement and overpredicts the load on the piles. The non linear analysis (NL), on the contrary, shows a good agreement with the experimental results, with the only exception of the data referring to piles with length L = 135 mm. Similar conclusions had been reached by Poulos (1994).

5. SUMMARY AND CONCLUSIONS

The accuracy of computer programs available for the analysis of piled rafts of practical engineering significance (large number of piles, layered subsoil) is probably not better than ± 20%, even for "rigorous" complete BEM methods. The use of simplified numerical procedures appears promising, but further assessment of the significance of their results is needed.

The mechanical characterization of the subsoil can be conveniently obtained by the results of a full scale load test on a single pile. It is shown, however, that the group effect depends on the subsoil model adopted to extrapolate from the single pile to the group. The success of settlement prediction for pile groups on the basis of the settlement of a single pile (Mandolini & Viggiani, 1996) depends hence on a careful consideration of the actual subsoil profile.

The analysis of the results of a centrifuge model test is used to demonstrate the influence of non linearity when the piles are loaded close to their bearing capacity.

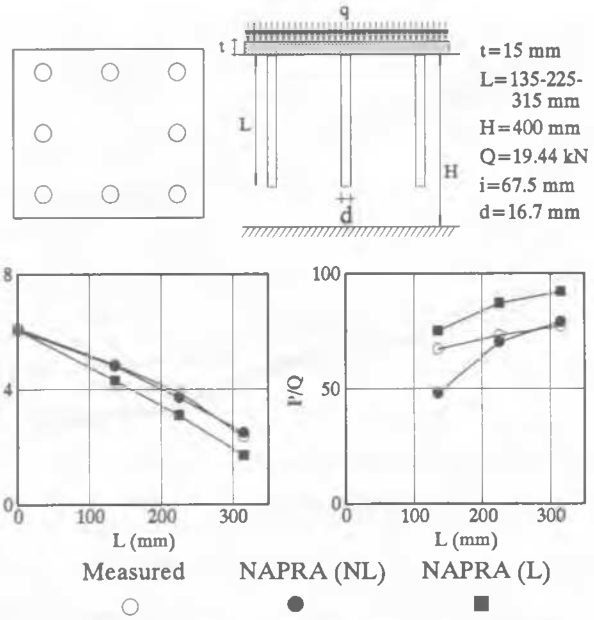


Fig. 5. Comparison between the centrifuge tests by Thaher & Jessberger (1991) and the results obtained by NAPRA with a linear (L) and non linear (NL) analysis

6. ACKNOWLEDGEMENTS

The HyPR code has been kindky supplied to the Authors by Prof. M.F. Randolph and Dr. P. Clancy of the U.W.A.

REFERENCES

Banerjee P.K., Butterfield R. (1981) *Boundary element method in engineering science.* McGraw Hill, New York

Burland J.B. (1995) Piles as settlement reducers. *Atti XXII Convegno Italiano di Geotecnica*, Pavia, vol. 2

Burland J.B., Broms B.B., De Mello V.F.B. (1977) Behaviour of foundations and structures. *Proc. IX ICSMFE,* vol. 2, p. 495-546. Tokyo

Butterfield R., Banerjee P.K. (1971) The problem of pile group-pile cap interaction. *Geotechnique*, vol. 21, n. 2, p. 135

Caputo V., Viggiani C. (1981) Pile foundations analysis: a simple approach to non linearity effects. *RIG*, vol. 18, n. 2, p. 32-51

Chow Y.K. (1986) Analysis of vertically loaded pile groups. *Int. Journal of Analytical and Numerical Methods in Geomechanics,* vol. 10, n. 1, p. 59-72

Clancy P. (1993) Numerical analysis of piled raft foundations. *PhD Thesis,* University of Western Australia

Clancy P., Randolph M.F. (1993) An approximate analysis procedure for piled raft foundations. *Int. Journal of Analytical and Numerical Methods in Geomechanics,* vol. 17, n. 12, p. 849-869

Kuwabara F. (1989) An elastic analysis for piled raft foundations in a homogeneous soil. *Soils and Foundations,* vol. 29, n. 2, p. 298

Mandolini A., Viggiani C. (1996) Settlement of piled foundations. Accepted for publication, *Géotechnique.*

Mandolini A., Viggiani C. (1992) Settlement prediction for piled foundations from loading tests on single piles. *Proc. Wroth Mem. Symp. on Predictive Soil Mech.*

Poulos H.G. (1989) Pile behaviour - theory and application. *Géotechnique,* vol. 39, p. 365-415

Poulos H.G. (1994) An approximate numerical analysis of pile-raft interaction. *Int. Journ. Numer. Analyt. Methods in Soil Mech.*, vol. 18, n. 2, p. 73-92

Poulos H.G., Davis E.H. (1980) *Pile foundations analysis and design.* J. Wiley and Sons, New York

Randolph M.F. (1994) Design methods for pile groups and piled rafts. *Proc. XIII ICSMFE,* New Delhi, vol. 5, p. 61-82

Randolph M.F., Wroth P.C. (1978) Analysis of deformation of vertically loaded piles. *Journ. Geot. Eng. Div., Proc. ASCE,* vol. 104, GT12, p. 1465-1488

Russo G. (1995) Interazione terreno struttura per piastre su pali. *PhD Thesis,* University of Napoli Federico II

Terzaghi K., Peck R.B. (1948) *Soil Mechanics in Engineering Practice.* J. Wiley & Sons, New York

Thaher M., Jessberger H.G. (1991) Investigation of the behaviour of pile-raft foundation by centrifuge modelling. *Proc. X Eur. CSMFE,* Florence. Vol. 1, p. 597-603