An example of pile walls analysis
Un exemple d'analyse d'un mur de pieux

L.Trauner, B.Žlender & S.Škrabl – *Faculty of Civil Engineering, University of Maribor, Slovenia*

ABSTRACT: Theoretical foundations for analyses of the stability of cantilever retaining pile walls, by methods of limit states, are presented in the paper. The impact of friction between massive support constructions and rear soils, as to the stability of cantilever retaining pile walls, is depicted. The method of limit equilibrium states, with simultaneous elasto-plastic analysis of particular constructions upon the Mohr-Coulomb elasto-plastic rheological model is used by way of analysis. A cantilever retaining pile wall is analysed by the theory of linear elasticity. The solution of interaction is given by the incremental iterational form. The method of finite elements is applied. The serviceability of the proposed model of numerical analysis is demonstrated upon the concrete example of the analysis of the stability of cantilever retaining pile walls.

RESUME: Les principes théoriques pour analyse de la stabilité des murs de pilots fixés par consoles selon les méthodes d'états limites sont exposés dans cet article. Il retrace l'influence de la friction entre les structures massives de soutènement et les sols de derrière sur la stabilité des murs de pilots fixés par consoles. Pour l'analyse a été appliqée la méthode des états limites d'équilibre avec l'analyse simultanée élasto-plastique de la construction en question selon le modèle rhéologique élasto-plastique de Mohr-Coulomb. Le mur de pilots fixé par consoles est analysé selon la théorie de l'élasticité linéaire. La solution de l'interaction est présentée en forme incrémentielle itérative. La méthode des éléments finis y est emloyée. L'application du modéle proposé de l'analyse numérique est illustrée sur le modéle concret de l'analyse de stabilité du mur de pilots fixés par consoles.

1 INTRODUCTION

Numerous factors have an impact upon the stability of geotechnical constructions, e.g. the rheological characteristics of soils, possibilities of friction between the construction and the soils, conditions for the drainage of the soils, the speed of load increases etc. In stability analyses methods of limit equilibrium states are often used.

While planning, we must take into consideration at least two types of limit states and in particular: the limit state of failure and the limit state of the construction serviceability.

The limit state of failure of the geotechnical objects is expressed by a failure or by a complete loss of the global stability of the said geotechnical object.

The limit state of serviceability is expressed with respect to the still acceptable deformations (movements), cracks, plastifications of the grounds and other impairments, which have an impact upon the serviceability of the object.

Pile walls are considered to be slender support constructions, as the contribution towards stability is, due to resistance of the grounds on the basic plane, substantially smaller than the contribution towards the lateral surfaces of the construction. The stability of pile walls is ensured by their fixity in the grounds and anchoring into the rear of the grounds.

Further, an example of the analysis of cantilever retaining pile walls is presented, considering in particular the friction appearing at the contact of the walls and the soils.

2 THE THEORETICAL BASIS

Two essential different limit states are distinguished in geotechnical analyses of pile walls: stability limit state and serviceability limit state.

2.1 *The stability limit state*

The stability of pile walls is ensured by its fixity in the soils. In geotechnical practice, a numerical model with its constituent active and passive soil pressures is commonly used. The shear fixity of soils is given according to the Mohr-Coulomb theory of failure:

$$\tau_m = c_m + \sigma tg\varphi_m \qquad 1$$

with mobilised shear strength, normal strain, cohesion and shear angle. To ensure the stability of the construction, the following equilibrium conditions must be met:

$$\sum H = 0$$

$$z^2(k_3 - k_5) + 2z(p_2 - a_3) + 2a_1 D + k_1 D^2$$
$$+2a_2 x + k_2 x^2 - 2p_1 x - k_4 x^2 = 0 \qquad 2$$

$$\sum M = 0$$

$$z^2(p_2 - a_3 - 3k_5 d \tan\vartheta) + z(4a_1 D + 2k_1 D^2$$
$$+4(a_2 - p_1)x + 2(k_2 - k_4)x^2 - 6a_3 d \tan\vartheta) +$$
$$a_1 D(3D + 6x) + k_1 D^2(D + 3x) + 3(a_2 - p_1)x^2 \qquad 3$$
$$+(k_2 - k_4)x^3 - \frac{p_1 x + k_4 x^2}{3} 6d \tan\vartheta = 0$$

Solving those systems of quadratic and cubic equations, depths (x, z) are evaluated. Limit values of the passive and active earth pressures are determined by the known procedures of potential failure of surfaces, using a chosen safety factor.

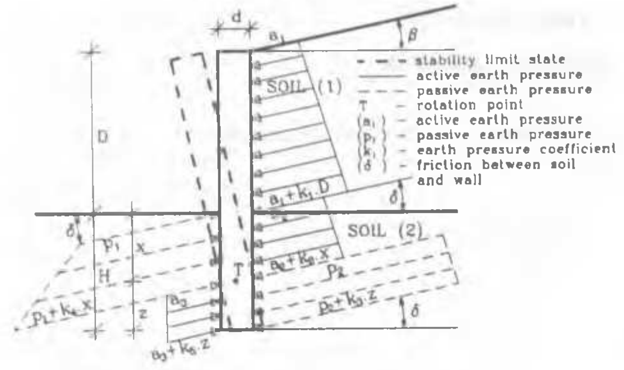

Figure 1. Limit state of pile wall.

2.2 The serviceability limit state

The serviceability limit state is expressed in geotechnical engineering by a number of criteria. The criterion of admissible deformations, i.e. those deformations, which still ensure the serviceability of the construction, is most commonly used. The criteria of the serviceability limit state, with respect to admissible cracks, plastification, criteria of freezing, durability of constructions, etc. are also common.

As to numerical analyses of expected deformations of the pile wall, we considered the soils as elasto-plastic materials, by the Mohr-Coulomb criterion of failure, whereas the support construction was considered to be linearly elastic.

The solution of the problem is given by the application of FEM, in the incremental-iteration form, with a system of linear equations. The analysis of strain and deformation with this problem is founded on the simultaneous solution of equilibrium equations (4), by taking into consideration compatibility (5) and constitutive relations (6):

$$\sigma_{ij},_k = \rho b_j \tag{4}$$

$$\varepsilon_{ij} = \frac{u_j,_i + u_i,_j}{2} \tag{5}$$

$$\dot{\sigma}_{ij} = C_{ijkl}\, \dot{\varepsilon}_{kl} \tag{6}$$

where σ_{ij} = stress tensor, ρ = density, ε_{ij} = strain tensor, b_j = body force vector, u_k = displacement vector.

We introduce the function of soils flow $F(s_{ij})$. It is in the range of elasticity.

$$F(\sigma_{ij}) \langle\, 0 \tag{7}$$

The constitutive tensor C_{ijkl} may be considered as constant in the analysis.

$$C_{ijkl} = E\, D_{ijkl} \tag{8}$$

where:

$$D_{ijkl} = \frac{\nu\delta_{ij}\delta_{kl}}{1-\nu-2\nu^2} + \frac{\delta_{ik}\delta_{jl}}{1+\nu} \tag{9}$$

where υ = Poisson's ratio, δ_{ij} = Cronecker Delta symbol, E = Young's modul.

On the yield surface tensor:

$$d\sigma_{ij}\, F,_{\sigma_{ij}} \langle\, 0 \tag{10}$$

D_{ijkl} is given by equation (9). In the case of loading:

$$F(\sigma_{ij}) = 0 \text{ and } d\sigma_{ij}\, F,_{\sigma_{ij}} \rangle 0 \tag{11}$$

The stress-strain tensor is evaluated:

$$C_{ijkl} = E\left\{ D_{ijkl} - \frac{D_{ijmn}\, G,_{\sigma_{mn}}\, F,_{\sigma_{pq}}\, D_{pqkl}}{A + F,_{\sigma_{pq}}\, D_{pqrs}\, G,_{\sigma_{rs}}} \right\} \tag{12}$$

where A = scalar term of hardening and softening, $G(\sigma_{ij})$ = plastic potential function. The yield function of a material with a Mohr Coulomb yield envelope is:

$$F = \frac{p\sin\varphi}{3} + \frac{q(\cos\varphi\sqrt{3} - \sin\theta\sin\varphi)}{\sqrt{2}} - c\cos\varphi \tag{13}$$

with angle of internal friction and cohesion of soils.

The sphere and distorsion component of the stress tensor and Lode's angle in the space of the major stress can be determined according to the following equations.

$$p = \frac{\delta_{ij}\sigma_{ij}}{3} \tag{14}$$

$$q = \sqrt{\frac{S_{ij}S_{ij}}{3}} \tag{15}$$

$$S_{ij} = \sigma_{ij} - \delta_{ij}p \tag{16}$$

$$\sin 3\theta = \frac{2\sigma_{ij}\sigma_{jk}\sigma_{kl}}{3q^3} \tag{17}$$

As for soils with associative flow models, the function of the plastic potential $G(\sigma_{ij})$ is equal to the function of the plastic flow $F(\sigma_{ij})$.

As for soils with a non-associative flow model, the function of the plastic potential may be expressed by the function of plastic flow, so that in the expression (13), the angle φ is replaced by the angle ψ $(0 \leq \psi \leq \varphi)$.

The relationshops between the changes of the loads and displacements, in the cases of space discretisation, may be evaluated by optional isoparametric finite elements:

$$K\ \Delta V = \Delta R \tag{18}$$

where K = stiffness matrix, ΔV = vector of displacement changes, ΔR = vector of load variations, Δt = loading icrement.

The accomplished system of linear equations is solved iterationally as for all chosen load increments by the SPACESOIL computer programme. The frontal method has been used. Since the optional loading increment, the flow of soils appears only in the limited area of the finite element, i.e. only at individual Gauss points, the programme envisages a respective correction of results, by taking into account residual forces.

Rheological parameters of soils were determined by laboratory examinations. On the basis of results of mono-axial and rotational shear examinations, we determined that deformational characteristics of the studied soils are linear in the range of strain states $q \leq 2q_f$ / , whereas at greater distorsion weights plastic flow appears. By the condition of plastic flow of soils, plastic deformation appearances are determined by strain levels. It may be expressed in the following generalised form:

$$F(\sigma_{ij}) = k(\kappa) \tag{19}$$

where k is a material parameter, which is determined experimentally.

On the base of results of laboratory tests we used the parameter in next form:

$$k(\kappa) = k(\varepsilon_p) = E(\varepsilon_l \ln \varepsilon_p - \varepsilon_p) \tag{20}$$

With known Young's modul, plastic deformations over the strain path and the limit plastic deformations. A scalar term of hardening is:

$$A = -\frac{k(\varepsilon_p)_{,\varepsilon_p} \, d\varepsilon_p}{d\lambda} \tag{21}$$

where $d\lambda$ is proportionality constant defined with relation:

$$d\varepsilon_{ij(p)} = d\lambda G_{,\sigma_{ij}} \tag{22}$$

3 A PRACTICAL EXAMPLE

The serviceability of the above presented theoretical foundation was tested on a concrete example of the cantilever retaining pile wall.

The soils were considered as a multi-strata semi-space with characteristics demonstrated in Figure 2.

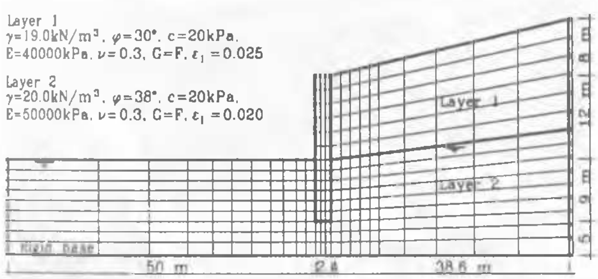

Figure 2. Stratigraphical and geometrical data.

Standard analyses of the stability of the console pile wall were made according to the limit state theory, whereas analyses of the stress-strain state was carried out by the method of the horizontal coeffitient of subgrade reactions.

The results of standard geostatical analyses were compared to results of analyses according to the above stated theoretical foundation, as to various examples of considering friction between the pile wall and the soils.

Later, the results were verified by measurements on the constructed pile wall. Measurements were done at all stages of the construction pit procedure, during the construction of facilities and after the completion of construction.

3.1 *Results of limit state stability analyses*

The limit state of failure was analysed according to the method of extremes for plane and deformed surfaces, where failure is possible. Possible combinations of various values of mobilised friction between the pile wall and the soils as well as the safety coefficients were analysed. The results of analyses are demonstrated in Figure 3.

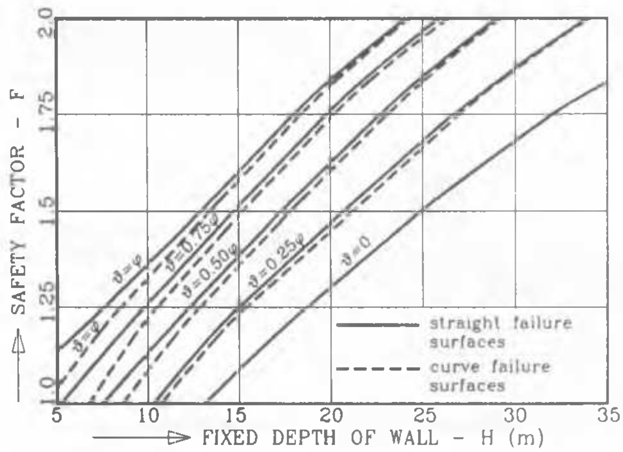

Figure 3. Depth of pile wall as a function of chosen safety factor and friction angle.

3.2 *Results of limit state deformation analyses*

The lateral side with the support construction was discretised for the purpose of numerical analysis by 325 isoparametric finite elements, with 16 degrees of freedom. At contact surfaces between the construction and the soils non-dimensional finite elements were used, as they simulate the state of discontinuities between the construction and the soils.

Three cases were analysed:

(i) Actual shear characteristics of soils without friction at the contact with the construction,

(ii) Actual shear characteristics of soils and the friction at the contact with the construction,

(iii) Mobilised shear characteristics of soils and the friction at the contact with the construction.

Movements of pile walls and the expansion of the area of plasticity for the above mentioned examples are presented on Figures 4 and 5.

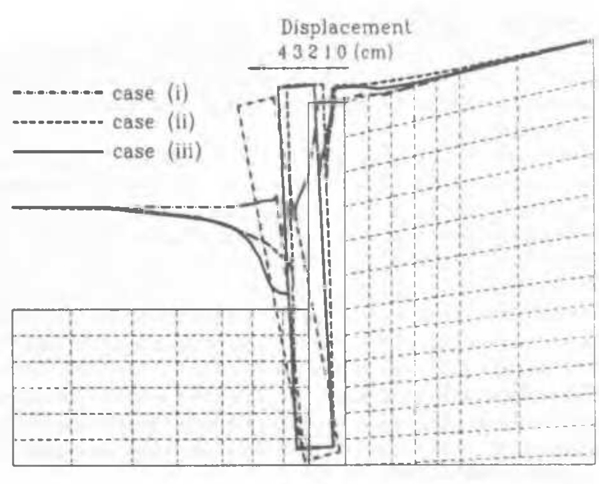

Figure 4. Displacement of the retaining construction.

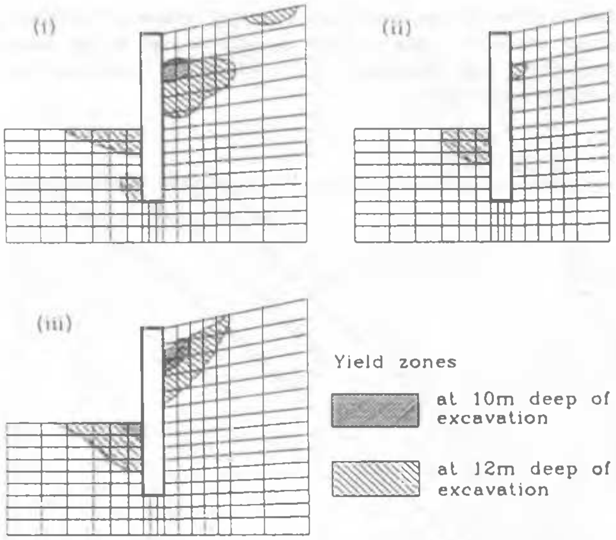

Figure 5. Yield zones of the soils at the excavation.

3.3 *Construction and measurements*

The pile wall was constructed within the framework of the construction of the "Termal hotel Habakuk" facilities in Maribor. During the construction, it served as protection of the construction pit, whereas it later received a permanent function as a support construction of sub-terrainian facilities.

It was built according to the diaphragm construction technology, so that the piles were carried out by rectangular cross-section, by way of interrupted excavations, protected by bentonite swill. The length of the pile wall is 150 m, the thickness is 2.40 m, the width of the piles is 0.80 m, interaxial distance among the piles is 2.0 m. The depth of the insertion varies between 7.0 and 11.0 m, the free height (excavation of the construction pit) is up to 14.0 m.

Simultaneously, movements were measured, which confirmed the realistic nature of the analyses results. Thus, maximum movements of the construction were measured to be 30 mm, while they computationally varied between 30 and 50 mm considering the friction at the outer layer of the pile and the degree of shear activation.

4 CONCLUSIONS

The results of the geostatical analyses and the verification by measurements upon the constructed pile wall confirm the hypothesis that the friction between the pile wall and the soils has a significant impact upon stability. This is particularly valid as to the stage of the opening of the construction pit, when shears appear at the contact surfaces between the soils and the construction, due to dis-burdening of the strain state under the construction pit.

As to analysis of the pile wall as a permanent construction, it is also necessary to consider the more complex marginal conditions, the mode of carrying out of construction work, the actual strain and deformational state in the grounds during all stages of construction, possible dynamic loads etc.

On the basis of the results of the analyses carried out and after the verification by measurements at the concrete facility, we assess that the impact of friction is certainly favourable as to the stability of the console pile walls. This assessment is valid in the case that the soils have sufficiently high shear characteristics. It is, therefore, significant that at planning pile walls, the actual possibilities of the activation of friction and the duration of its impact are carefully analysed for each individual case.

Figure 6. Some phases of the pile wall construction

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